Bipartite time-energy uncertainty relation for quantum metrology with noise

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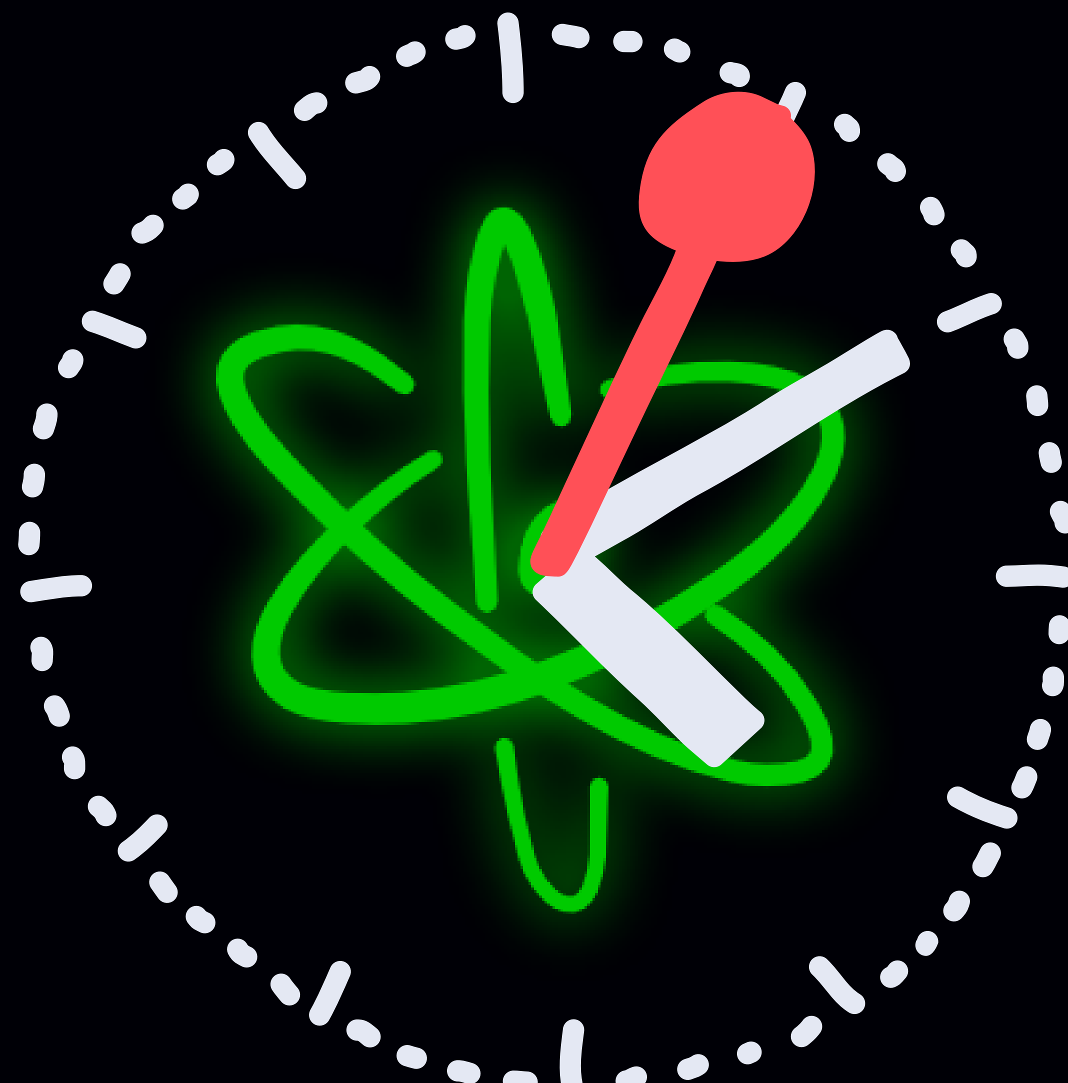
unitary time evolution
of $|\psi\rangle$ according to H

measurement reveals
information about
elapsed time



SI definition
of the second
Woods+ AHP 2019
Bartlett+ RMP 2006
...

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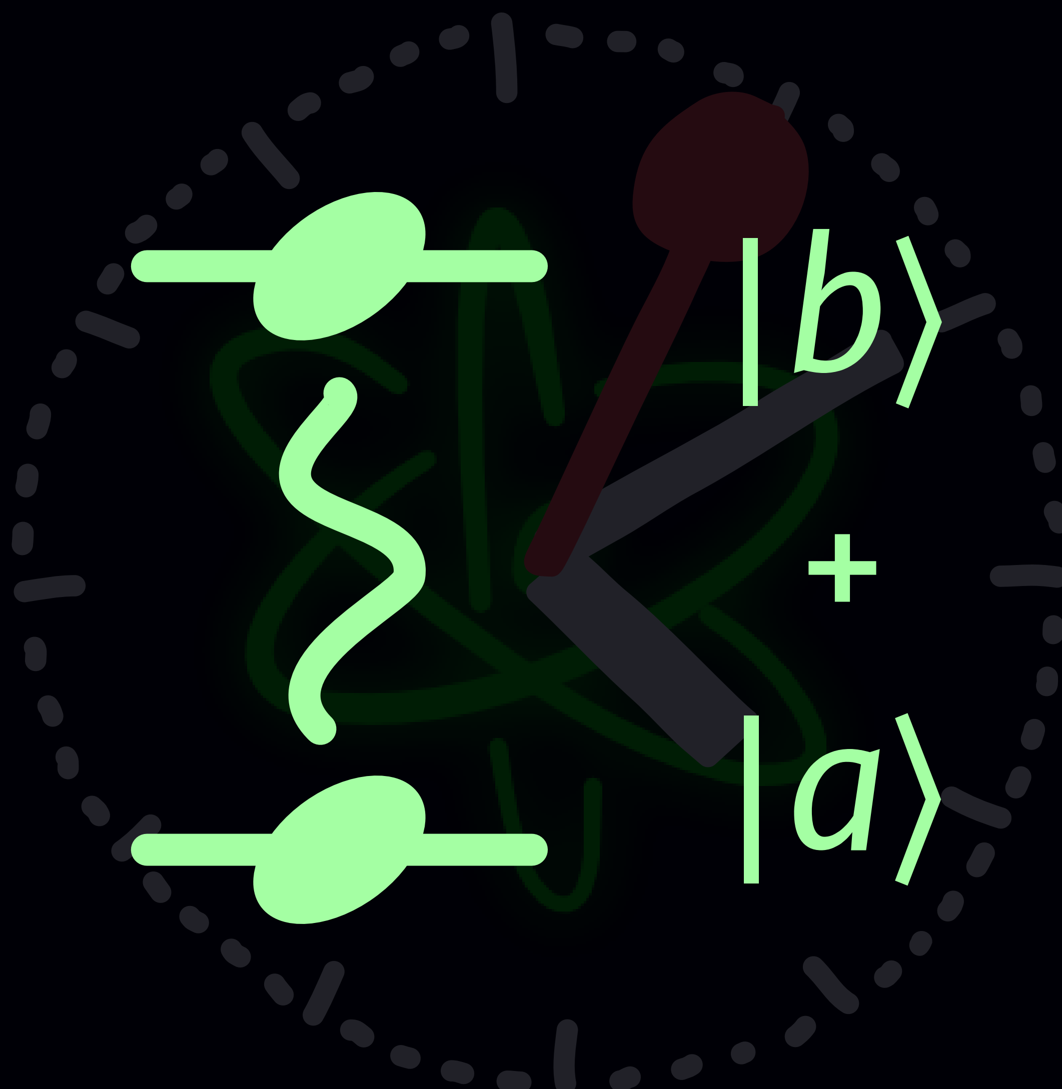

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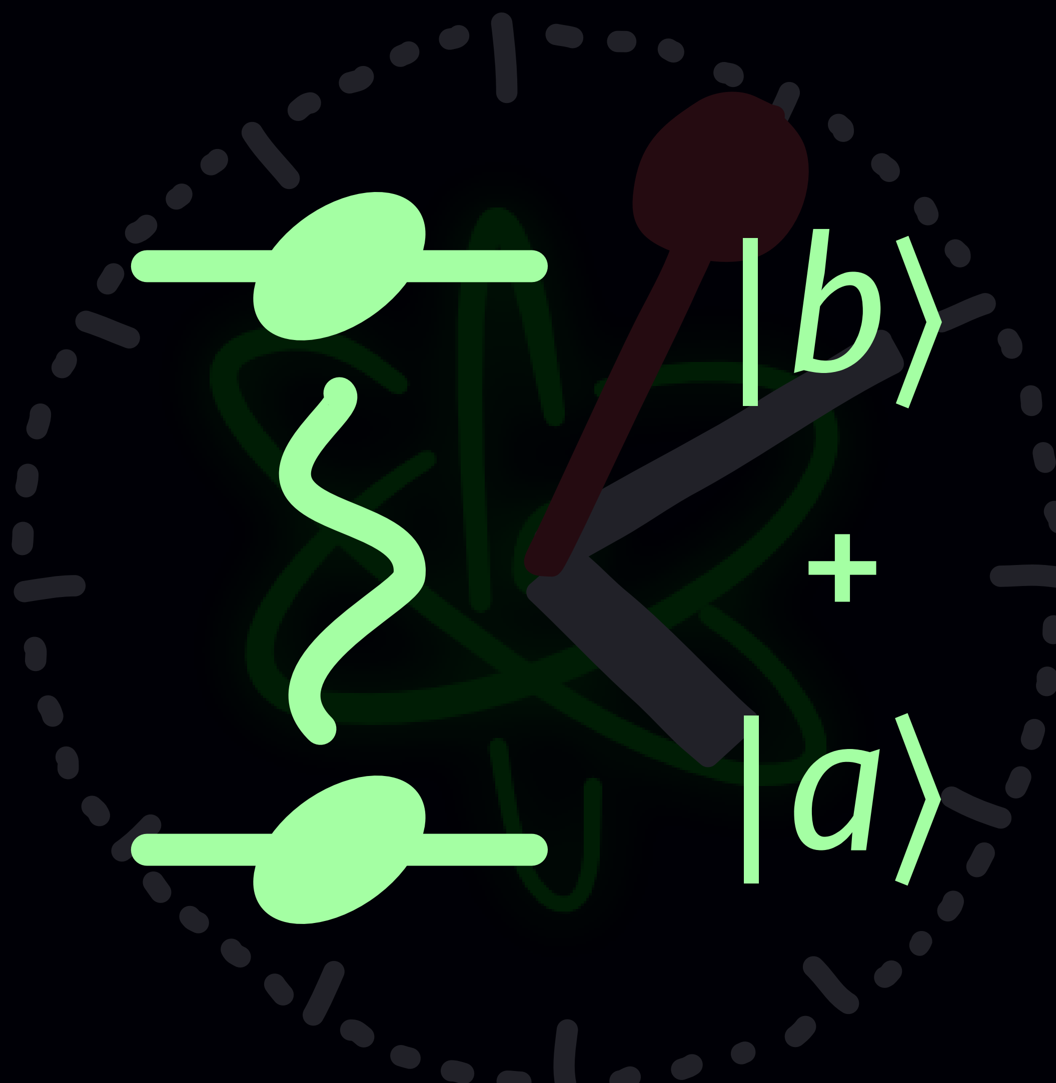

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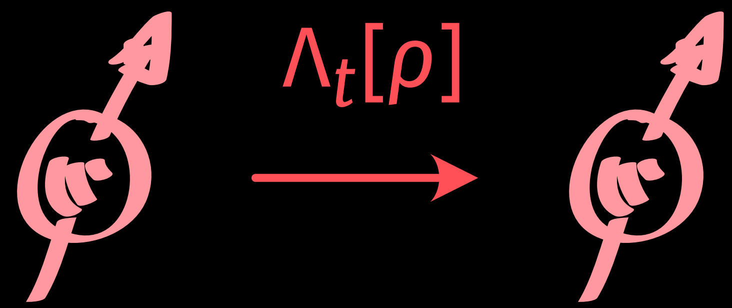
How does noise affect the sensitivity of the clock?

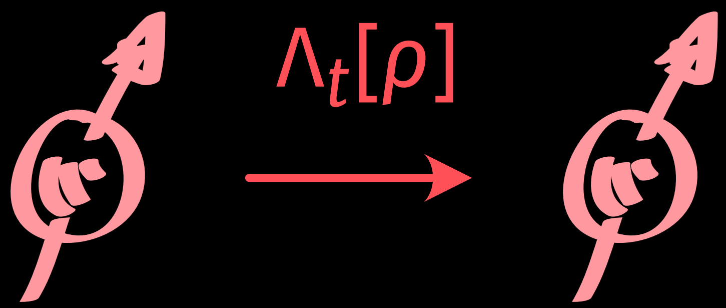


Achieve same sensitivity scaling as without noise (Heisenberg scaling)

$\Leftrightarrow$

The signal Hamiltonian is not parallel to the noise (Hamiltonian not in Lindblad span)

Demkowicz-Dobrzański+ N. Comm. 2011; Escher+ N. Phys 2011;
Kessler+ / Arrad+ / Dür+ PRL 2014;
Zhou+ N. Comm. 2018; Layden+ PRL 2019; …



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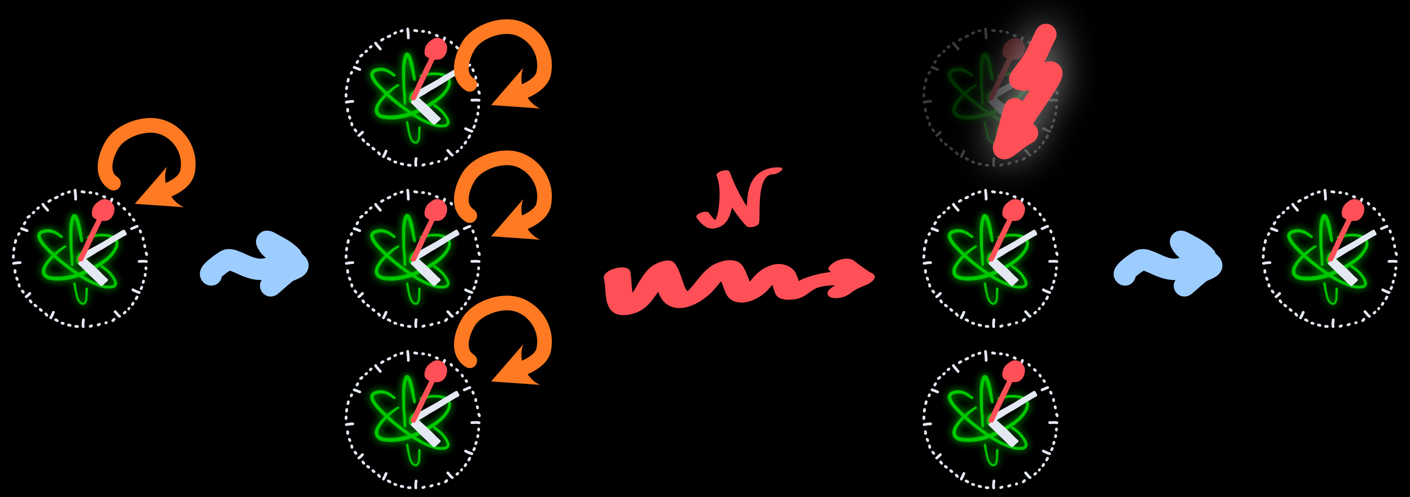
use
time-covariant
error-correcting
code

Hayden *et al.*, 1709.04471



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use approximate
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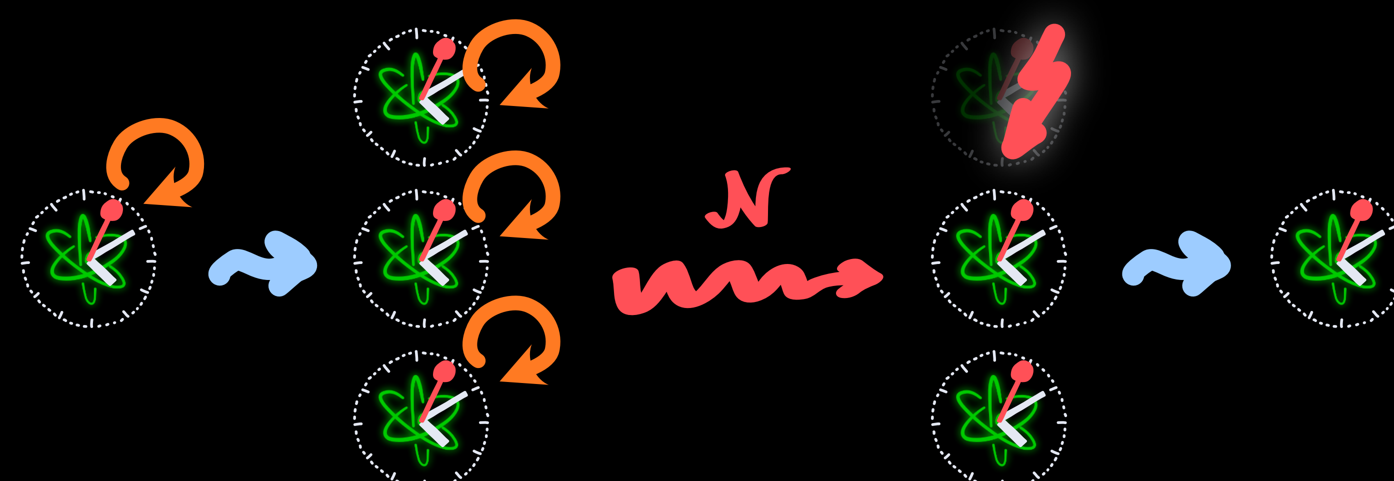
code



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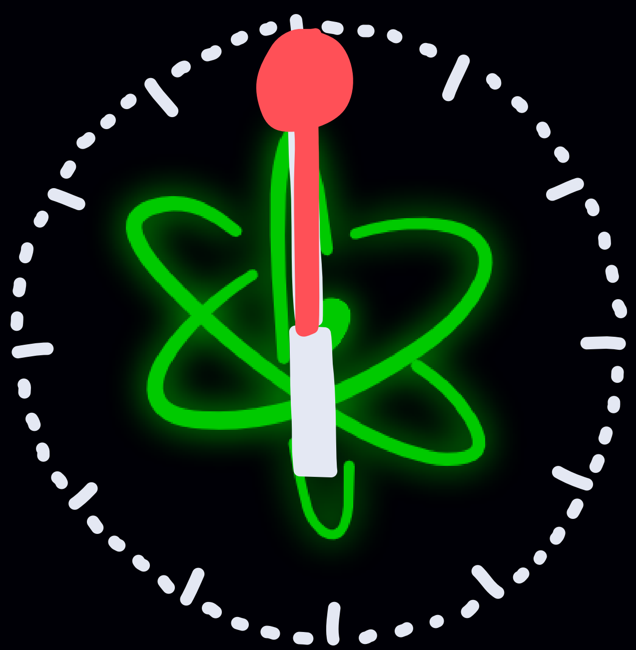
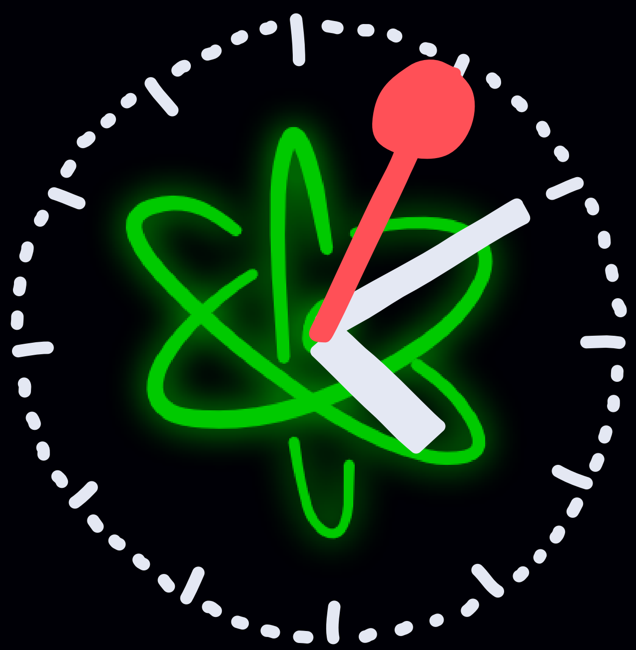
PhF+ PRX 2020; Woods & Alhambra *Quantum* 2020;
Kubica+ 2004.11893; Zhou+ 2005.11918; Yang+ 2007.09154



$t = 0$



$$t = t_0$$

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How much sensitivity is left?

? Uncertainty relation for sensitivity:

noisy clock w.r.t. time <> environment w.r.t. energy

Expression for sensitivity loss

? Uncertainty relation for sensitivity:

noisy clock w.r.t. time < > environment w.r.t. energy

Expression for sensitivity loss

(methods from semidefinite programming)

- alternative sensitivity bounds
- uncertainty relation for any two parameters

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01 Necessary and sufficient conditions for zero sensitivity loss

- “metrological codes”

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Expression for sensitivity loss

(methods from semidefinite programming)

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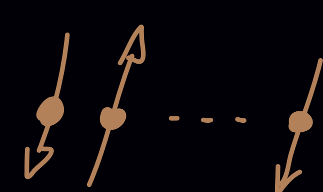
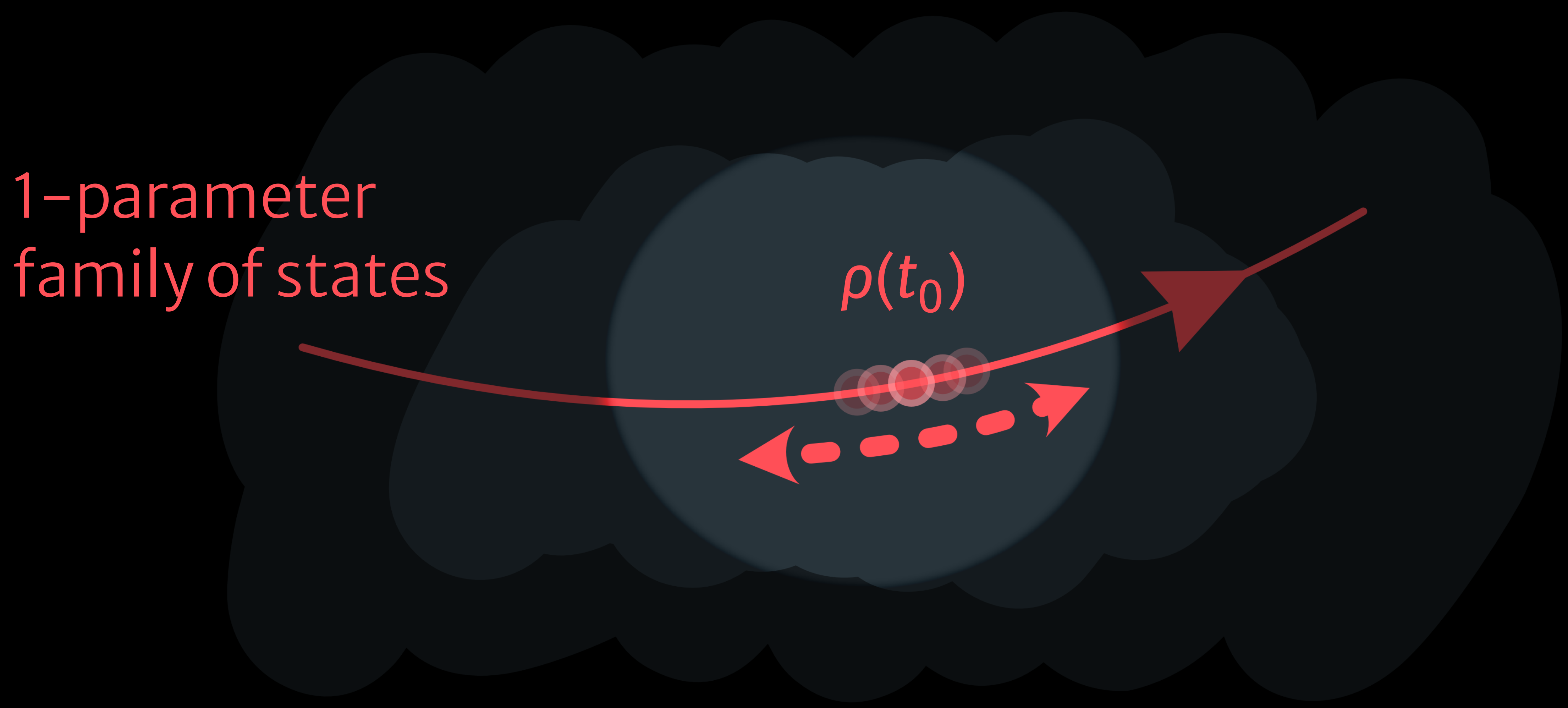
Ø1 Necessary and sufficient conditions for zero sensitivity loss

- “metrological codes”



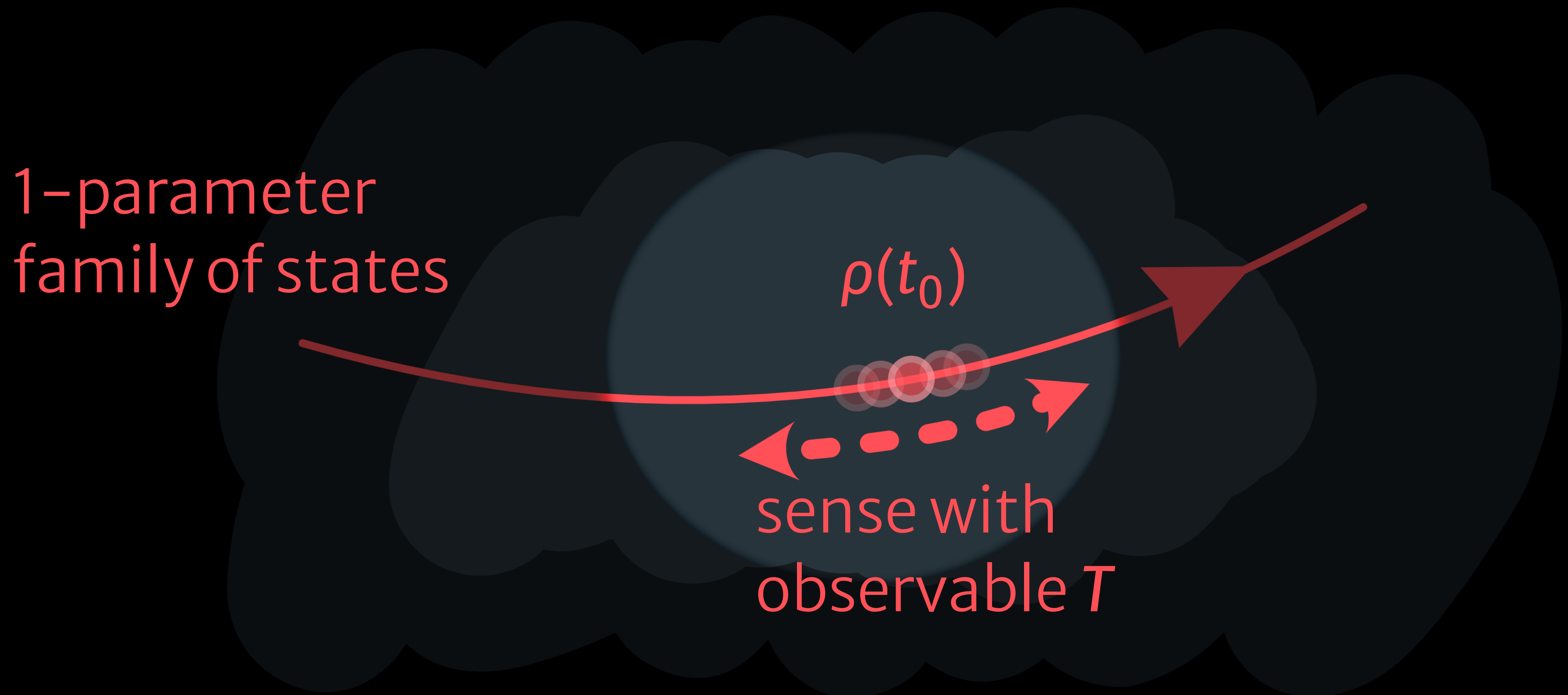
Example: Ising Hamiltonian & amplitude damping noise



Braunstein/Caves PRL 1994; Holevo 2011; …

want: observable T with minimal variance at $\rho(t_0)$, such that:

$$\langle T\rangle_{\rho(t_0+dt)} = t_0 + dt + O(dt^2)$$



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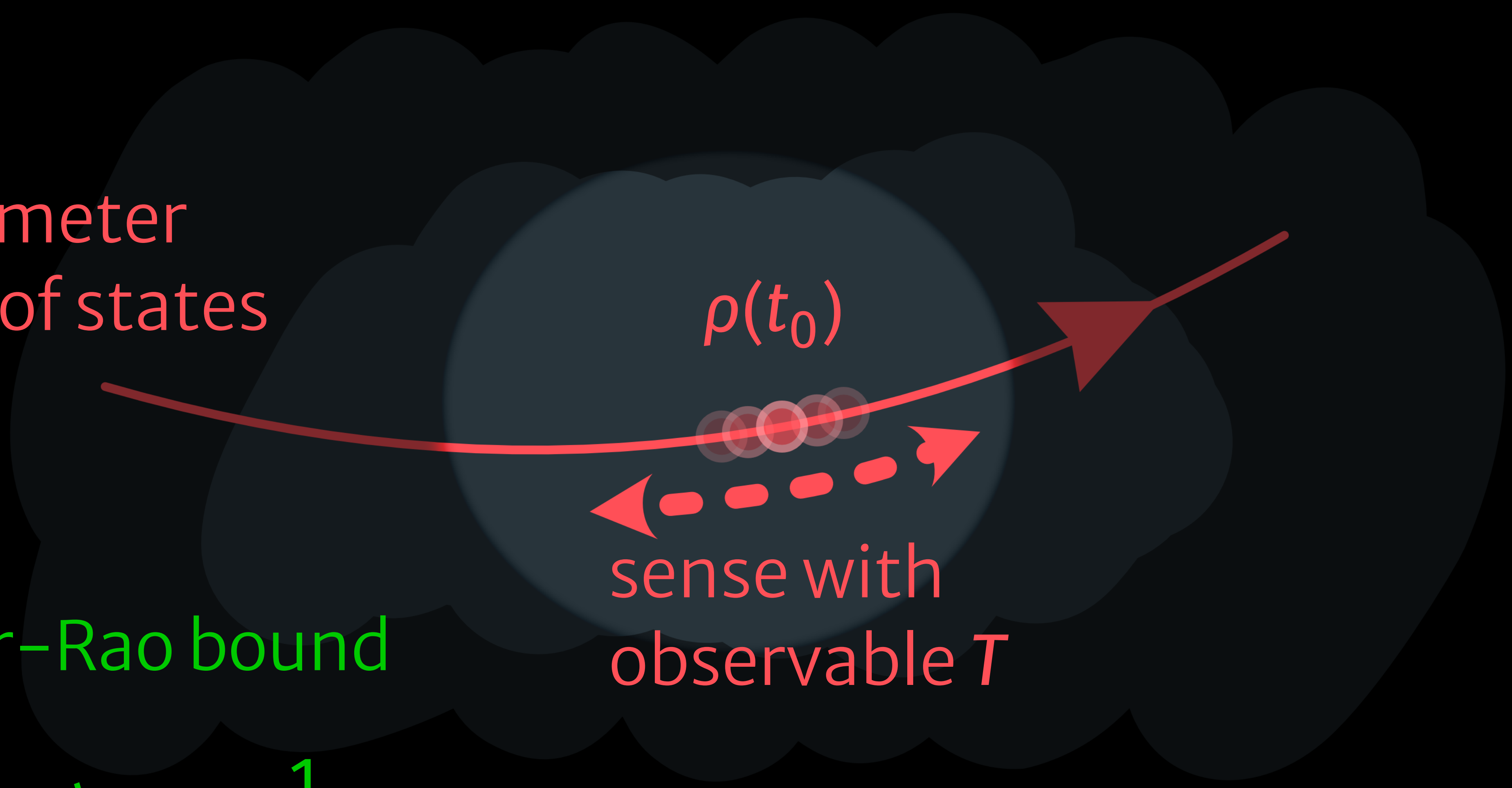
1-parameter family of states



Cramér–Rao bound

$$\left\langle (T - t_0)^2 \right\rangle \geq \frac{1}{F(\rho; \partial_t \rho)}$$

Fisher information

(here: defined via optimal T)

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SDP Alert!

cf. H. Fawzi's tutorial

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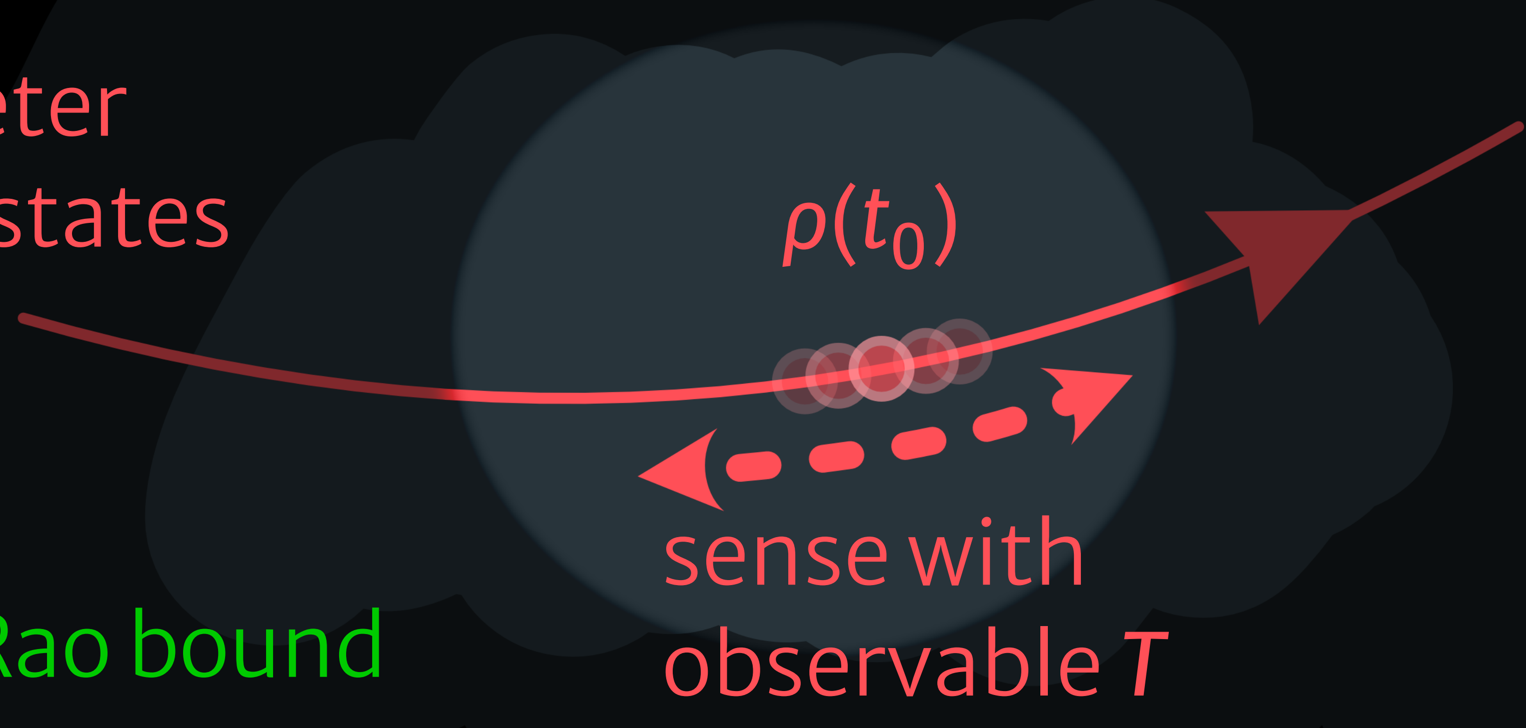

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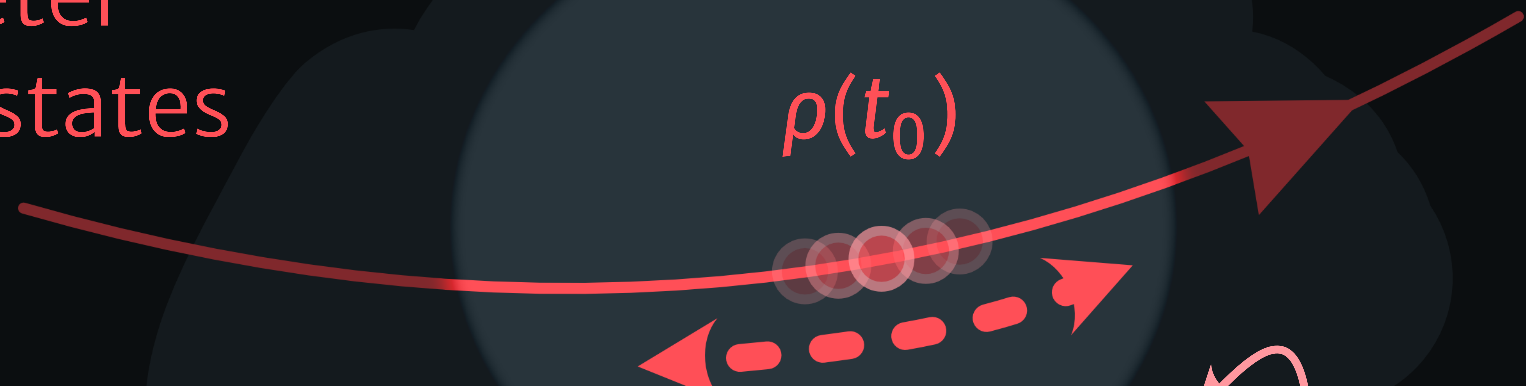

sense with observable T

T is optimal when $\{\rho, T\} \propto \partial_t \rho$

(*"symmetric log. derivative"*)

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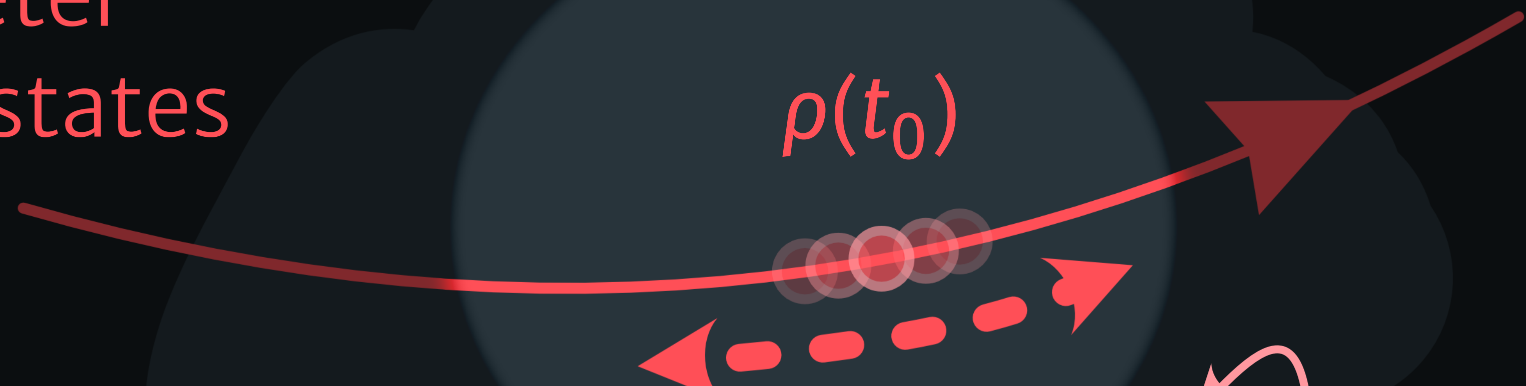

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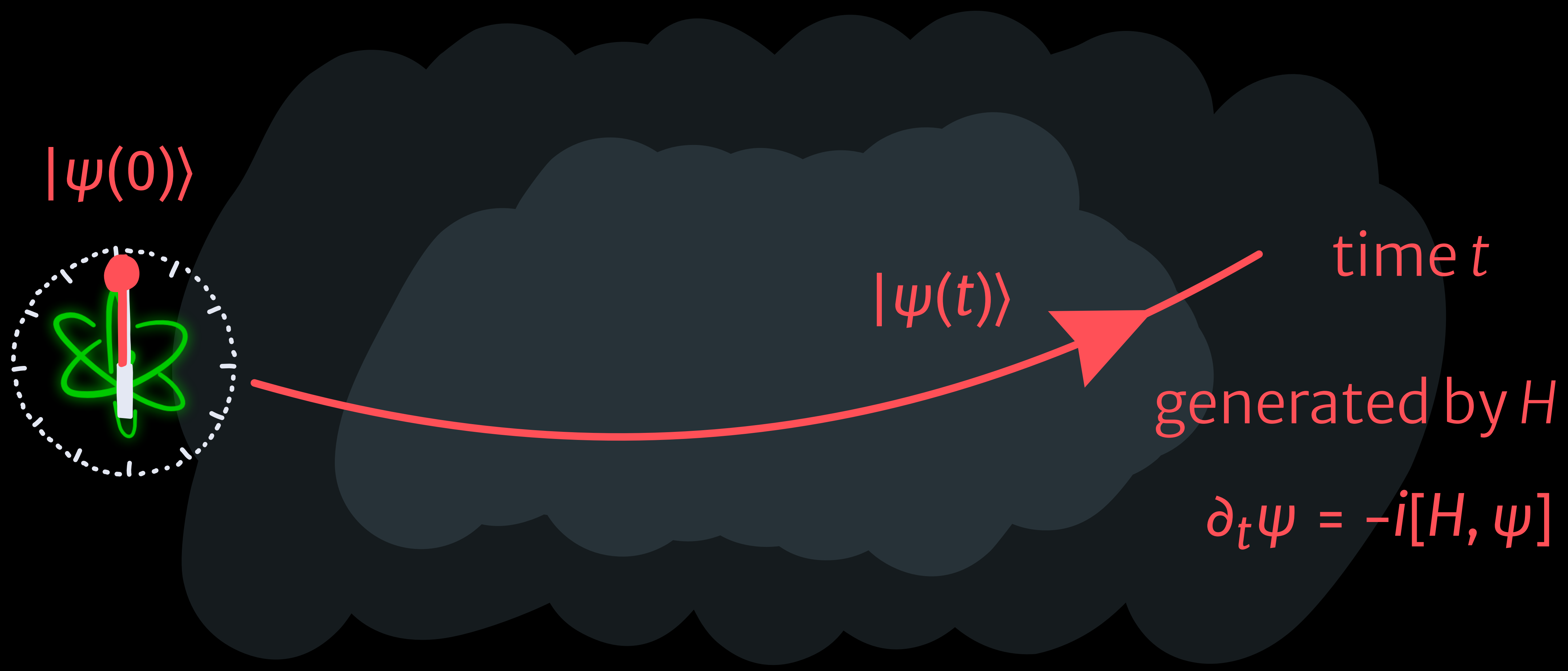
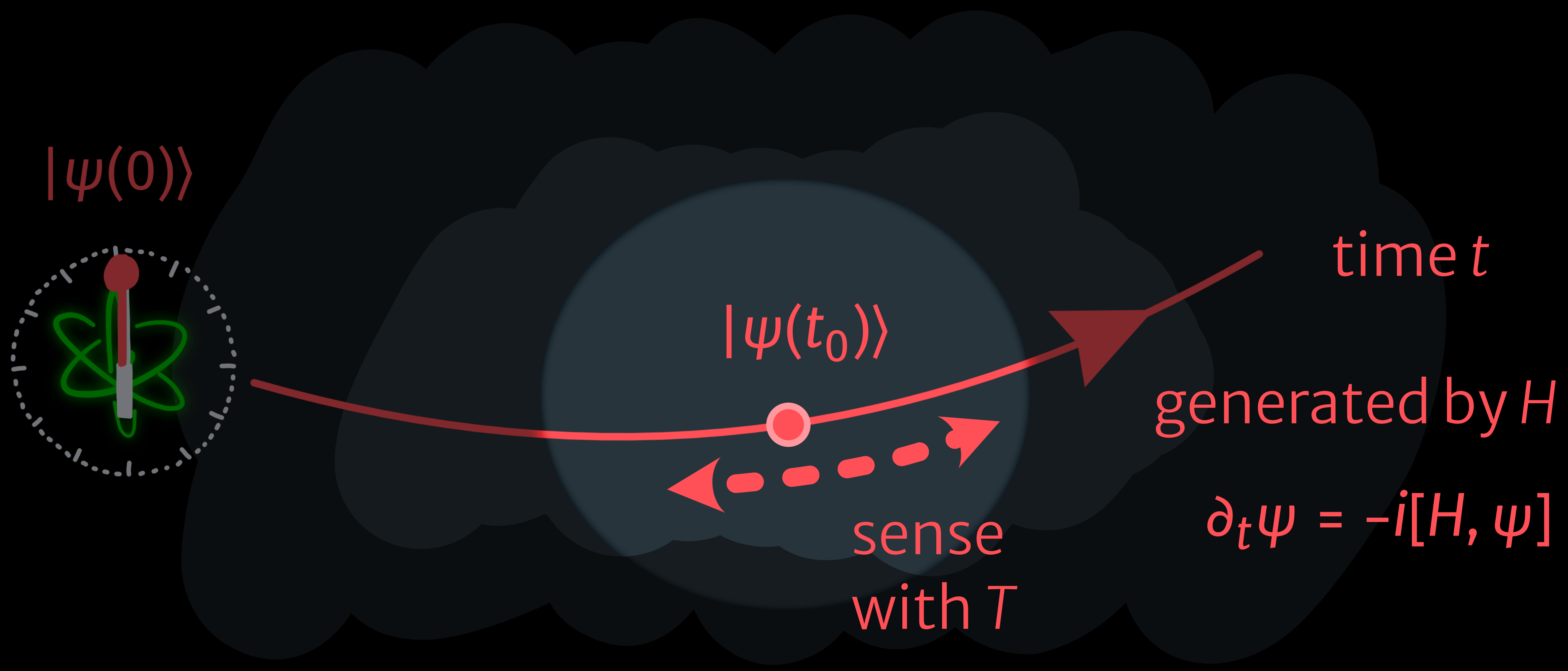
$F(\rho; \partial_t \rho)$ & T can be tricky to determine for mixed ρ

Braunstein/Caves PRL 1994; Holevo 2011; …



$|\psi(0)\rangle$
$|\psi(t)\rangle$
time t
generated by H
$\partial_t \psi = -i[H, \psi]$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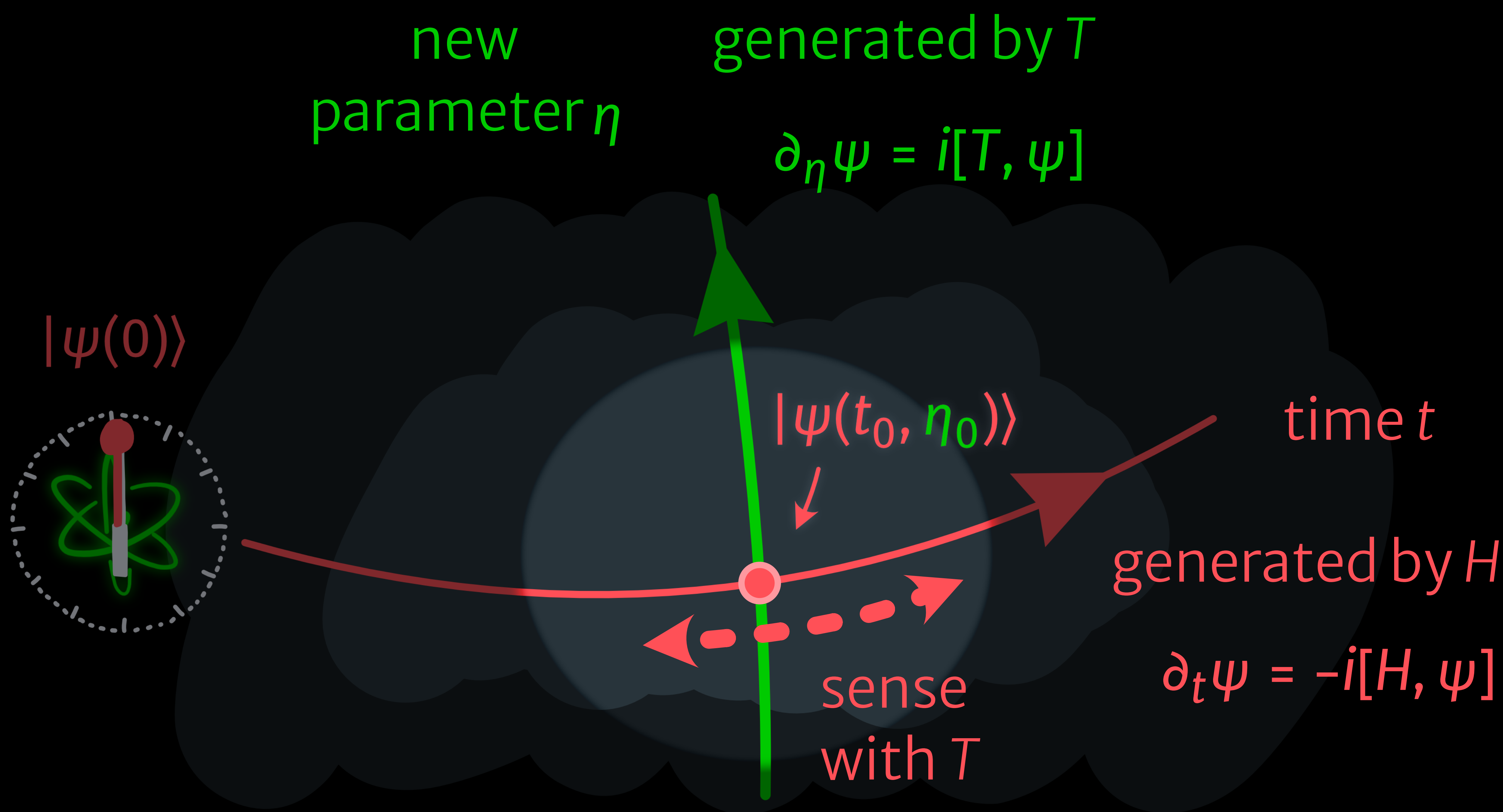
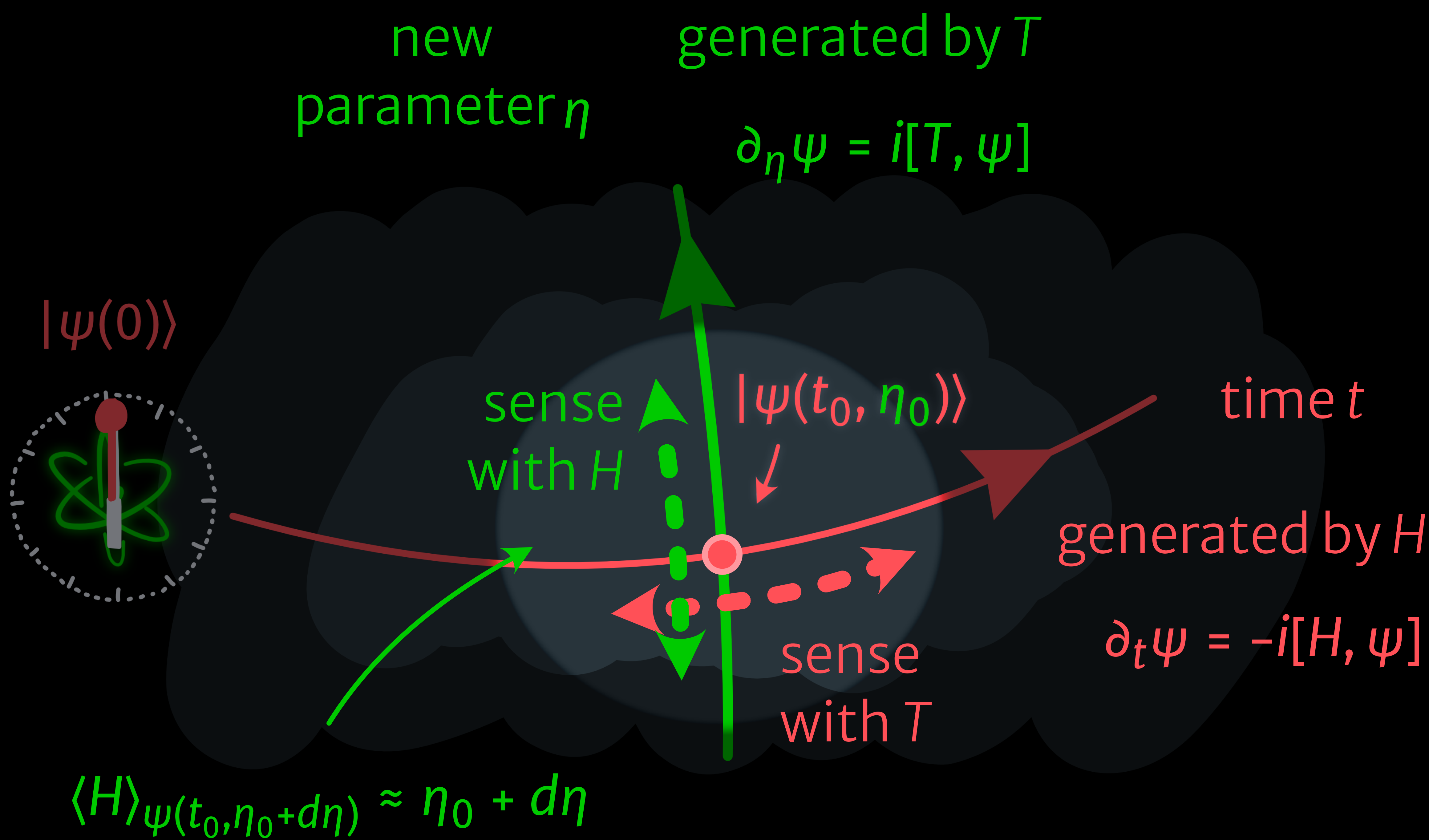


new
parameter η
generated by T
$\partial_\eta \psi = i[T, \psi]$
$|\psi(0)\rangle$
$|\psi(t_0, \eta_0)\rangle$
time t
generated by H
$\partial_t \psi = -i[H, \psi]$
sense
with T



new parameter η
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time t
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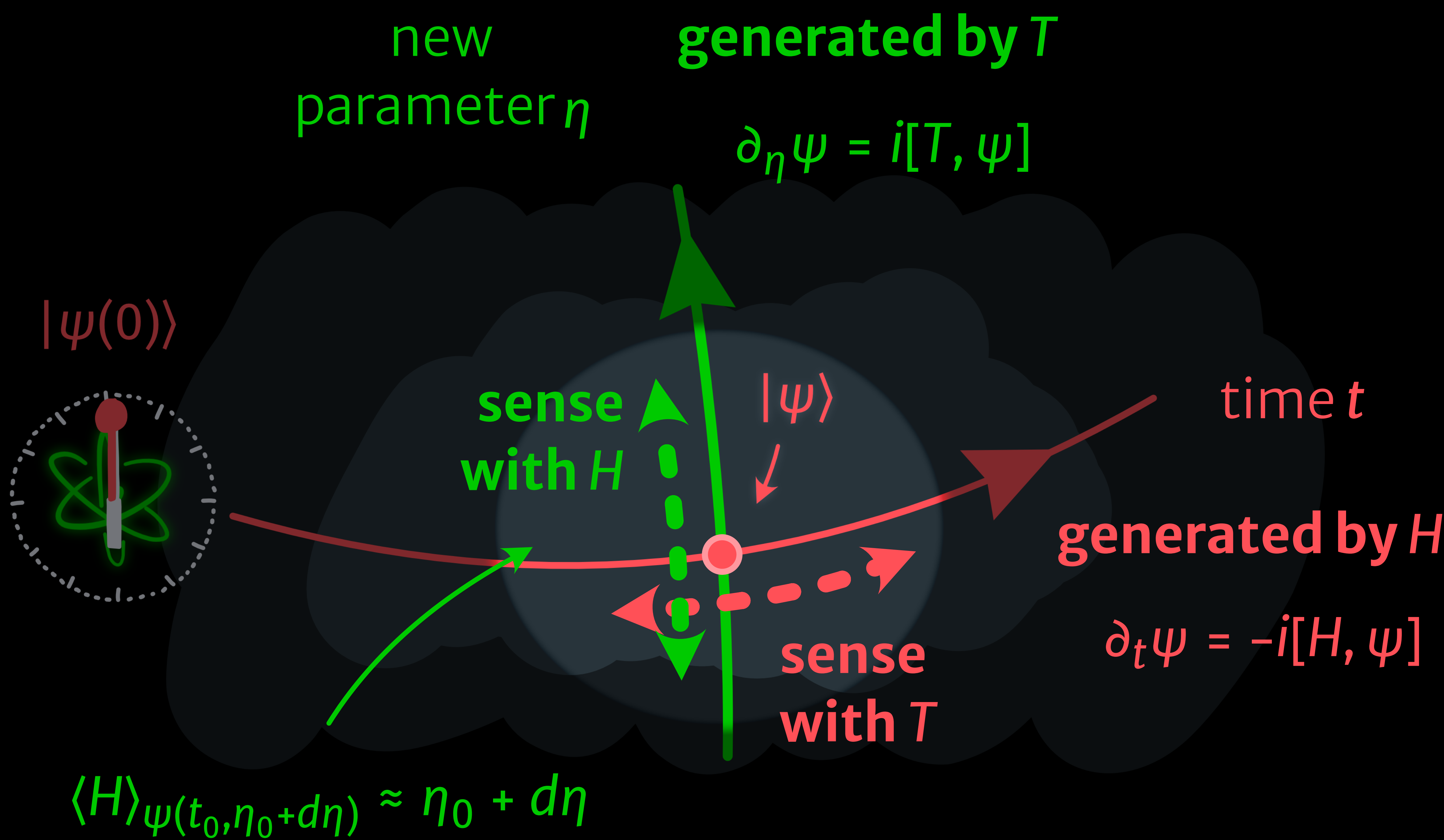
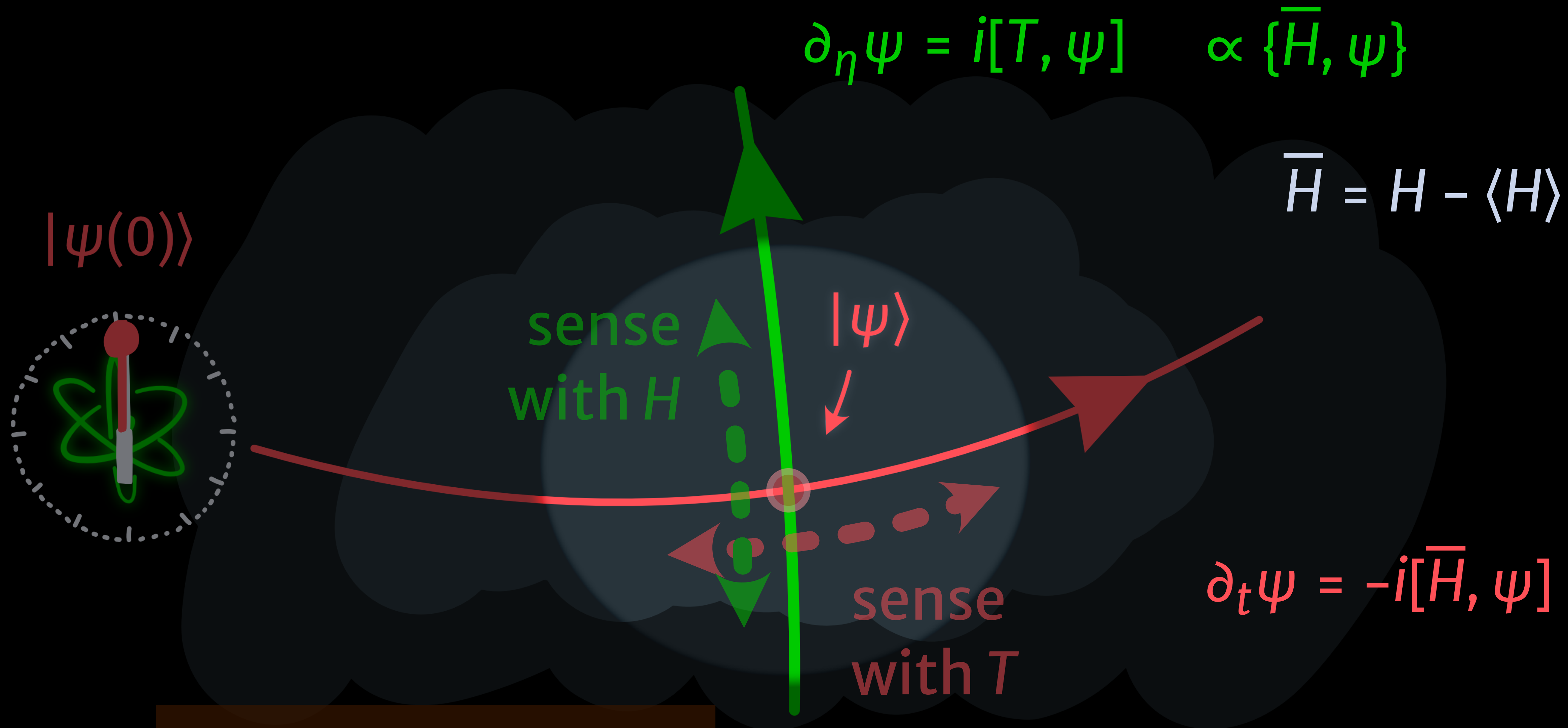
sense with T
$\langle H \rangle_{\psi(t_0, \eta_0 + d\eta)} \approx \eta_0 + d\eta$



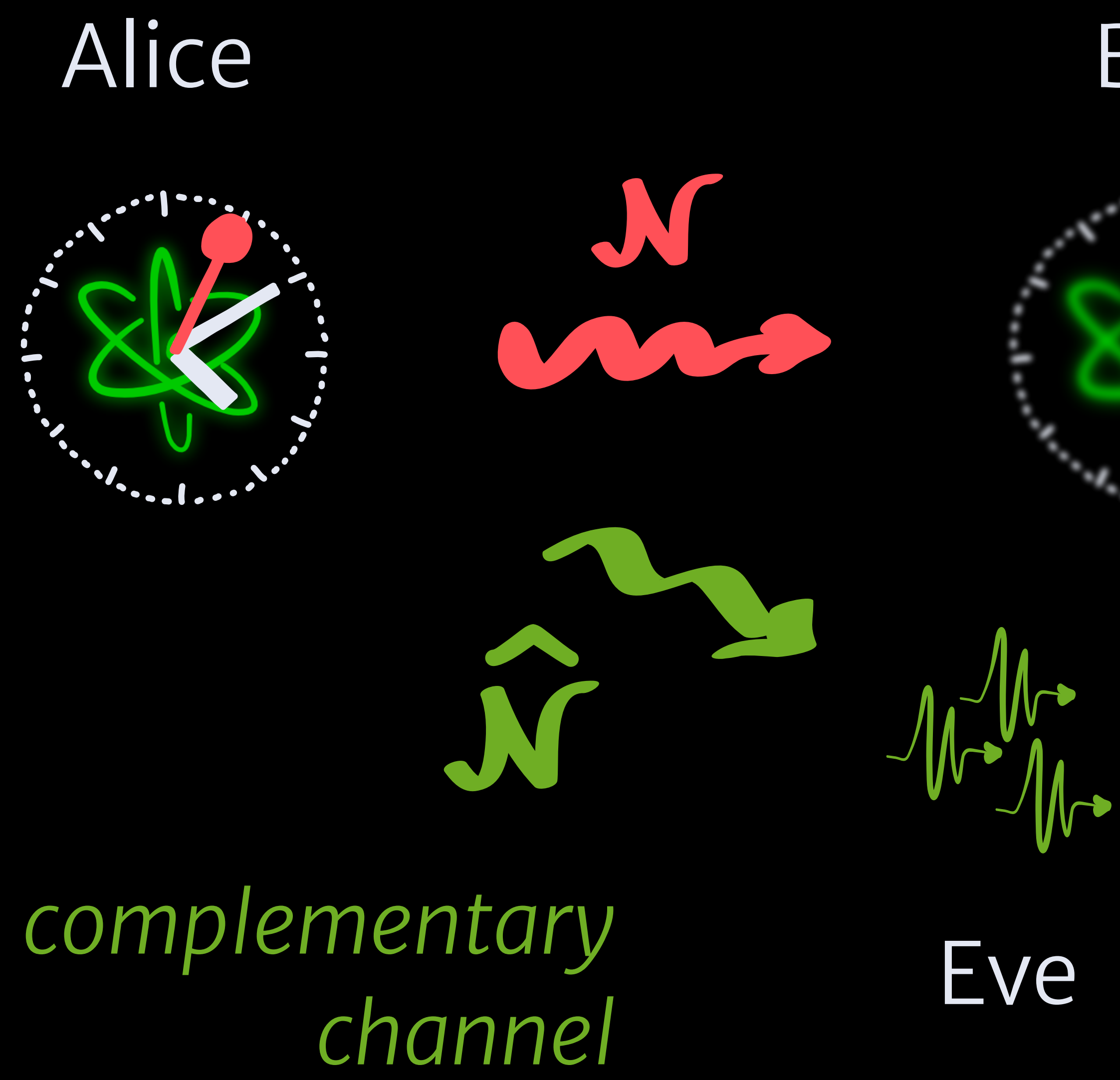
The η parameter represents **energy**, and is complementary to **time** t locally at $|\psi\rangle$



$$F(\psi; \partial_t \psi) = 4\sigma_H^2$$

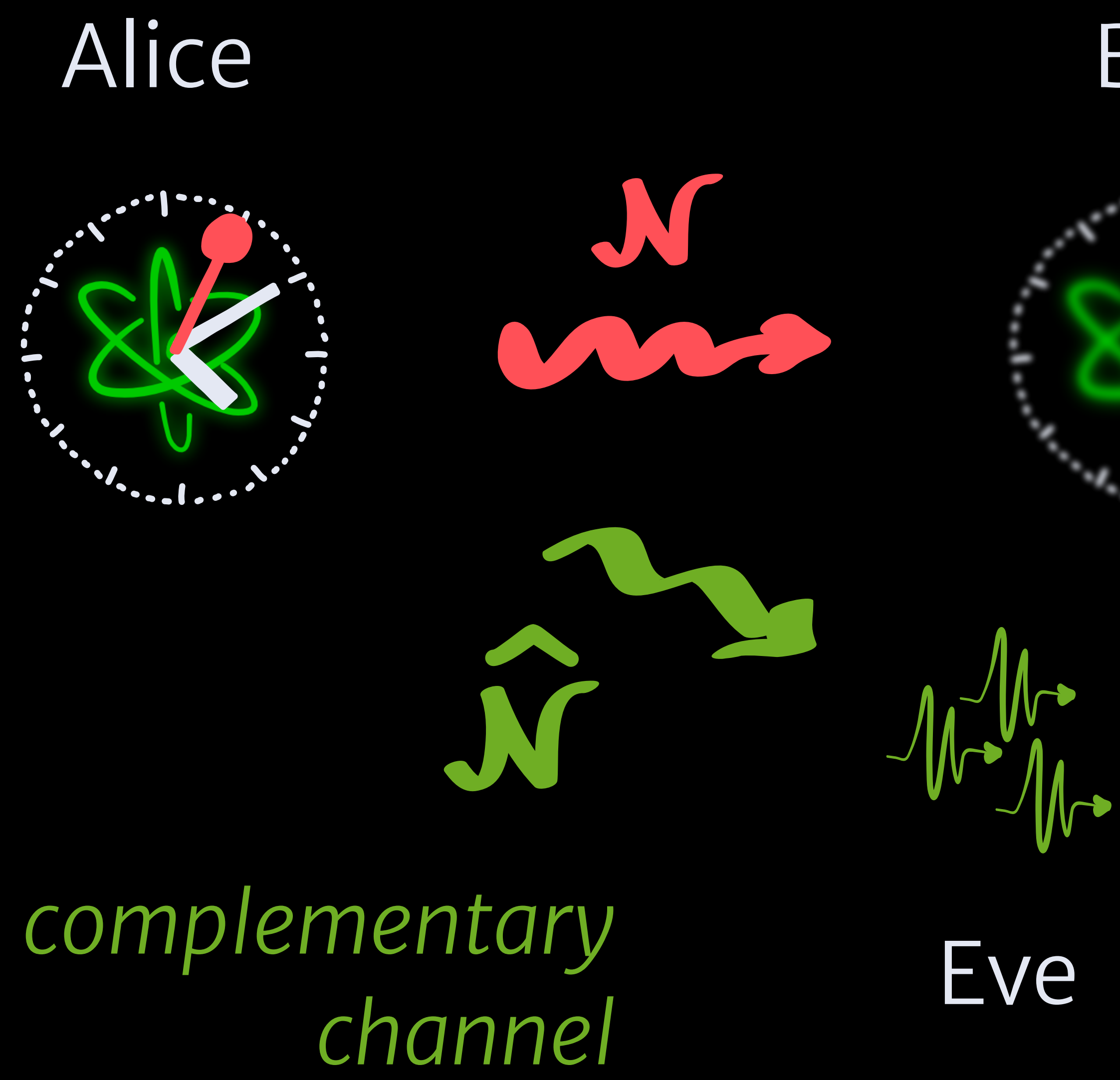
$$F(\psi, \partial_\eta \psi) = \frac{1}{\sigma_H^2}$$

$$\sigma_H^2 = \langle H^2 \rangle - \langle H \rangle^2$$



$$F_{\mathrm{Bob},t} = F(\mathcal{N}(\psi); \mathcal{N}(\partial_t \psi))$$

$$F_{\mathrm{Eve},\eta} = F(\mathcal{N}(\psi); \mathcal{N}(\partial_\eta \psi))$$



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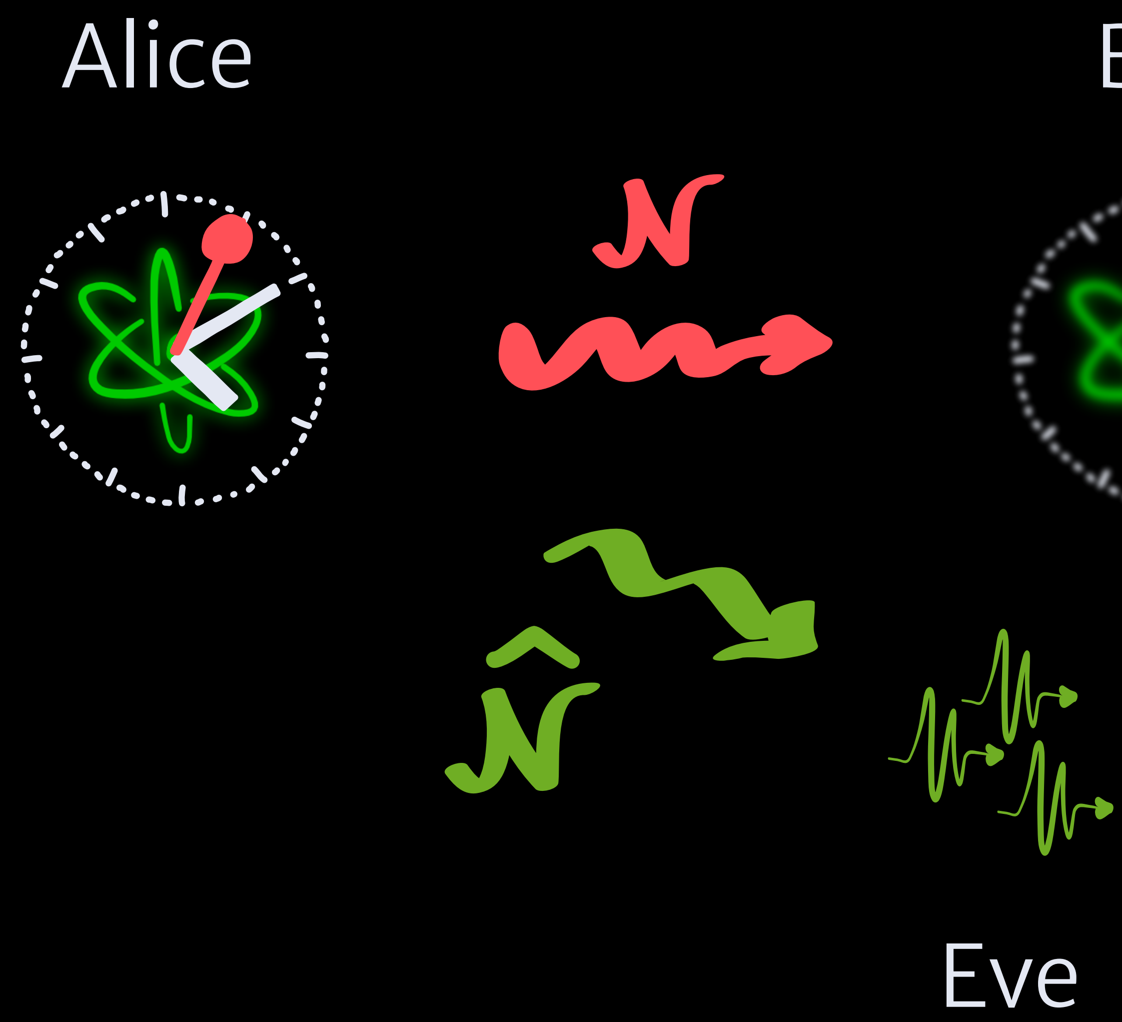
Main Result

$$\boxed{\frac{F_{\mathrm{Bob},t}}{F_{\mathrm{Alice},t}} + \frac{F_{\mathrm{Eve},\eta}}{F_{\mathrm{Alice},\eta}} = 1}$$

Fisher information tradeoff



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$F_{\mathrm{Eve},\eta}$ =
$F(\mathcal{N}(\psi); \mathcal{N}(\partial_\eta \psi))$

Bob's sensitivity to time

Eve's sensitivity to energy

Main Result

$$\boxed{\frac{F_{\mathrm{Bob},t}}{F_{\mathrm{Alice},t}} + \frac{F_{\mathrm{Eve},\eta}}{F_{\mathrm{Alice},\eta}} = 1}$$

Fisher information tradeoff

make units consistent

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$$F_{\mathrm{Alice},t} = 4\sigma_H^2 \qquad F_{\mathrm{Alice},\eta} = \sigma_H^{-2}$$

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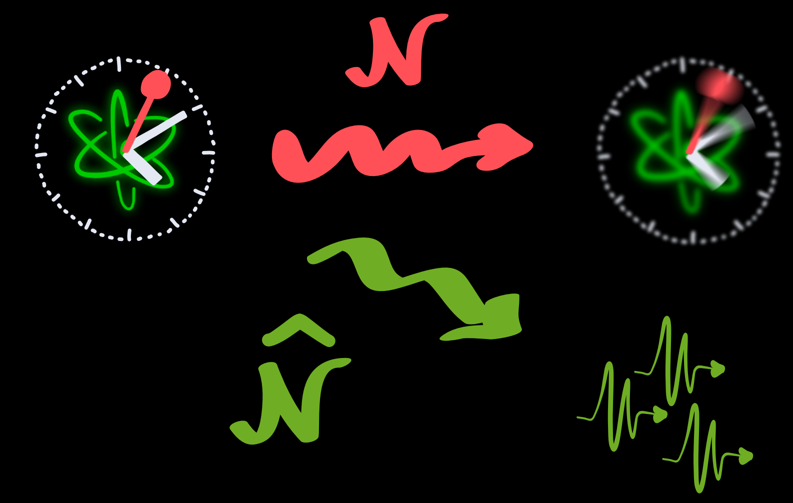

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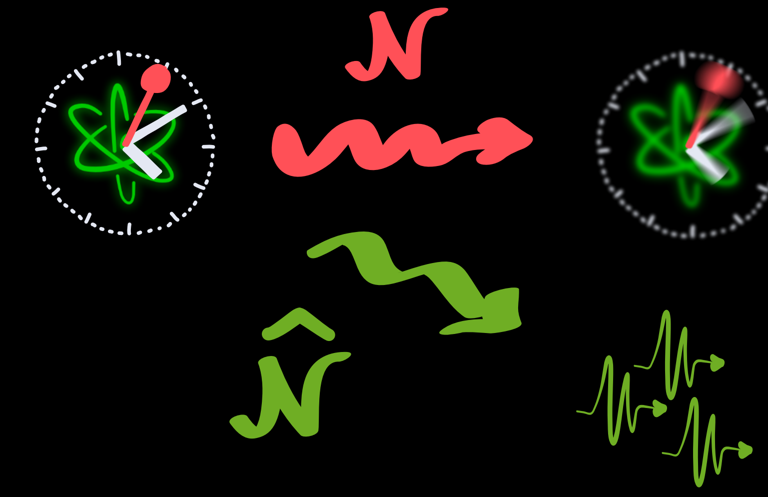

$$\partial_\eta \psi = \frac{\{\overline{H}, \psi\}}{2\sigma_H^2}$$

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$$G = 4\sigma_H^4 F_{\mathrm{Eve},\eta}$$
$$= F(\hat{\mathcal{N}}(\psi); \hat{\mathcal{N}}(\{\overline{H}, \psi\}))$$

- Decrease in Fisher information can be computed from Eve's system

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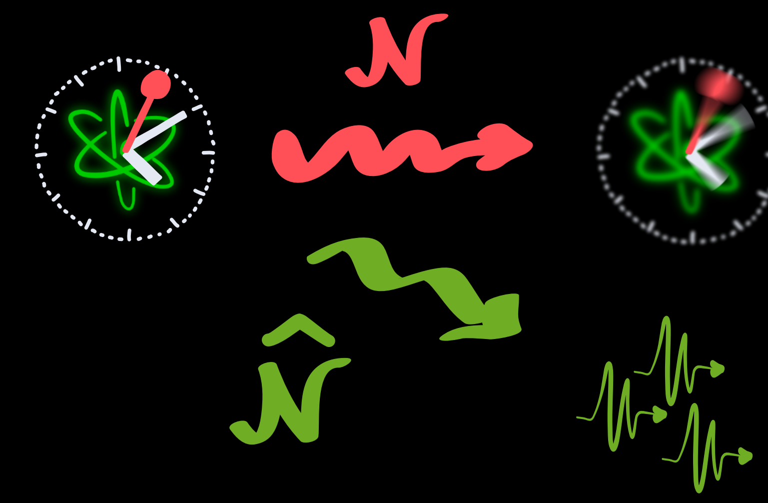

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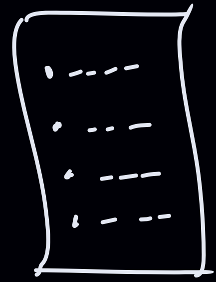
- *e.g. interesting for local, weak noise on a many-body system*



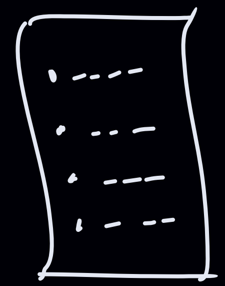
Main proof ingredients

Variational characterization of the Fisher information

$$\frac{1}{4}F(\rho; D) = \max_{S=S^\dagger} \left[\mathrm{tr}(DS) - \mathrm{tr}(\rho S^2)\right]$$

$$= \min \left\{\mathrm{tr}(LL^\dagger) :\ \rho^{1/2}L + L^\dagger\rho^{1/2} = D\right\}$$

Macieszczak 1312.1356; Holevo 2011; …



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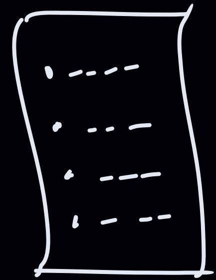
Connection to Fidelity / Bures metric

$$F(\rho(t_0); \partial_t\rho(t_0)) = -8 \left.\frac{d^2}{dt^2}\right|_{t_0} F(\rho(t_0), \rho(t))$$

Fidelity of quantum states



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Fidelity of quantum states

Uhlmann:

$$F(\mathcal{N}(\psi), \mathcal{N}(\psi')) = \max_{W_E} |\langle\psi|V^\dagger(\mathbb{1}\otimes W_E)V|\psi'\rangle|$$

unitary on environment

dilation of $\mathcal{N}$

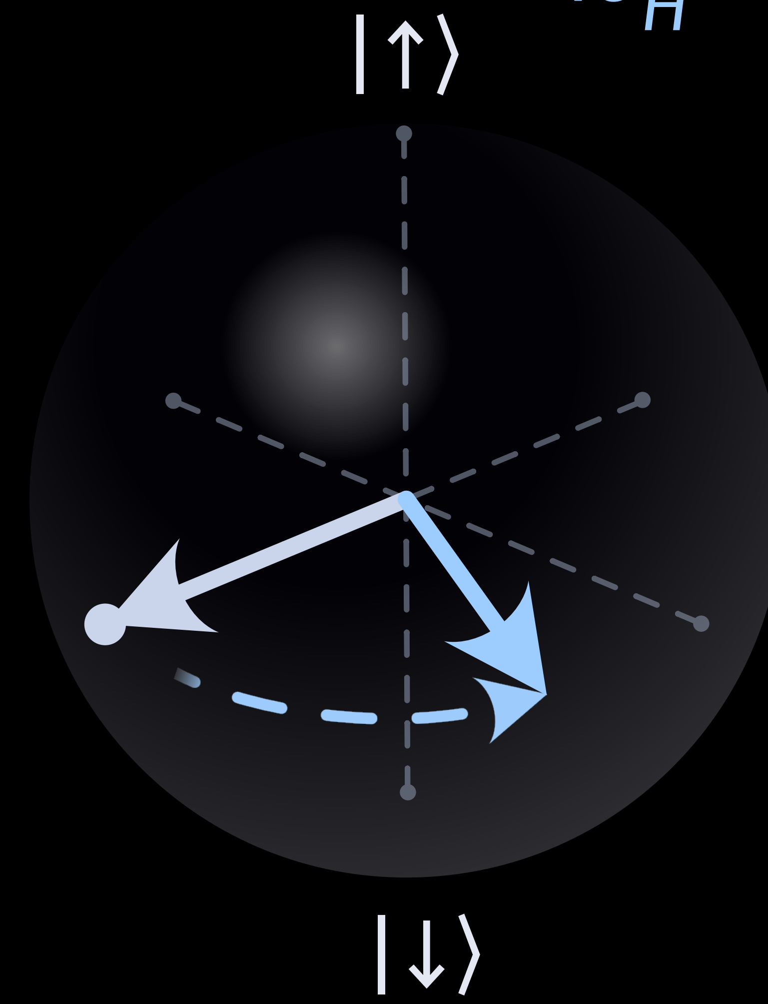
optim. over W_E

$\leftrightarrow$ optim. over S

Example: 1 qubit $\quad H = \frac{\omega}{2}\sigma_Z \quad |\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle + |\downarrow\rangle\right]$

$4\sigma_H^2 = \omega^2$

Evolve for t_0: $\quad \psi_t = \frac{1}{2}\begin{bmatrix} 1 & e^{-i\omega t} \\ e^{i\omega t} & 1 \end{bmatrix}$



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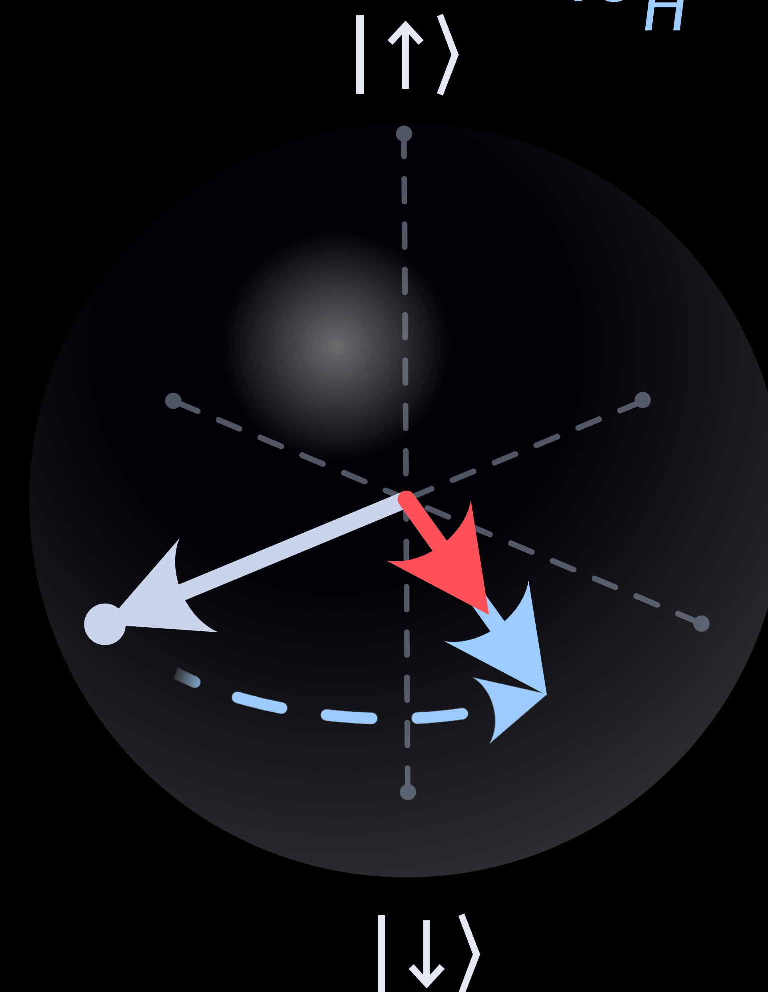
$$\mathcal{D}_p[\rho] = \begin{bmatrix} \rho_{00} & (1-p)\rho_{01} \\ (1-p)\rho_{10} & \rho_{11} \end{bmatrix}$$



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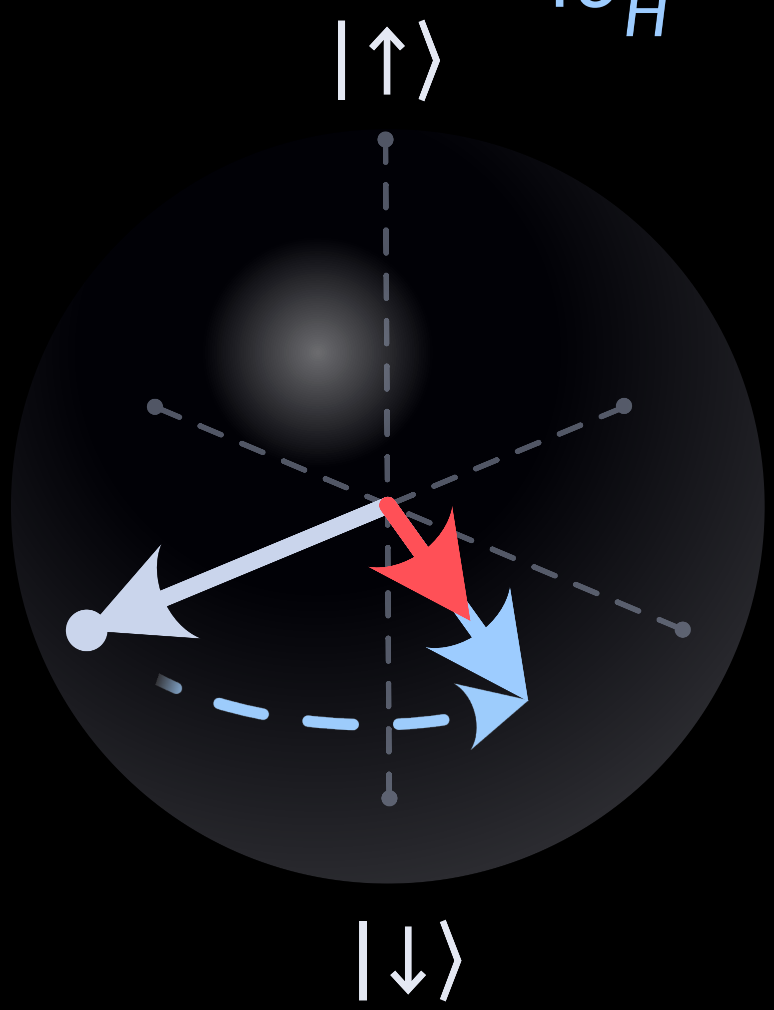
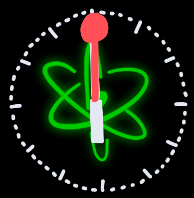


Eve:

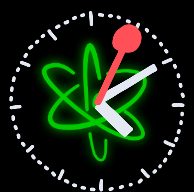
$$\hat{\mathcal{D}}_p[\rho] = \begin{bmatrix} \left(1-\frac{p}{2}\right)\mathrm{tr}(\rho) & \sqrt{\frac{p}{2}\left(1-\frac{p}{2}\right)}\,\mathrm{tr}(\sigma_Z\rho) \\ \sqrt{\frac{p}{2}\left(1-\frac{p}{2}\right)}\,\mathrm{tr}(\sigma_Z\rho) & \frac{p}{2}\,\mathrm{tr}(\rho) \end{bmatrix}$$



Example: 1 qubit

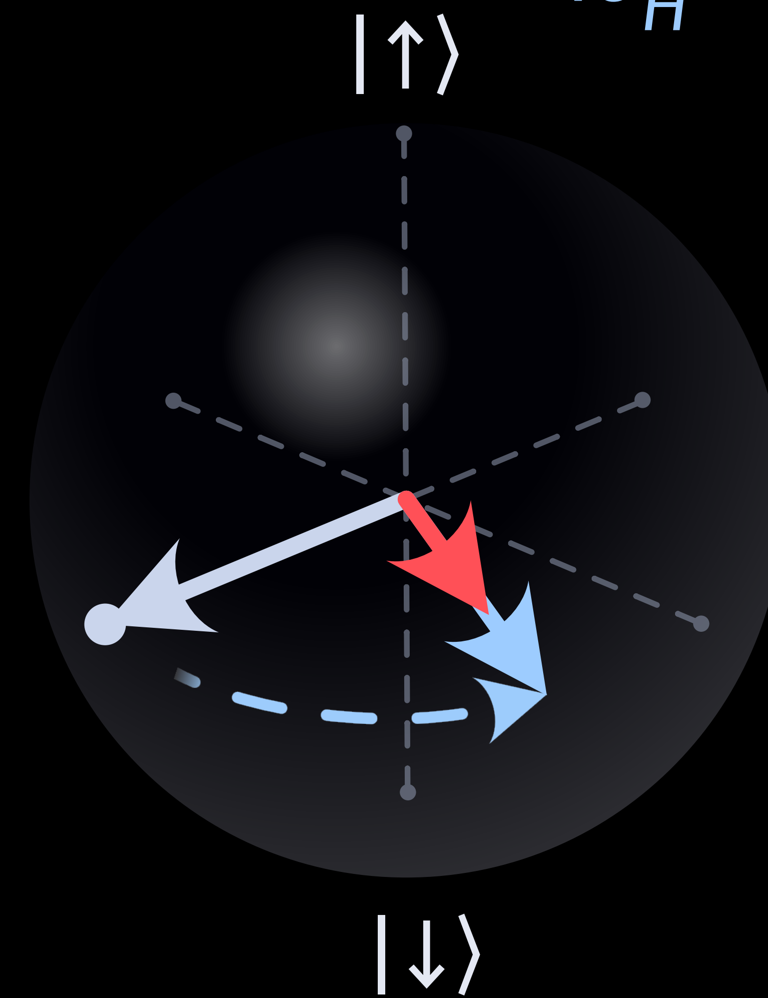
$$H = \frac{\omega}{2}\sigma_Z \quad |\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle + |\downarrow\rangle\right]$$

$$4\sigma_H^2 = \omega^2$$



Evolve for t_0:

$$\psi_t = \frac{1}{2}\begin{bmatrix} 1 & e^{-i\omega t} \\ e^{i\omega t} & 1 \end{bmatrix}$$



Apply partial dephasing

$$\mathcal{D}_p[\rho] = \begin{bmatrix} \rho_{00} & (1-p)\rho_{01} \\ (1-p)\rho_{10} & \rho_{11} \end{bmatrix}$$

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$$\hat{\mathcal{D}}_p[\rho] = \begin{bmatrix} \left(1-\frac{p}{2}\right)\mathrm{tr}(\rho) & \sqrt{\frac{p}{2}\left(1-\frac{p}{2}\right)}\,\mathrm{tr}(\sigma_Z\rho) \\ \sqrt{\frac{p}{2}\left(1-\frac{p}{2}\right)}\,\mathrm{tr}(\sigma_Z\rho) & \frac{p}{2}\,\mathrm{tr}(\rho) \end{bmatrix}$$

$$\{\overline{H}, \psi\} = \frac{\omega}{2}\sigma_Z$$

sensitivity loss:

$$G = F(\hat{\mathcal{N}}(\psi); \hat{\mathcal{N}}(\{\overline{H}, \psi\})) = \omega^2[1-(1-p)^2]$$

$$\begin{pmatrix} 1-\frac{p}{2} & 0 \\ 0 & \frac{p}{2} \end{pmatrix} \qquad \omega\sqrt{\frac{p}{2}\left(1-\frac{p}{2}\right)}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Example: 1 qubit $\quad H = \frac{\omega}{2}\sigma_Z \quad |\psi\rangle = \frac{1}{\sqrt{2}}\left[|\uparrow\rangle + |\downarrow\rangle\right]$

$4\sigma_H^2 = \omega^2$

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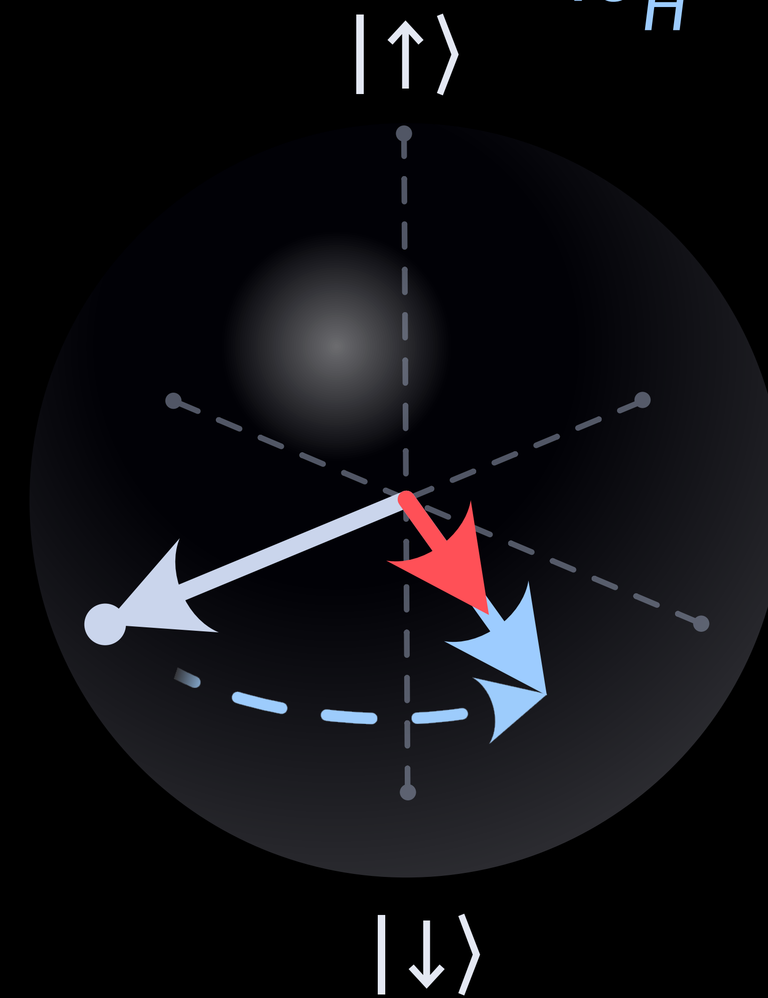

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Direct calculation: $\quad F_{\mathrm{Bob},t} = \omega^2(1-p)^2 \qquad F_{\mathrm{Alice},t} = \omega^2$ ✔

? Uncertainty relation for sensitivity:

noisy clock w.r.t. time <> environment w.r.t. energy

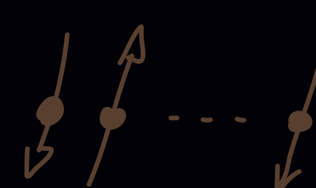
Expression for sensitivity loss

(methods from semidefinite programming)

- alternative sensitivity bounds
- uncertainty relation for any two parameters

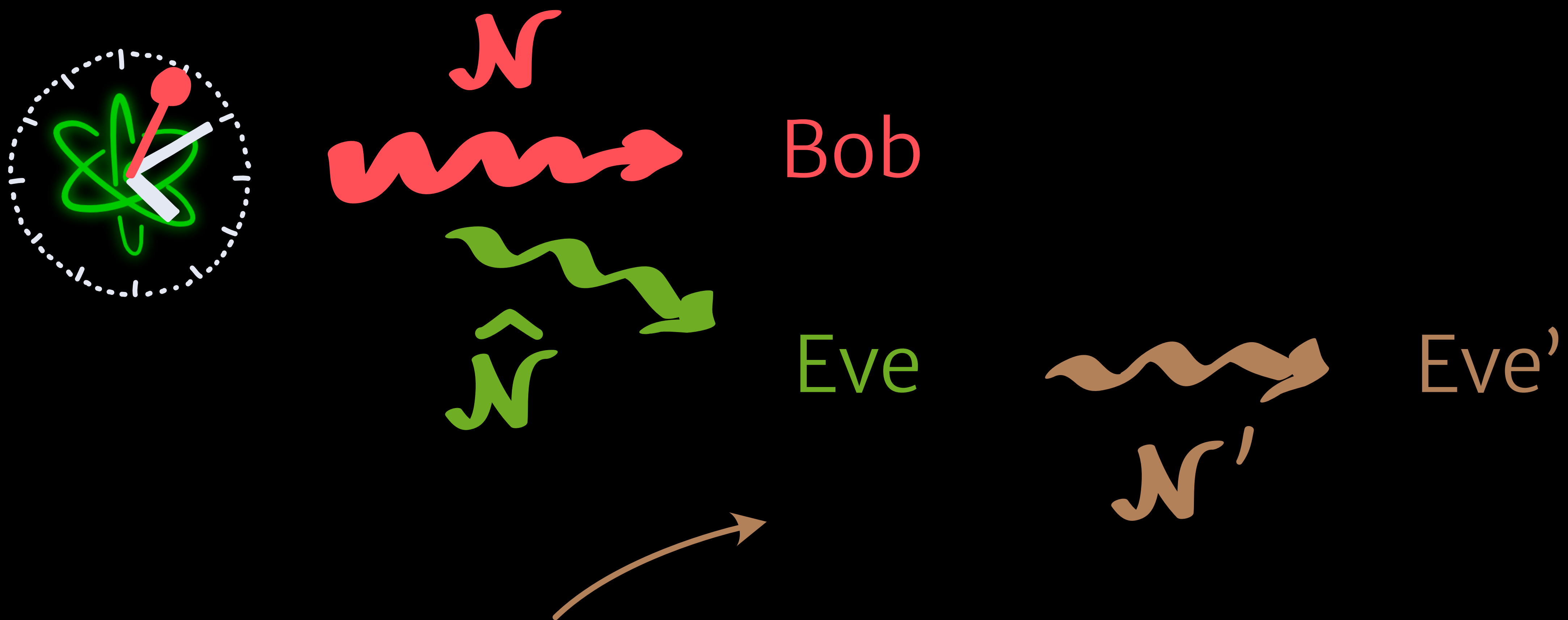
01 Necessary and sufficient conditions for zero sensitivity loss

- “metrological codes”



Example: Ising Hamiltonian & amplitude damping noise


Bob
Eve



e.g., decohere in fixed basis
→ simpler bound for
amplitude damping noise

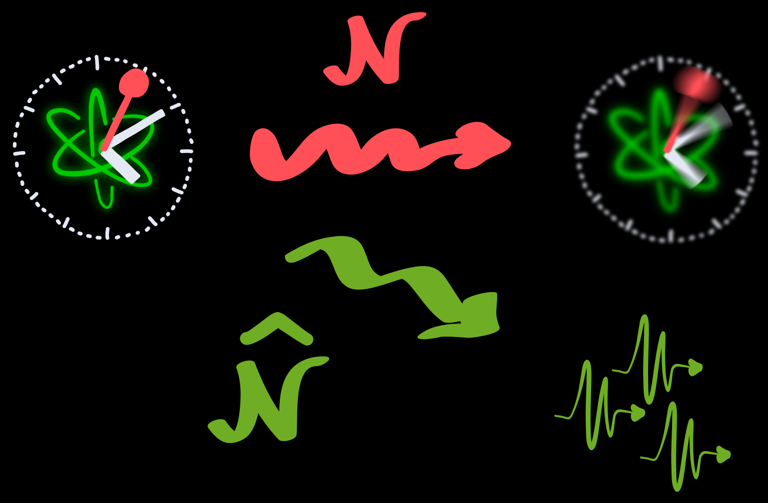
$$F_{\mathrm{Eve}'}(\eta) \leqslant F_{\mathrm{Eve}}(\eta)$$

e.g., decohere in fixed basis
→ simpler bound for
amplitude damping noise

$$\rightarrow \quad \frac{F_{\mathrm{Bob}}(t)}{F_{\mathrm{Alice}}(t)} \leqslant 1 - \frac{F_{\mathrm{Eve}'}(\eta)}{F_{\mathrm{Alice}}(t)}$$

any two generators H, Z:

$$\partial_t \psi = -i[H, \psi]$$
$$\partial_z \psi = -i[Z, \psi]$$



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$$\frac{F_{\mathrm{Bob}}(t)}{F_{\mathrm{Alice}}(t)} + \frac{F_{\mathrm{Eve}}(z)}{F_{\mathrm{Alice}}(z)} \leq 1 + 2\sqrt{1 - \frac{\left\langle i[H, Z]\right\rangle^2}{4\,\sigma_H^2 \sigma_Z^2}}$$

Fisher information
tradeoff for any
two parameters

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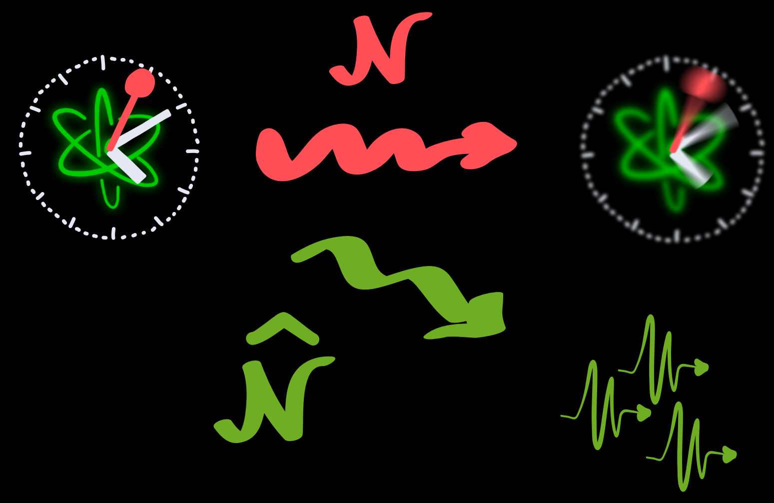

$$\frac{F_{\mathrm{Bob}}(t)}{F_{\mathrm{Alice}}(t)} + \frac{F_{\mathrm{Eve}}(z)}{F_{\mathrm{Alice}}(z)} \leq 1 + 2\sqrt{1 - \frac{\langle i[H, Z]\rangle^2}{4\,\sigma_H^2 \sigma_Z^2}}$$

Fisher information tradeoff for any two parameters

= 0 if H, Z saturate the Robertson uncertainty relation

$$\sigma_H \sigma_Z \geq \frac{1}{2}\left|\langle i[H, Z]\rangle\right|$$

(e.g., energy & time)

? Uncertainty relation for sensitivity:

noisy clock w.r.t. time < > environment w.r.t. energy

Expression for sensitivity loss

(methods from semidefinite programming)

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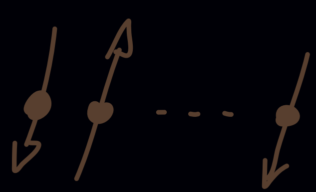
Ø1 Necessary and sufficient conditions for zero sensitivity loss

- “metrological codes”



Example: Ising Hamiltonian & amplitude damping noise

when is no sensitivity lost?

$$F_{\mathrm{Eve},\eta} = 0 \qquad ?$$

when is no sensitivity lost?

$$F_{\mathrm{Eve},\eta} = 0$$

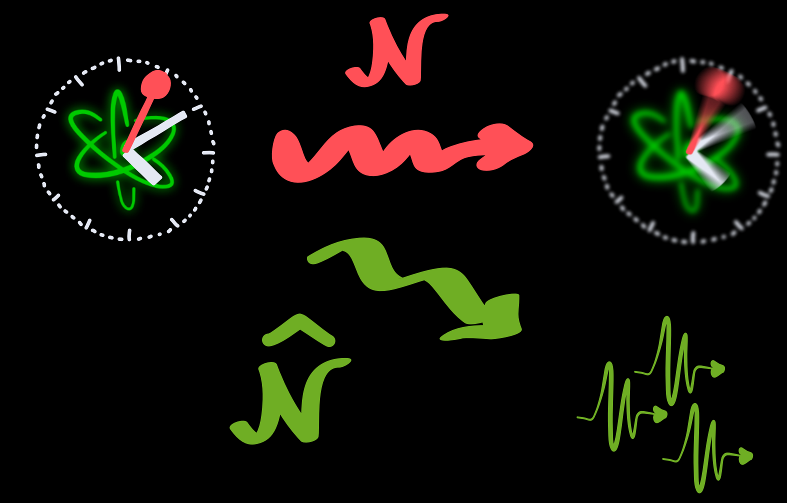
$$\Leftrightarrow$$

like Knill–Laflamme, but weaker

$$\mathbf{tr}\left[Z\,\Pi\,E_{k'}^{\dagger}E_k\,\Pi\right] = 0$$

"code qubit" spanned by $|\psi\rangle =: |+\rangle$ and $\bar{H}|\psi\rangle$

when is no sensitivity lost?



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like
Knill–Laflamme,
but weaker

"code qubit"
spanned by
$|\psi\rangle =: |{+}\rangle$ and $\bar{H}|\psi\rangle$

closely related to:

Hamiltonian not in
Lindblad span

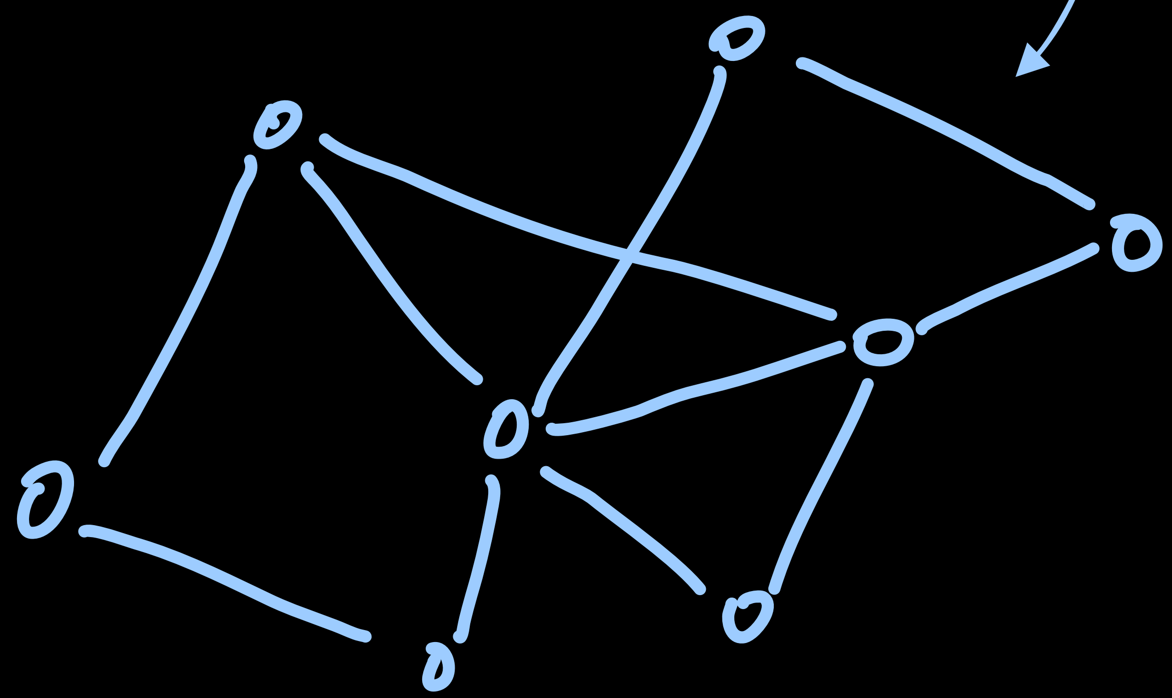
Demkowicz-Dobrzański+ PRX 2017;
Zhou+ N. Comm. 2018; Layden+ PRL 2019;
…

quantum error
correction for algebras

Bény+ PRL 2007

ZZ interactions

(can also include XX & YY)



cf. e.g. Ouyang 1908.02378; …

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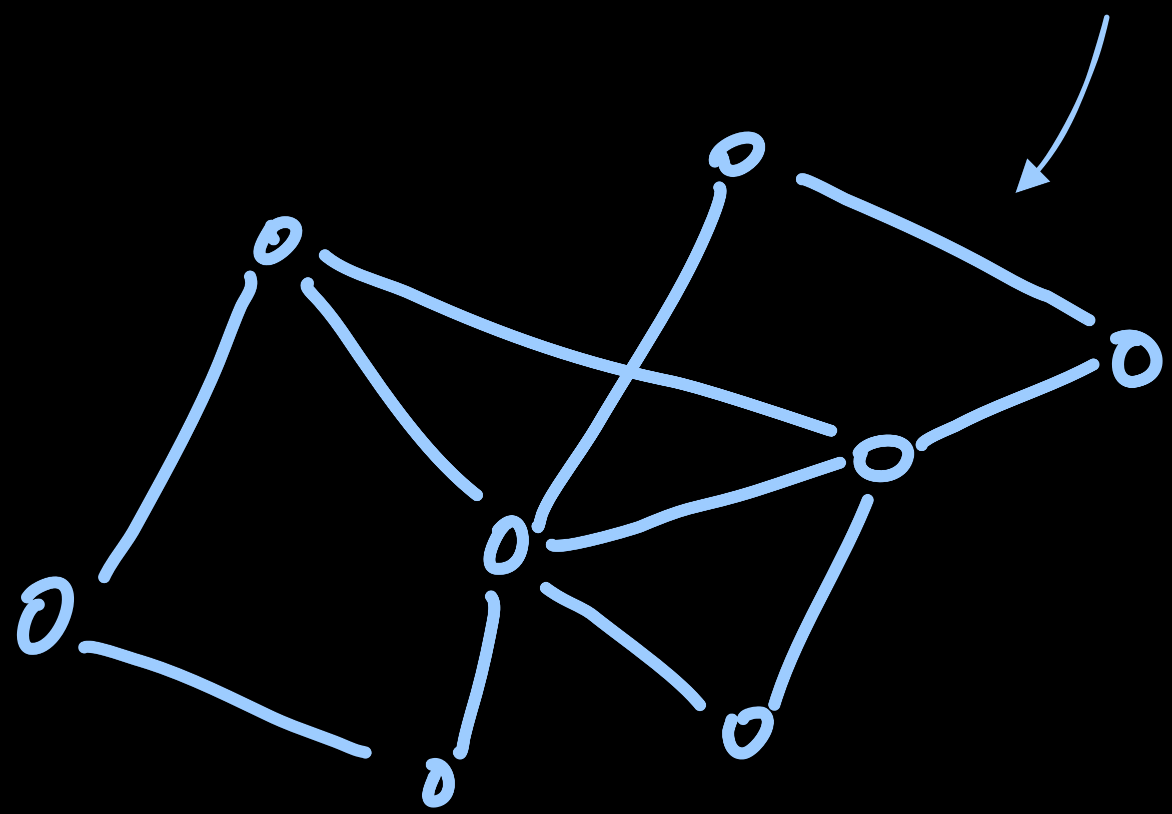


cf. e.g. Ouyang 1908.02378; ...

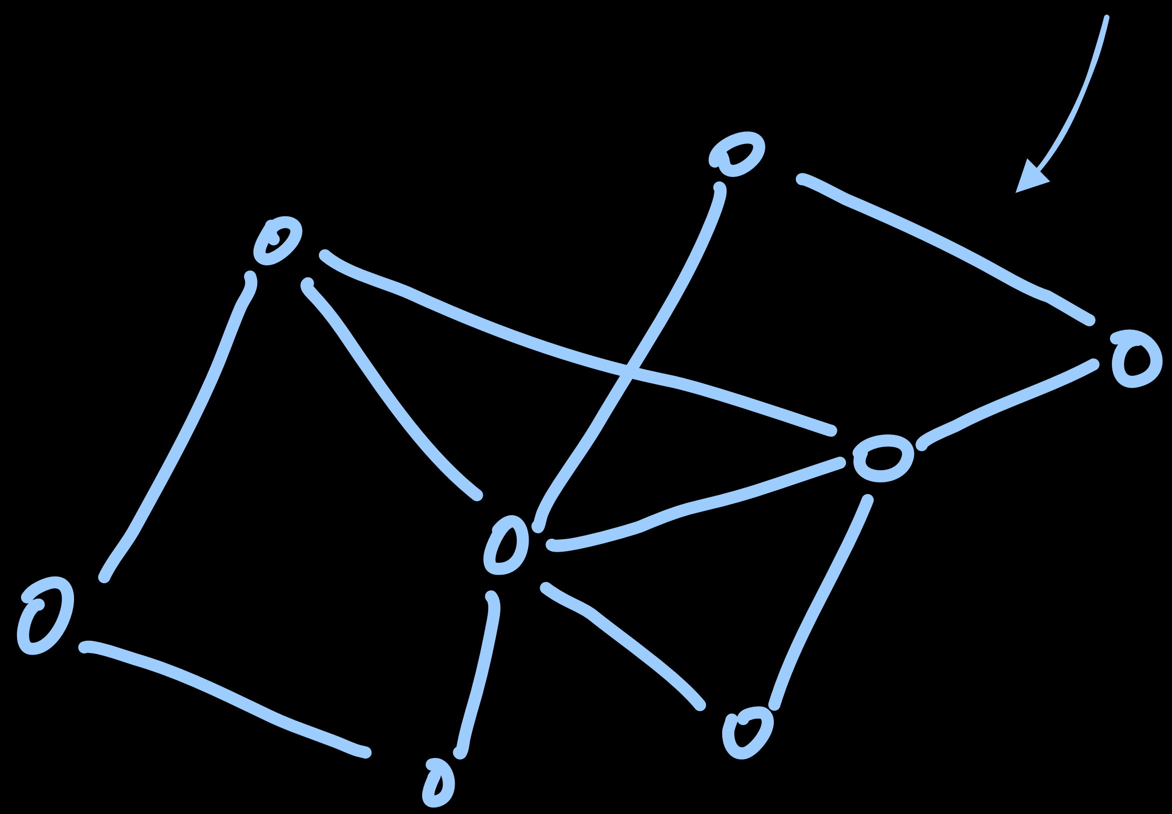
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|0\ldots 0\rangle + |c^n\rangle\right]$$

c^n = configuration that violates many interaction terms

$$\mathrm{tr}\left[Z\,\Pi\,E_{k'}^{\dagger}E_k\,\Pi\right] \neq 0$$ ✗

prone to sensitivity loss

ZZ interactions
(can also include XX & YY)



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c^n = configuration that violates many interaction terms

$$\mathrm{tr}\big[Z\,\Pi\,E_{k'}^{\dagger}E_k\,\Pi\big] = 0 \quad ✔$$

no sensitivity loss in case of a single localized error

? Uncertainty relation for sensitivity:

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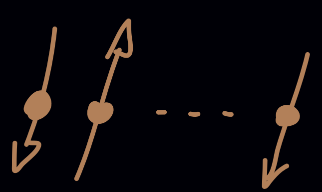
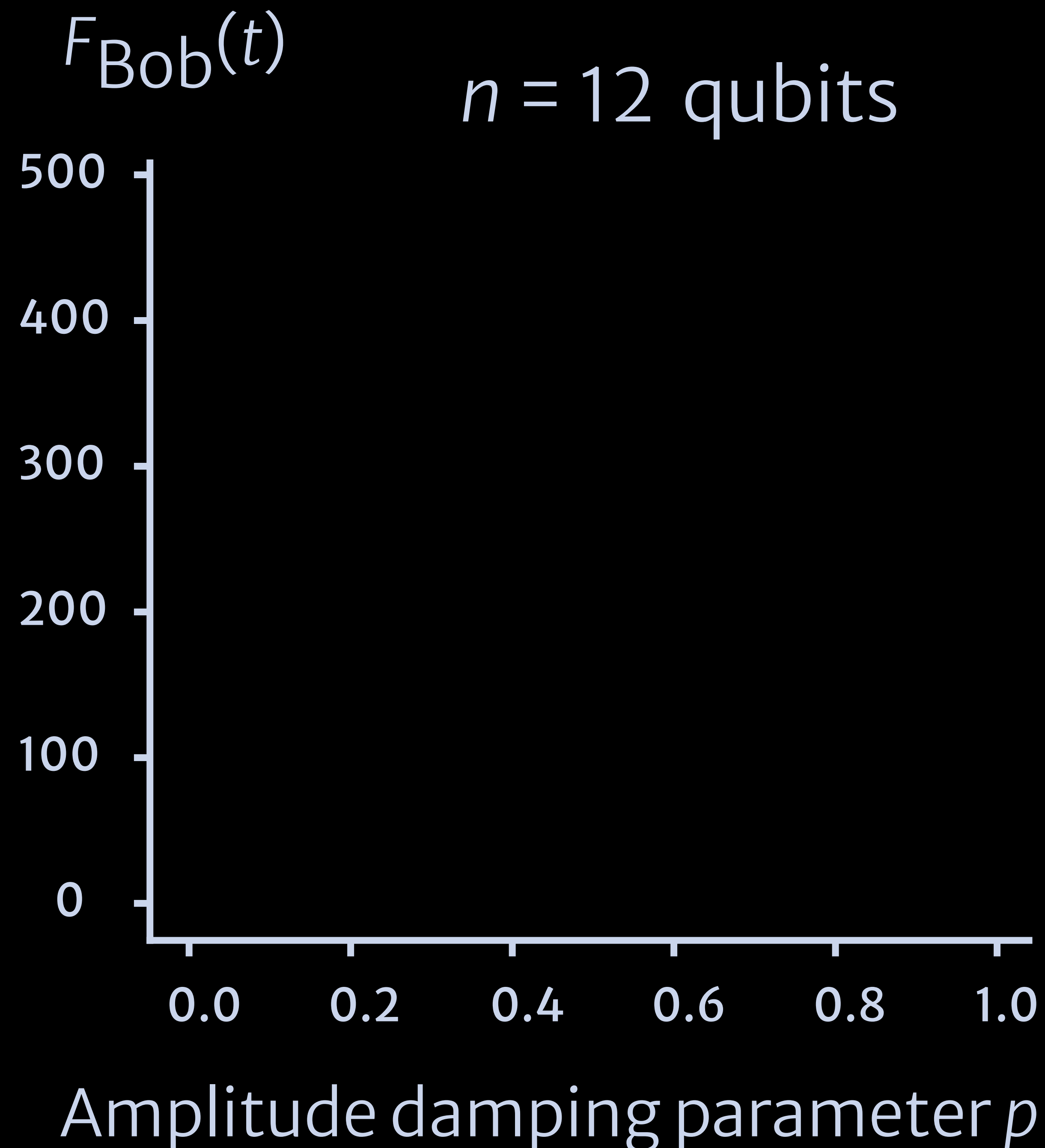


Example: Ising Hamiltonian & amplitude damping noise

$H = \sum \sigma_Z^{(j)} \sigma_Z^{(j+1)}$ 1D spin chain



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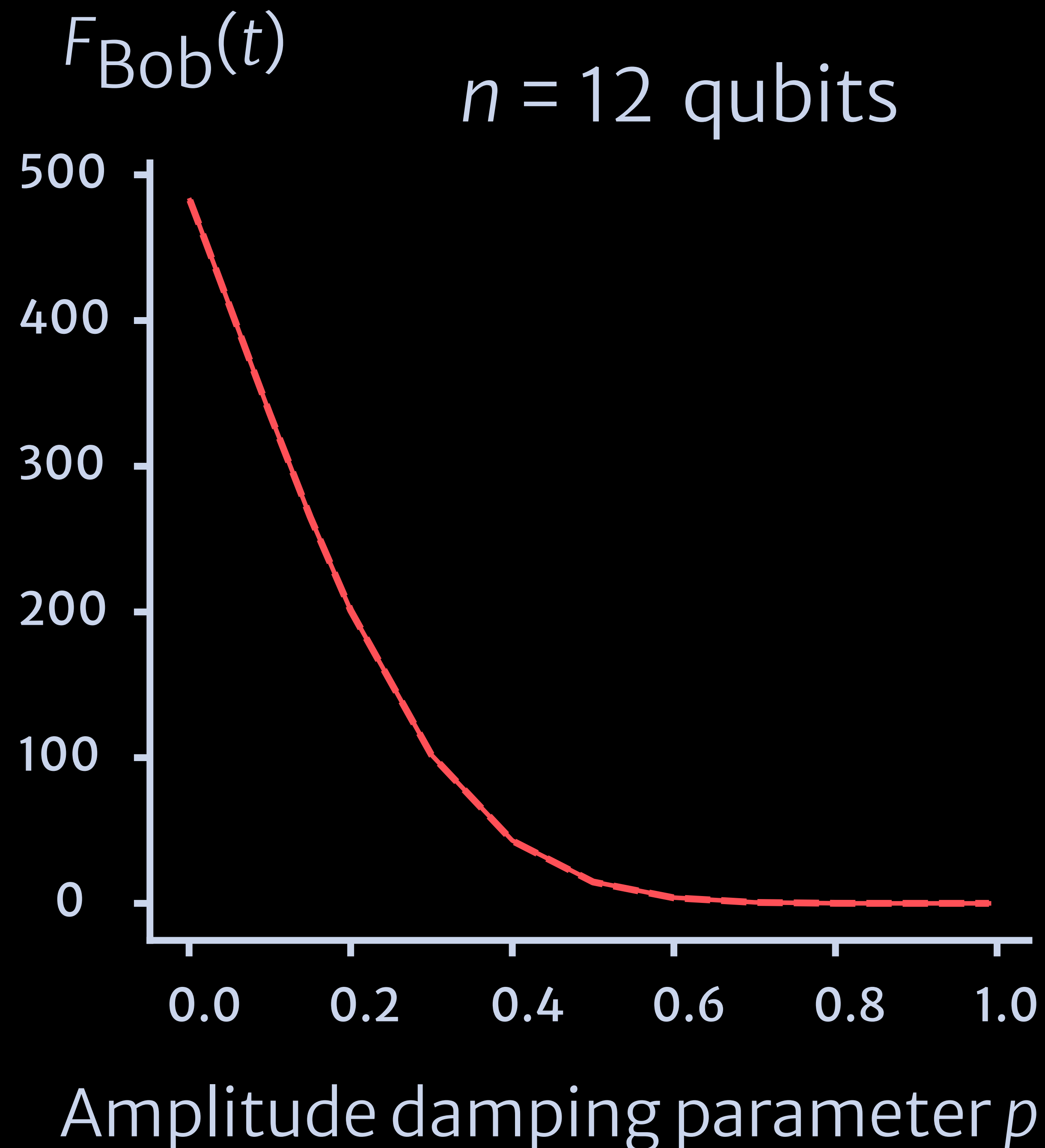

ferromagnet–antiferromagnet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00 \ldots 00\rangle + |01 \ldots 01\rangle\right)$$

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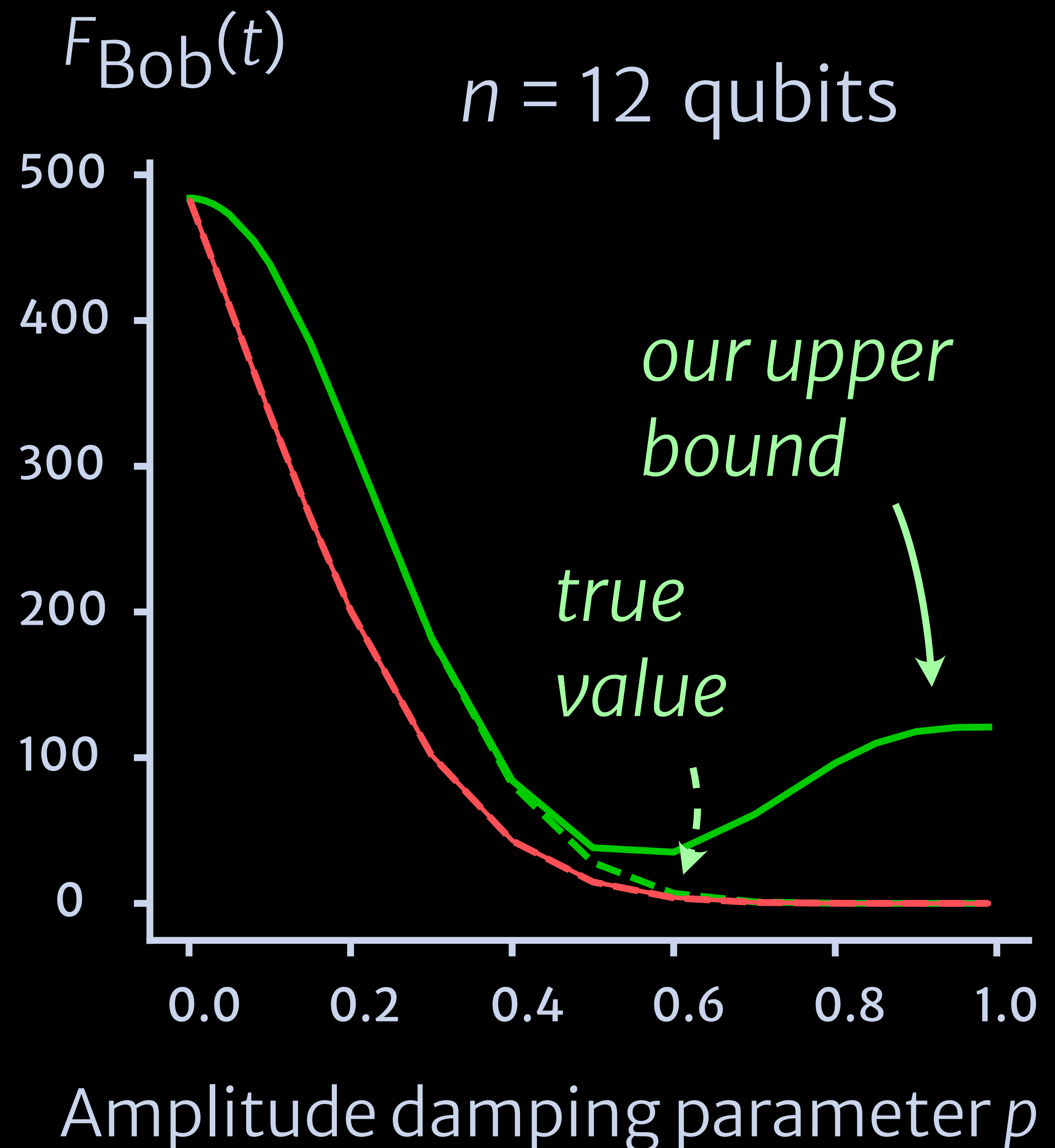

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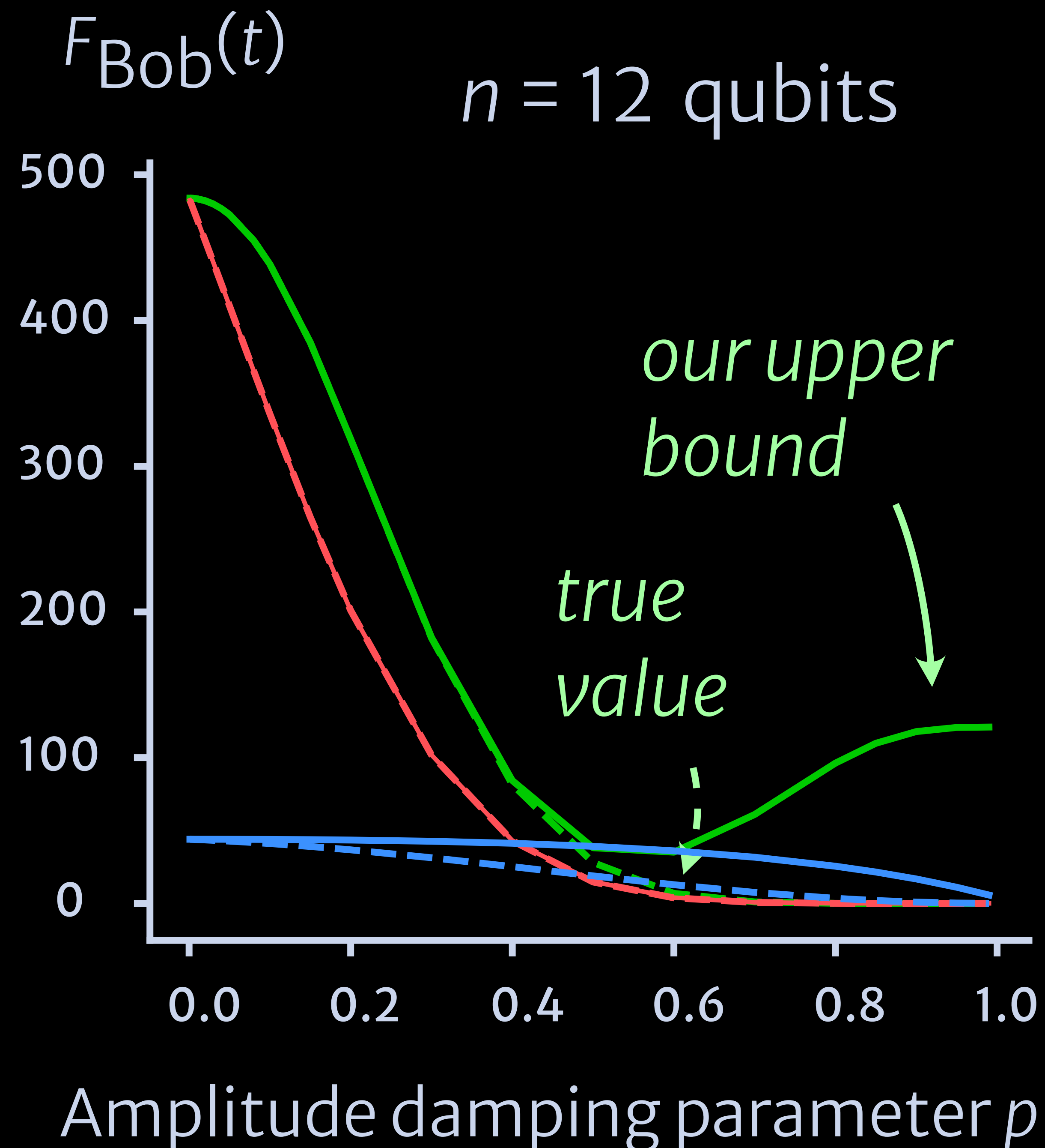

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spin-coherent (clock) state

$$|\psi\rangle = |+\rangle^{\otimes n}$$

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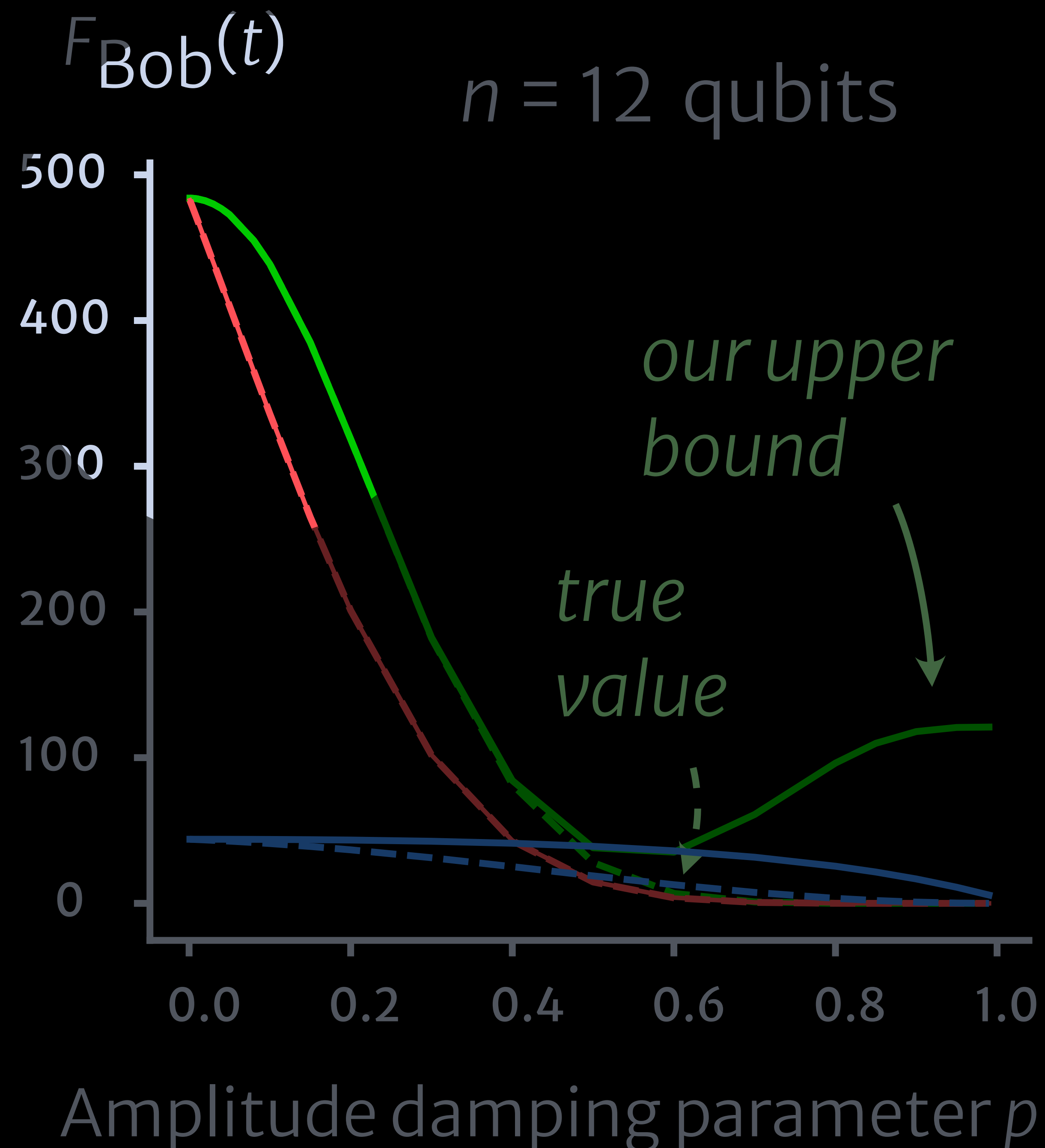

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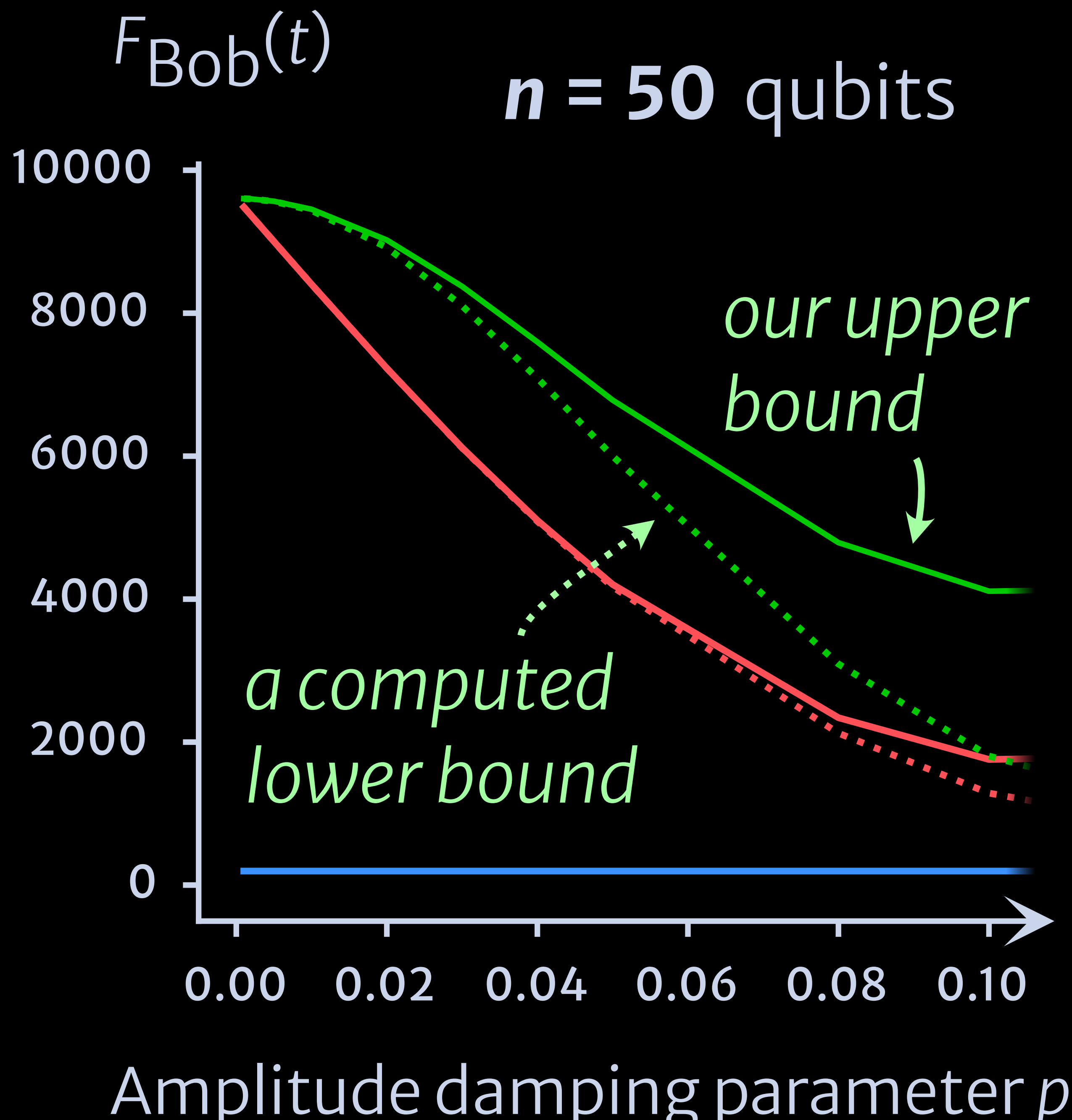

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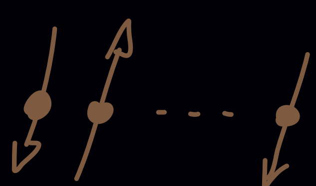

Example: Ising Hamiltonian & amplitude damping noise

- Better clock states = those that hide energy from Eve
- Our results hold for any fixed state $|\psi\rangle$
- Our uncertainty relations extend to infinite dimensions
- Also applicable for sensing an unknown parameter in the Hamiltonian
- Also applicable for continuous Markovian noise, under some additional assumptions

- Information tradeoffs / uncertainty relations give insight into the structure of quantum theory
- Fisher information counterpart to entropic uncertainty relations

  Coles+ PRL 2019
- Characterization of the Fisher information in intermediate regimes of *n* (non-asymptotic)
- general Fisher information bounds can be hard to obtain

  Fujiwara & Imai JPA 2008; Escher+ Nat Phys 2011; …
- strongly interacting many-body probes might offer better sensitivity in quantum metrology

  NV centers — Zhou+ PRX 2020

  lattice atomic clocks — Goban+ Nat. 2018

Applications to cryptography?

Coles+ RMP 2017; …

Multi-parameter metrology?

Gorecki+ Quantum 2020; …

sensitivity loss of "metrological codes" for weak i.i.d. noise?

Thank you for your attention!

$\partial_t \rho = \mathcal{L}(\rho)$ $\quad$ $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \longrightarrow \mathcal{E}_t = e^{t\mathcal{L}}$ full evolution

noise $\mathcal{L}_1(\rho) = \sum_j \left[L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right]$

unitary $\mathcal{L}_0(\cdot) = -i[H, (\cdot)]$

Decompose $\mathcal{E}_t$ as unitary followed by "effective noise channel" $\mathcal{N}_t$:

$$\mathcal{E}_t = \mathcal{N}_t \, e^{t\mathcal{L}_0}$$

$$\hookrightarrow \mathcal{N}_t = \mathcal{E}_t \, e^{-t\mathcal{L}_0}$$

$$|\psi_0\rangle \xrightarrow{e^{t\mathcal{L}}} \rho(t)$$

$$|\psi_0\rangle \longrightarrow U_t|\psi_0\rangle \xrightarrow{\mathcal{N}_t} \rho(t)$$

Assume

$\partial_t \mathcal{N}_t \ll \mathcal{N}_t(\partial_t \psi)$

$\longrightarrow$ setting of our main result

$$F(\rho; \partial_t \rho) \approx 4\sigma_H^2 - F(\mathcal{E}_t(\psi_0); \mathcal{E}_t(\partial_t \psi_0))$$