Quantum error correction thresholds for the universal Fibonacci Turaev-Viro code

arXiv:2012.04610

Alexis Schotte [1] Guanyu Zhu [2] Lander Burgelman [1] Frank Verstraete [1]

[1] Ghent University

[2] IBM Quantum

Outline

Topological quantum computation

Use anyons to perform fault-tolerant quantum computation [Kitaev '97]

Passive TQC

Majorana wires, FQH states, ...

- Information encoded in **groundspace** of **physical system**
- Protected by energy gap at $T = 0$
- Need error correction for thermal noise
- non-Abelian anyons are possible

Active QEC

surface code, color code, ...

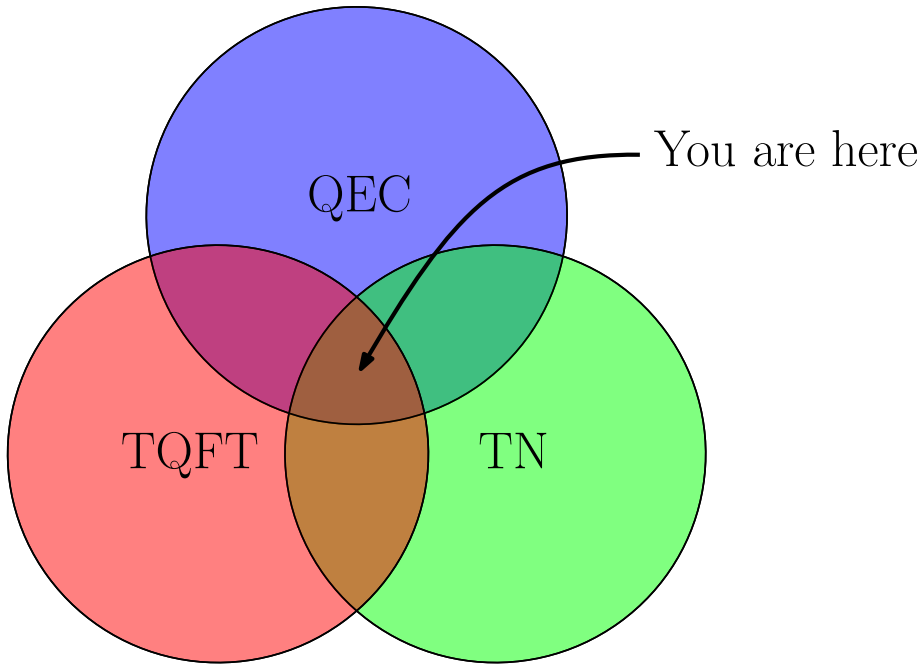
- Information encoded in **codespace** of system of **qubits**
- Actively measure stabilizers
- Error correction when stabilizers are violated

Active Non-Abelian quantum error correction

- Codespace of system of qubits
- Actively measure set of commuting projectors
- Noise $\rightarrow$ Non-abelian anyons

Our goals:
1. Construction of a non-Abelian code of qubits
2. Classical simulation of this code

Necessary ingredients:



Abelian vs non-Abelian codes

Abelian

- \+ Low-weight local stabilizer codes
- \+ Well studied
- \+ High thresholds

- − No native non-Clifford gate in 2D
  - Magic state distillation
  - Code-switching to higher dimensional codes

  $\rightarrow$ high overhead

Non-Abelian

- \+ Native universal gate set in 2D
- \+ Beyond stabilizer codes

- − Complicated
- − Hard to simulate classically

Previous work

- König, Kuperberg, Reichardt: Turaev-Viro codes
  arXiv:1002.2816
- Bonesteel, DiVincenzo: Measurement circuits for Levin-Wen Operators
  arXiv:1206.6048
- Feng, Bonesteel: Fixing violated vertices in Fibonacci Levin-Wen code
  PhD Thesis W. Feng
- Burton, Brell, Flammia: Threshold for phenomenological model of Fib anyons
  arXiv:1506.03815

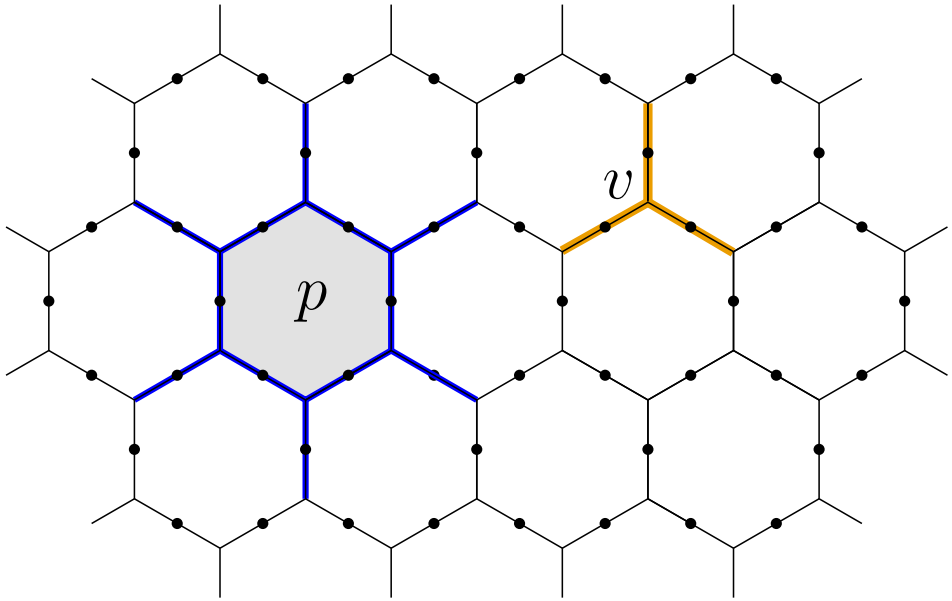
Outline

The Levin-Wen model

$$H_\Lambda = -\sum_v Q_v - \sum_p B_p$$

$$Q_v \left| \begin{array}{ccc} & j & \\ i & & k \end{array} \right\rangle = \delta_{ijk} \left| \begin{array}{ccc} & j & \\ i & & k \end{array} \right\rangle$$

$$B_p = \frac{1}{\mathcal{D}^2} \sum_s d_s O_p^s$$



Defined by a Unitary Fusion Category $\mathcal{C}$:

- **string types** $\{\mathbf{1}, i_2, \ldots, i_N\}$ $\rightarrow$ qudit states $|0\rangle, |1\rangle, \ldots, |N\rangle$
- **branching rules** δ_{ijk} $\rightarrow$ determine Q_v
- **numerical data** $\{d_i,\ F\}$ $\rightarrow$ determine B_p

The Levin-Wen model

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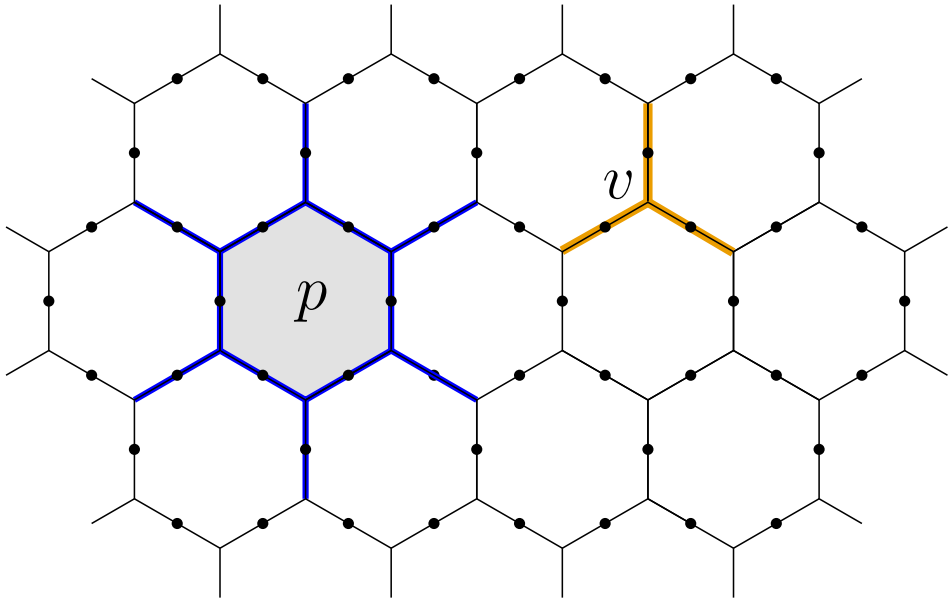
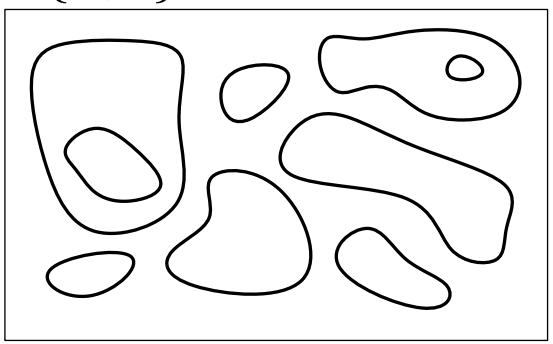


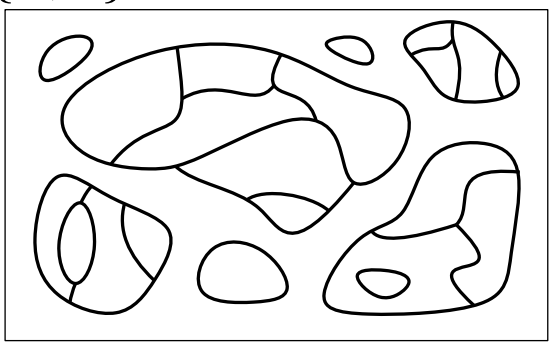
surface code: $\mathcal{C} = \mathbb{Z}_2$

$\{\mathbf{1}, e\}$ $\quad e \times e = \mathbf{1}$



$\boxed{\mathcal{C} = \mathrm{FIB}}$

$\{\mathbf{1}, \tau\}$ $\quad \tau \times \tau = \mathbf{1} + \tau$



The Levin-Wen model

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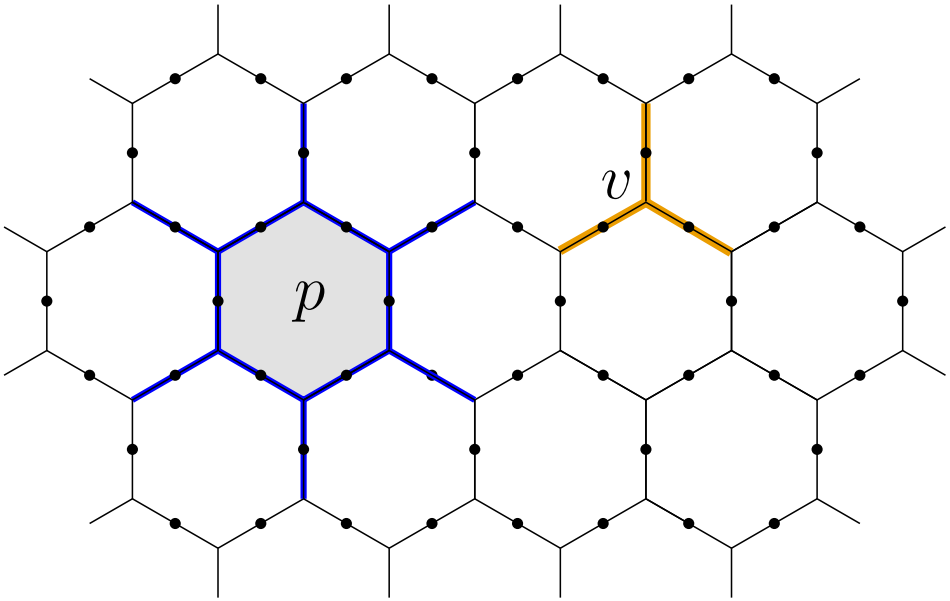
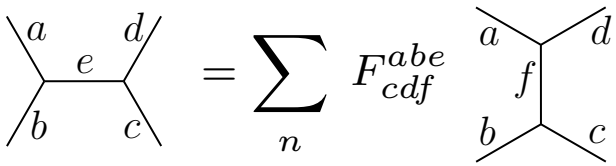
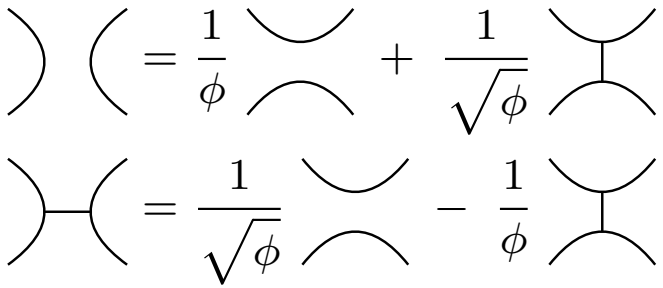




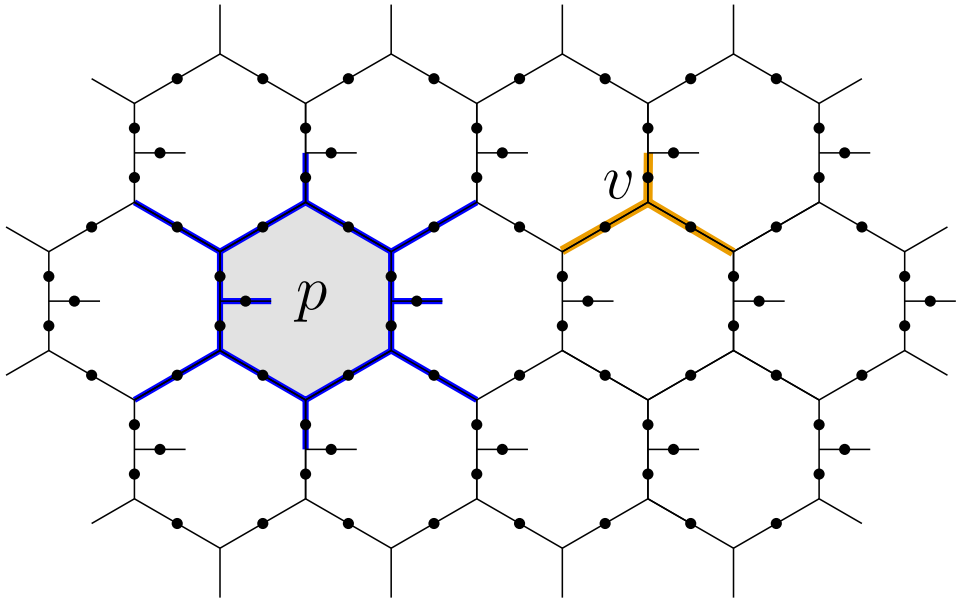


The extended Levin-Wen model

$$H_\Lambda = -\sum_v Q_v - \sum_p B_p$$

$$Q_v \left| \begin{smallmatrix} & & j & & \\ & i & & k & \end{smallmatrix} \right\rangle = \delta_{ijk} \left| \begin{smallmatrix} & & j & & \\ & i & & k & \end{smallmatrix} \right\rangle$$

$$B_p = \frac{1}{\mathcal{D}^2} \sum_s d_s O_p^s$$



- Localized excitations follow DFIB anyonic statistics: $\{\mathbf{1}\mathbf{1}, \mathbf{1}\tau, \tau\mathbf{1}, \tau\tau\}$

- Inside $\mathcal{H}_{\mathrm{s.n.}} = \{|\psi\rangle \mid \prod_v Q_v |\psi\rangle = |\psi\rangle\}$, the anyon charge of a plaquette is a well-defined quantum-number.

The tube algebra

$$O_{x\alpha y\beta}\left|\begin{array}{l}\\ \\ \\ \\ \\ \end{array}\right\rangle = \delta_{x,x'}\left|\begin{array}{l}\\ \\ \\ \\ \\ \end{array}\right\rangle$$

$$= \delta_{x,x'}\frac{v_\alpha v_\beta}{v_y}\sum_{k_1,\dots,k_{r+1}} F^{j_1 j_{r+1} x}_{\alpha\beta k_{r+1}}\left(\prod_{\nu=1}^{r} F^{m_\nu j_\nu j_{\nu+1}}_{\alpha k_{\nu+1} k_\nu}\right) F^{k_{r+1} k_1 y}_{\alpha j_1 \beta}\left|\begin{array}{l}\\ \\ \\ \\ \\ \end{array}\right\rangle$$

Central idempotents project on anyonic sectors

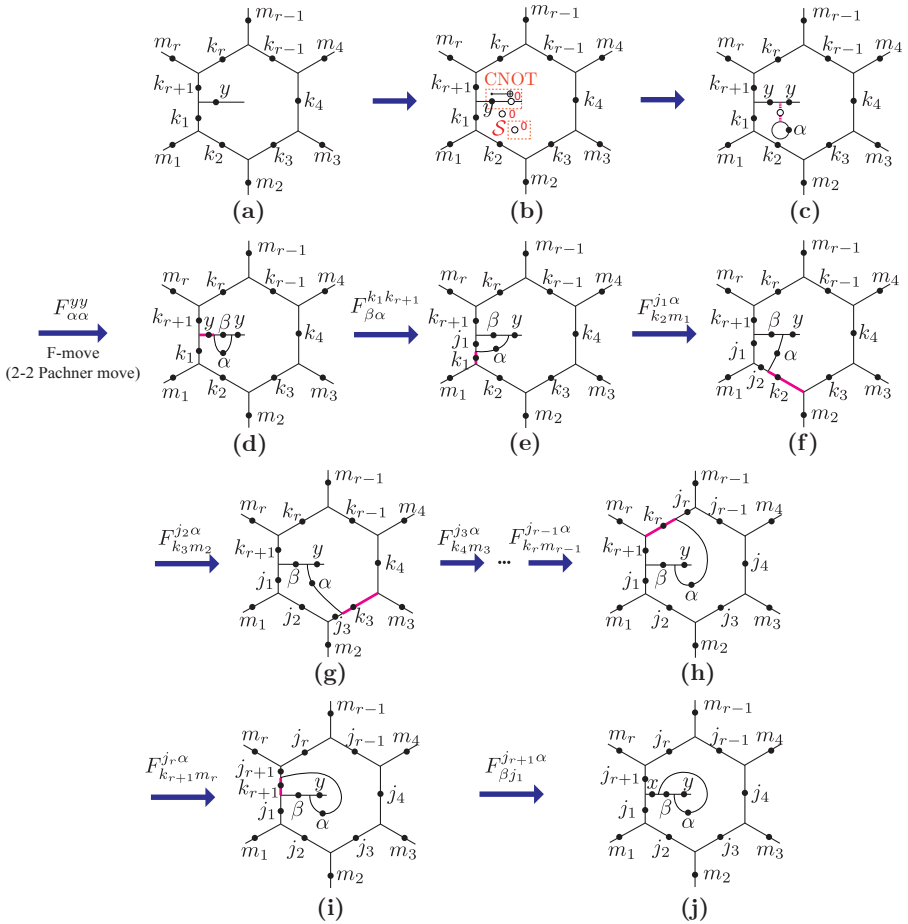
$$B_p = \mathcal{P}^{\mathbf{1}\mathbf{1}} \sim$$

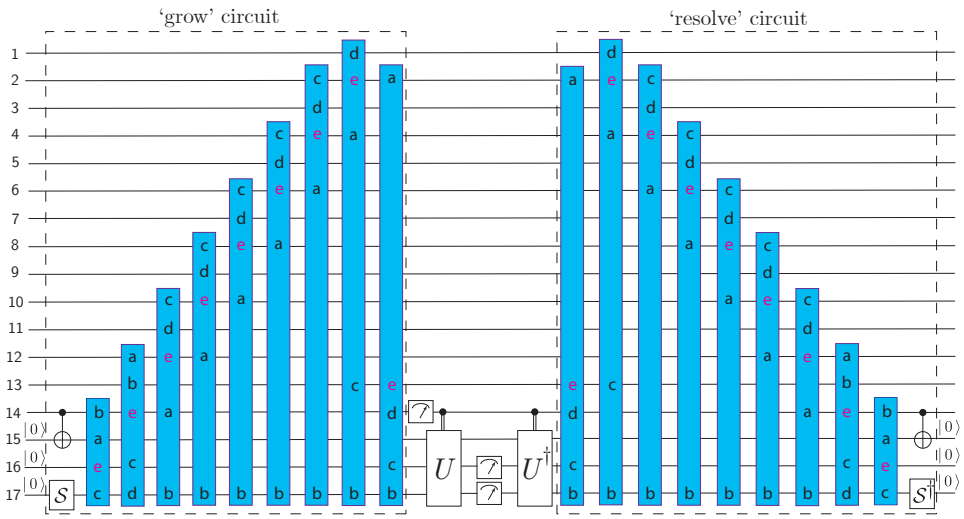
$$\mathcal{P}^{\mathbf{1}\tau} \sim + \sqrt{\phi}\mathrm{e}^{-3\pi\mathrm{i}/5} + \mathrm{e}^{4\pi\mathrm{i}/5}$$

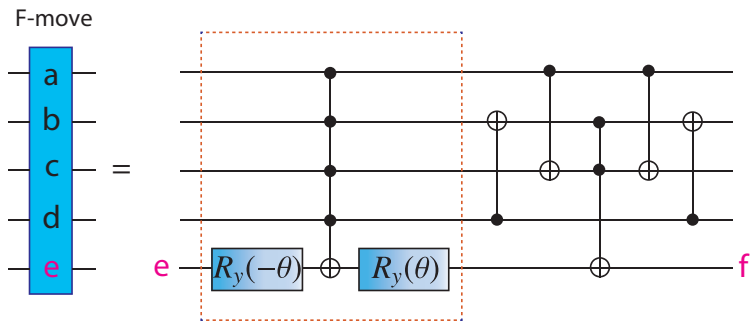
$$\mathcal{P}^{\tau\mathbf{1}} = \overline{\mathcal{P}^{\mathbf{1}\tau}}$$

$$\mathcal{P}^{\tau\tau} \sim \phi^2 - \phi + \phi + \frac{1}{\sqrt{\phi}} + (1+\frac{1}{\phi})$$

Measuring anyon charges



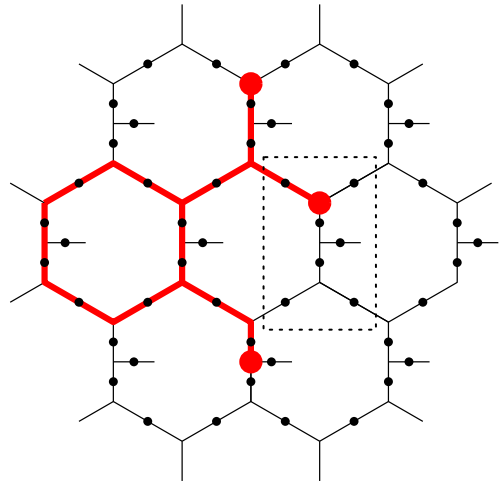




Vertex correction

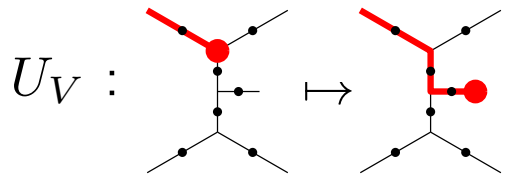
Some errors σ^e can take the initial state out of the string-net subspace:

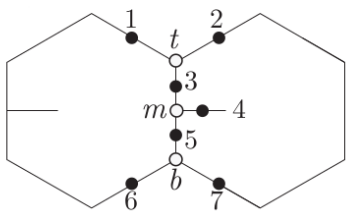
$$[\sigma^e, \prod_v Q_v] \neq 0 \quad \Rightarrow \quad \sigma^e |\Psi_0\rangle \notin \mathcal{H}_{\text{s.n.}}$$

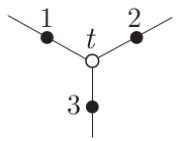


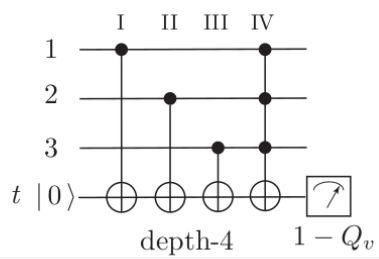
After measuring all Q_v projectors with outcomes V the state is returned to $\mathcal{H}_{\text{s.n.}}$ by unitary U_V conditioned on V

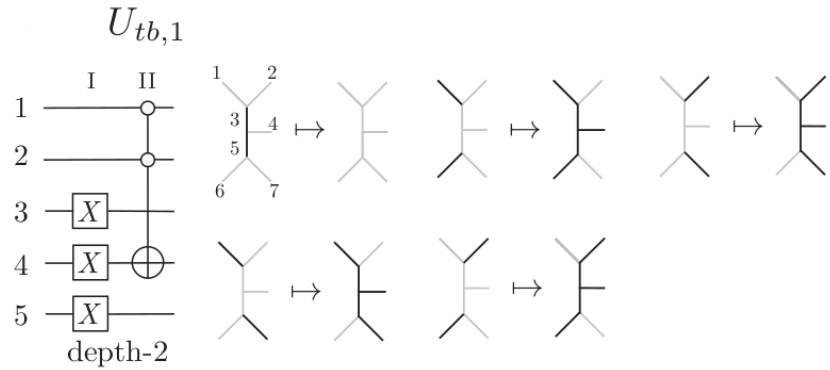
$$U_V |\Psi_V\rangle \in \mathcal{H}_{\text{s.n.}}$$











Error correction protocol

Apply noise

Measure vertex projectors + tails

Apply vertex correction

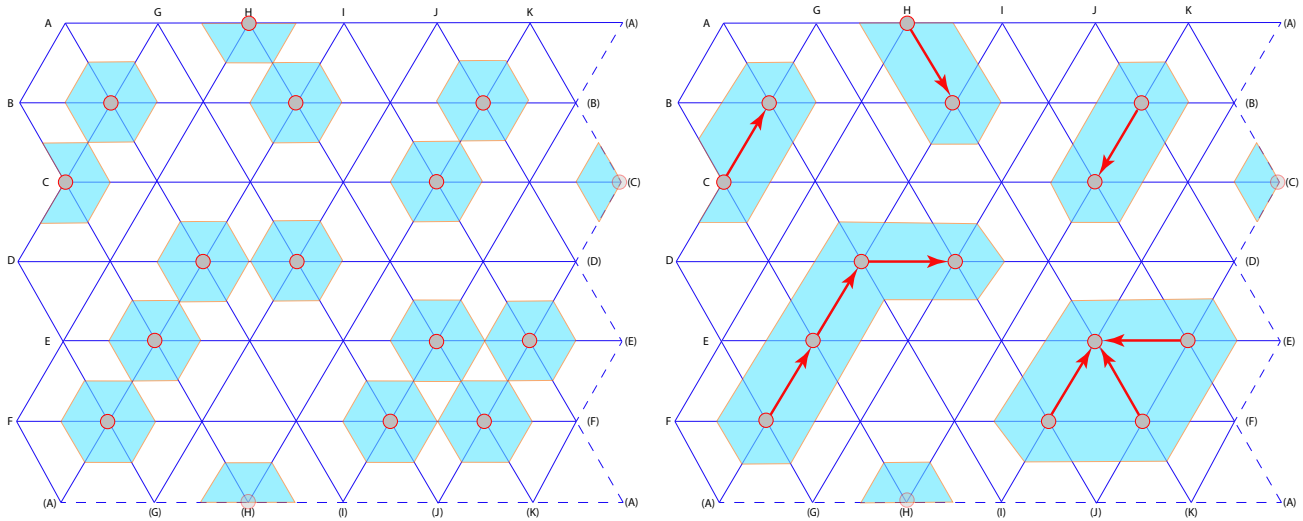
Measure anyon charge in all plaquettes

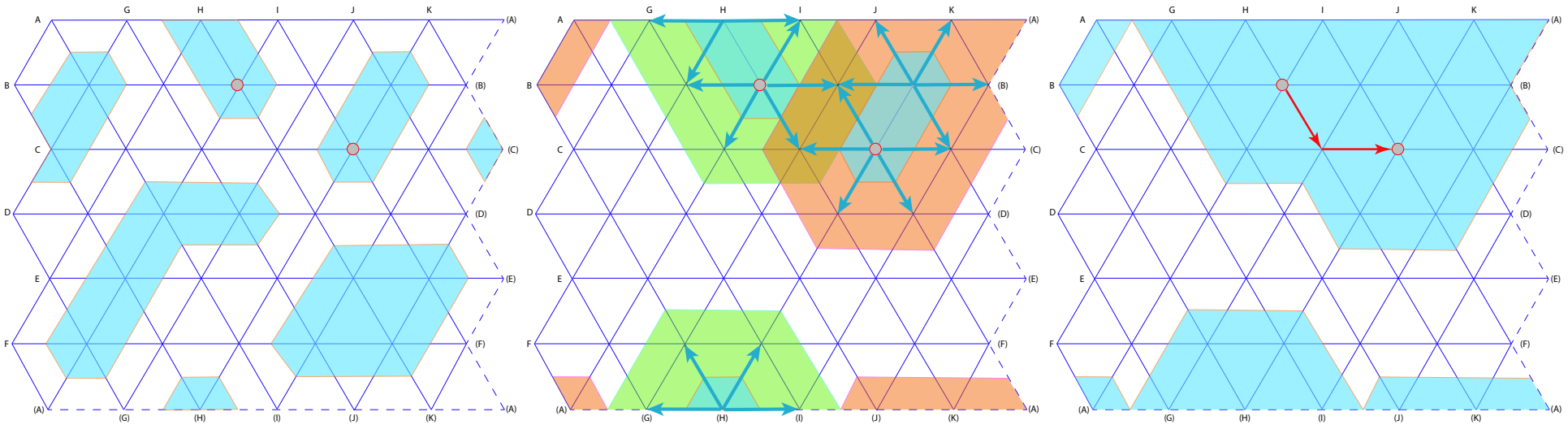
while syndrome is nonempty

1. Fuse pairs of anyons as dictated by decoding algorithm
2. Measure anyon charges

The clustering decoder

Decoding graph:



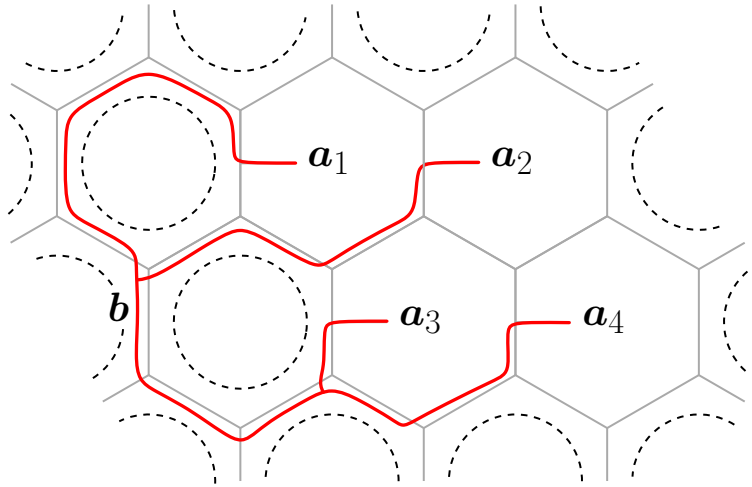


Outline

Translating qubit errors to anyonic processes

- Non-stabilizer code $\rightarrow$ Pauli noise is not natural here!
- Fusion states of anyons in plaquettes $\leftrightarrow$ unique lattice qubit states:

$$|\vec{a}, \vec{b}\rangle =$$



- The states $\{|\vec{a}, \vec{b}\rangle\}$ form an orthonormal basis of $\mathcal{H}_{\text{s.n.}}$
- Express joint action of noise in fusion basis:

vertex measurement

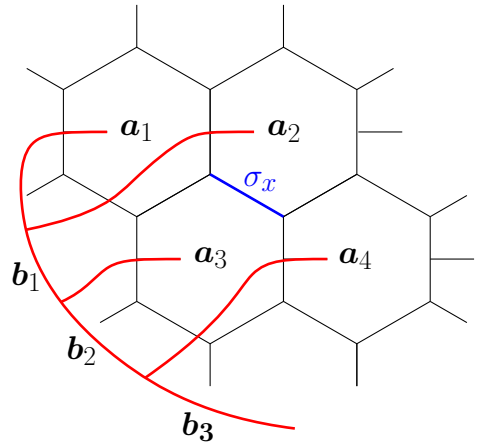
vertex correction

$$\langle \vec{a}', \vec{b}' |\, U_V \; P_V \; \sigma^e \; |\vec{a}, \vec{b}\rangle$$

$$\langle \vec{a}', \vec{b}' |\, \sigma^e \; P_V \; \sigma^e \; |\vec{a}, \vec{b}\rangle$$

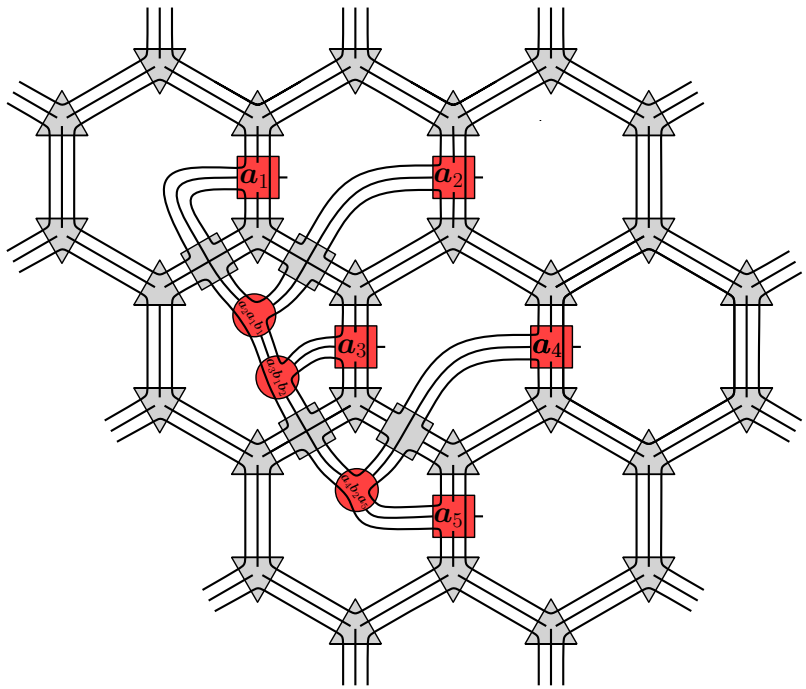
Computing the matrix elements

- $U_V P_V \sigma^e |\vec{a}, \vec{b}\rangle$ and $\sigma^e P_V \sigma^e |\vec{a}, \vec{b}\rangle$ only depend on local properties of $|\vec{a}, \vec{b}\rangle$



- Tensor network representation of anyonic fusion states:

$|\vec{a}, \vec{b}\rangle =$



Computing the matrix elements

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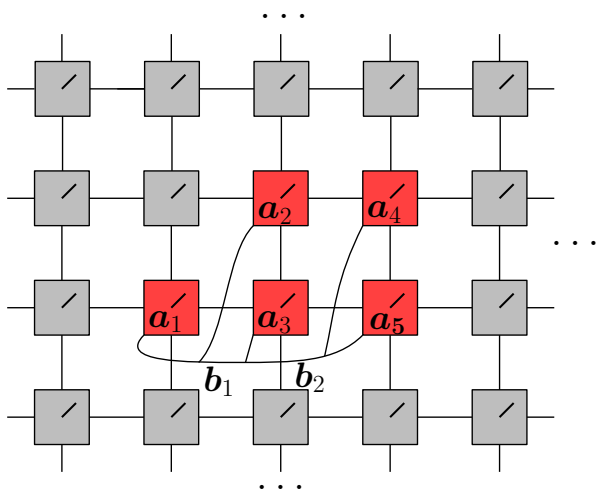
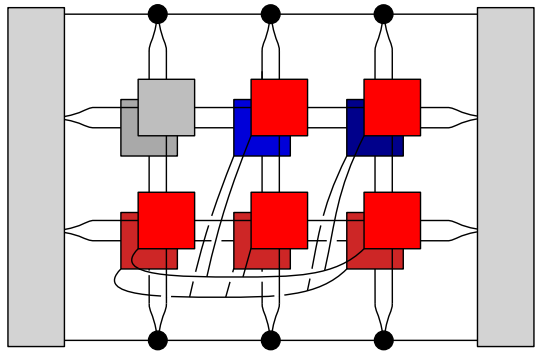
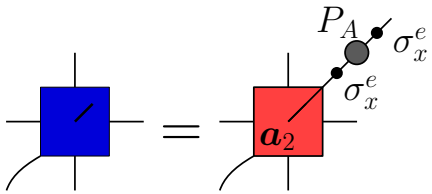


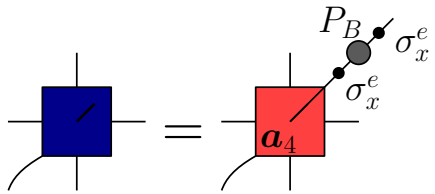
$$\langle \vec{a}', \vec{b}' | \sigma_x^e P_V \sigma_x^e | \vec{a}, \vec{b} \rangle =$$



where



and



Assumptions for the classical simulation

- Noise = random Pauli errors
- Total number of errors $T \sim \mathrm{Poisson}(|E|\, p)$
- Perfect measurements
- Perfect recovery operations

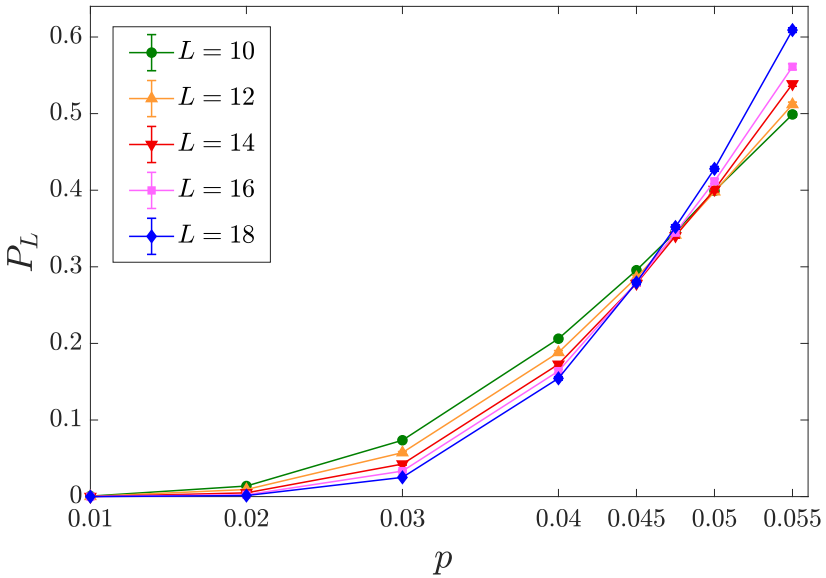
We simulate the dynamics of the system for T timesteps

Individual time step in the simulation

1. An edge e is selected at random
2. A random Pauli operator σ_i^e is applied to the state
3. All vertex projectors $\{Q_v\}$ and tail qubits are measured
4. Depending on the outcome V, vertex correction U_V is applied
5. The anyon charge of all plaquettes is measured.

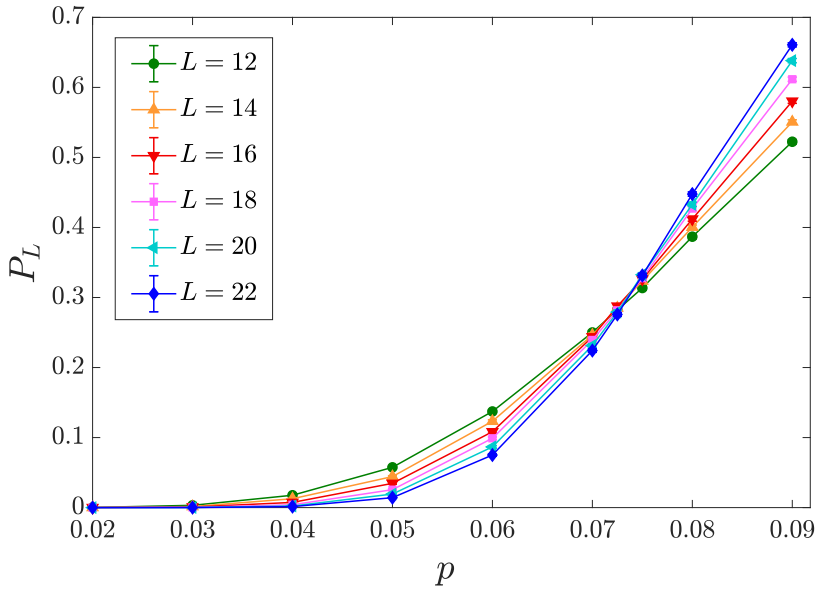
Results

Clustering decoder, depolarizing noise



$p_c \approx 0.0475$

Clustering decoder, dephasing noise



$p_c \approx 0.0725$

Comparable to surface code thresholds under similar assumptions!

Outline

Summary

- Active Non-Abelian QEC $\rightarrow$ system of qubits
- Code space defined using modified Levin-Wen Hamiltonian
- 3 ingredients:
  - Quantum error correction
  - Topological quantum field theory $\rightarrow$ tube algebra + anyonic fusion basis
  - Tensor networks $\rightarrow$ translating qubit errors to anyons
- First threshold for 2D code which *natively* supports a universal gate set
- Threshold comparable to surface code threshold under similar assumptions

Outlook

- More natural noise model?
- Simulating fully fault-tolerant setup
- Finding better decoders
- Applying our techniques to different non-Abelian codes (with lower-depth measurement circuits)
- Planar string-net codes
- Road to experimental implementation
  $\rightarrow$ hardware-efficient multi-qubit gates