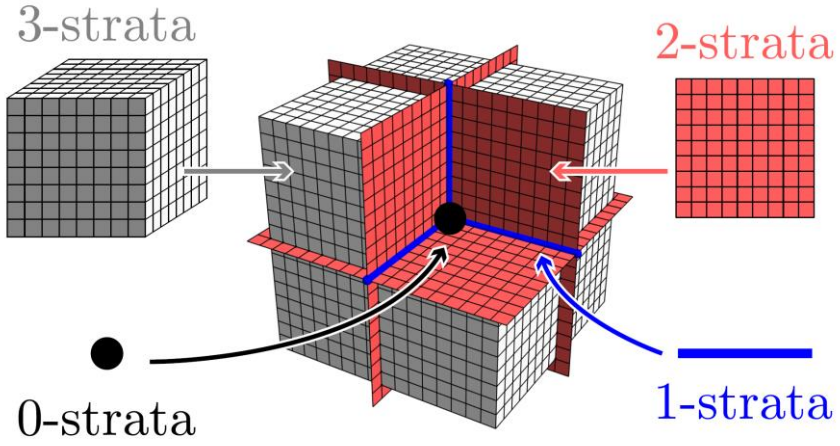
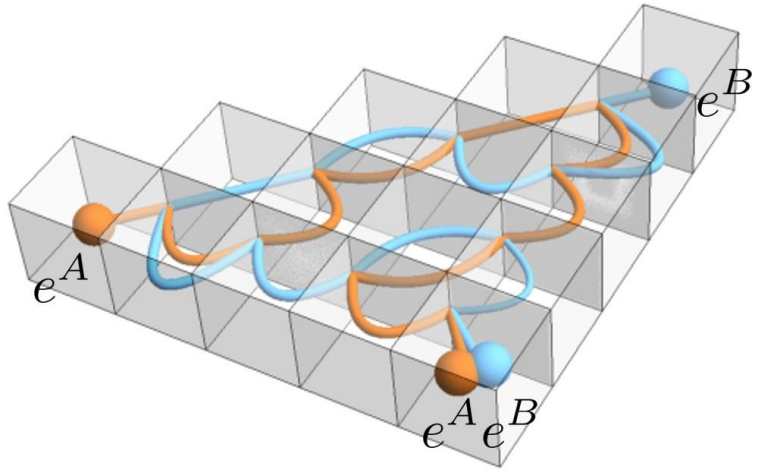
Topological defect networks for fractons of all types

Dominic Williamson

Stanford University





PRR (2020)
2002.05166

Thanks to:

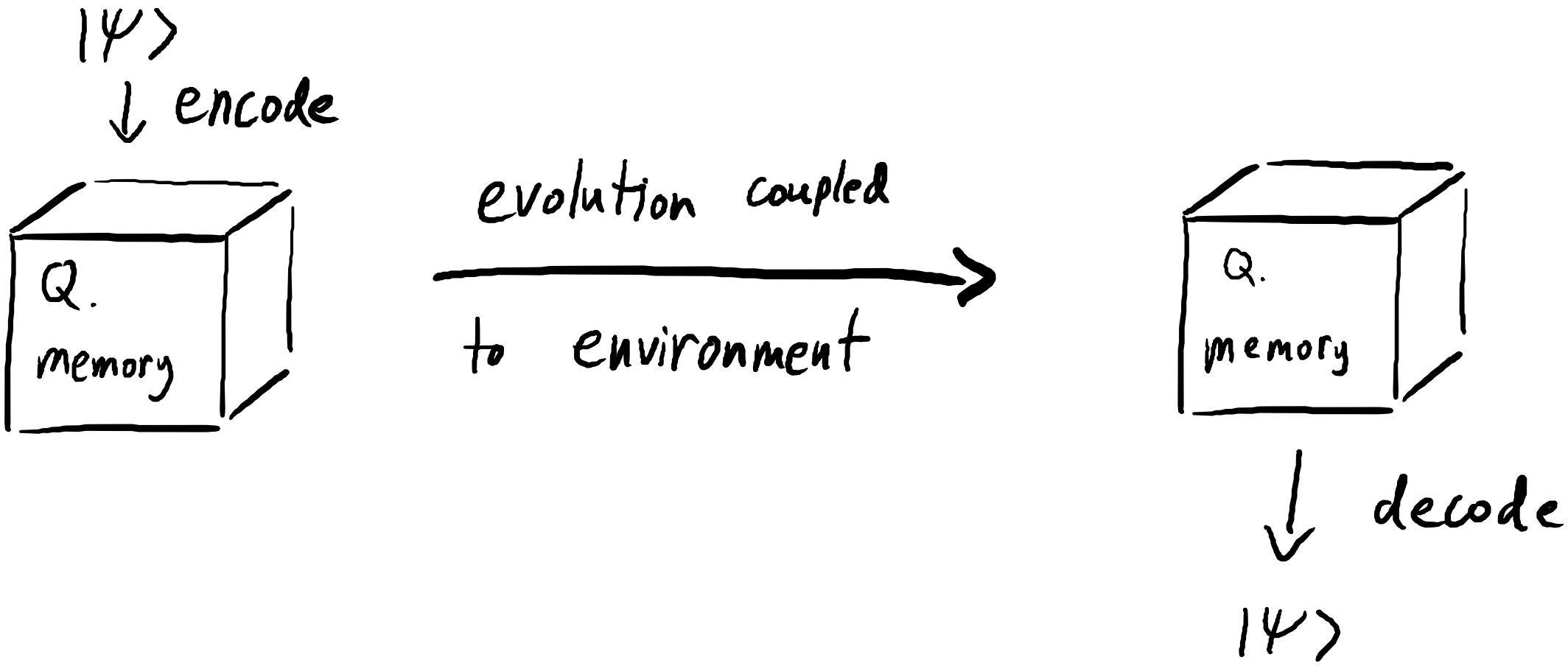
Dave Aasen
KITP/Station Q

Danny Bulmash
UMD

Abhinav Prem
Princeton

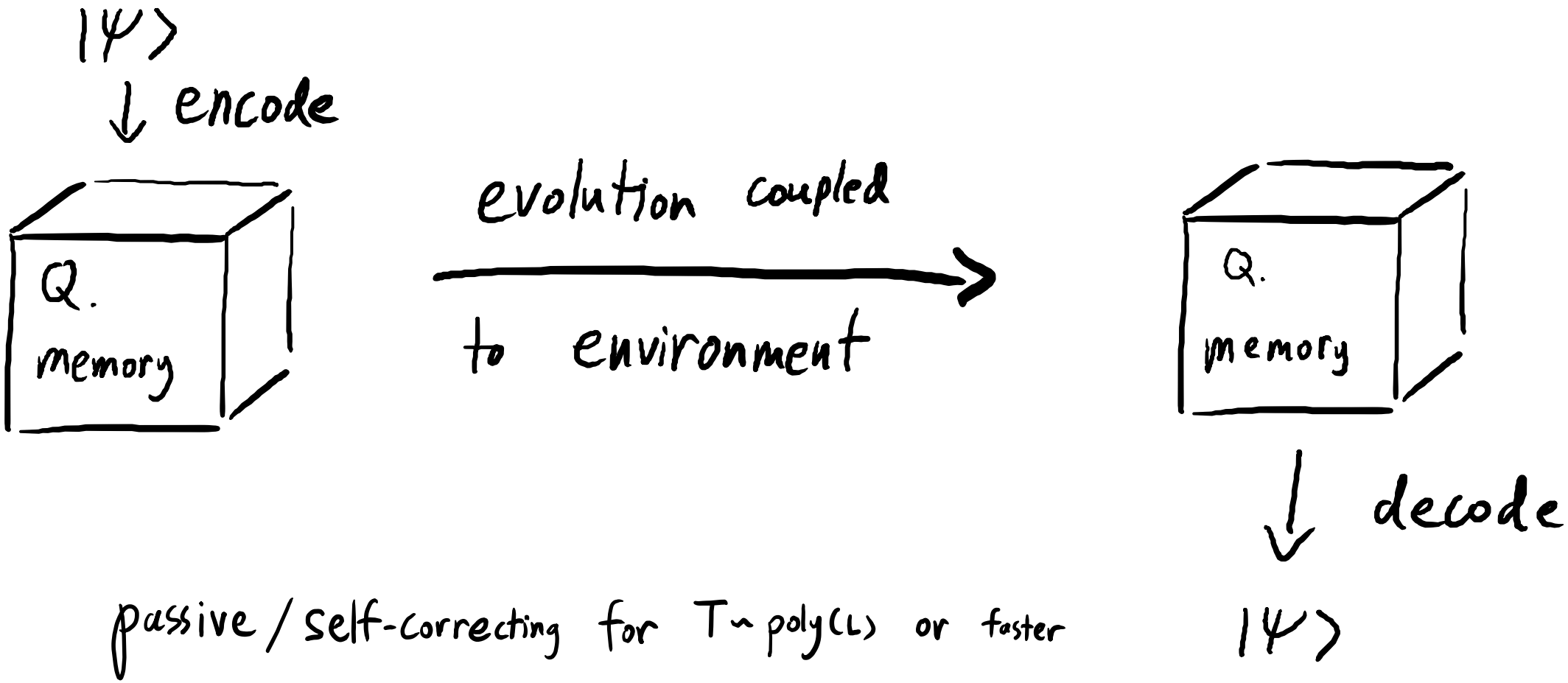
Kevin Slagle
Caltech

Topological quantum memory



Kitaev 97

Topological quantum memory



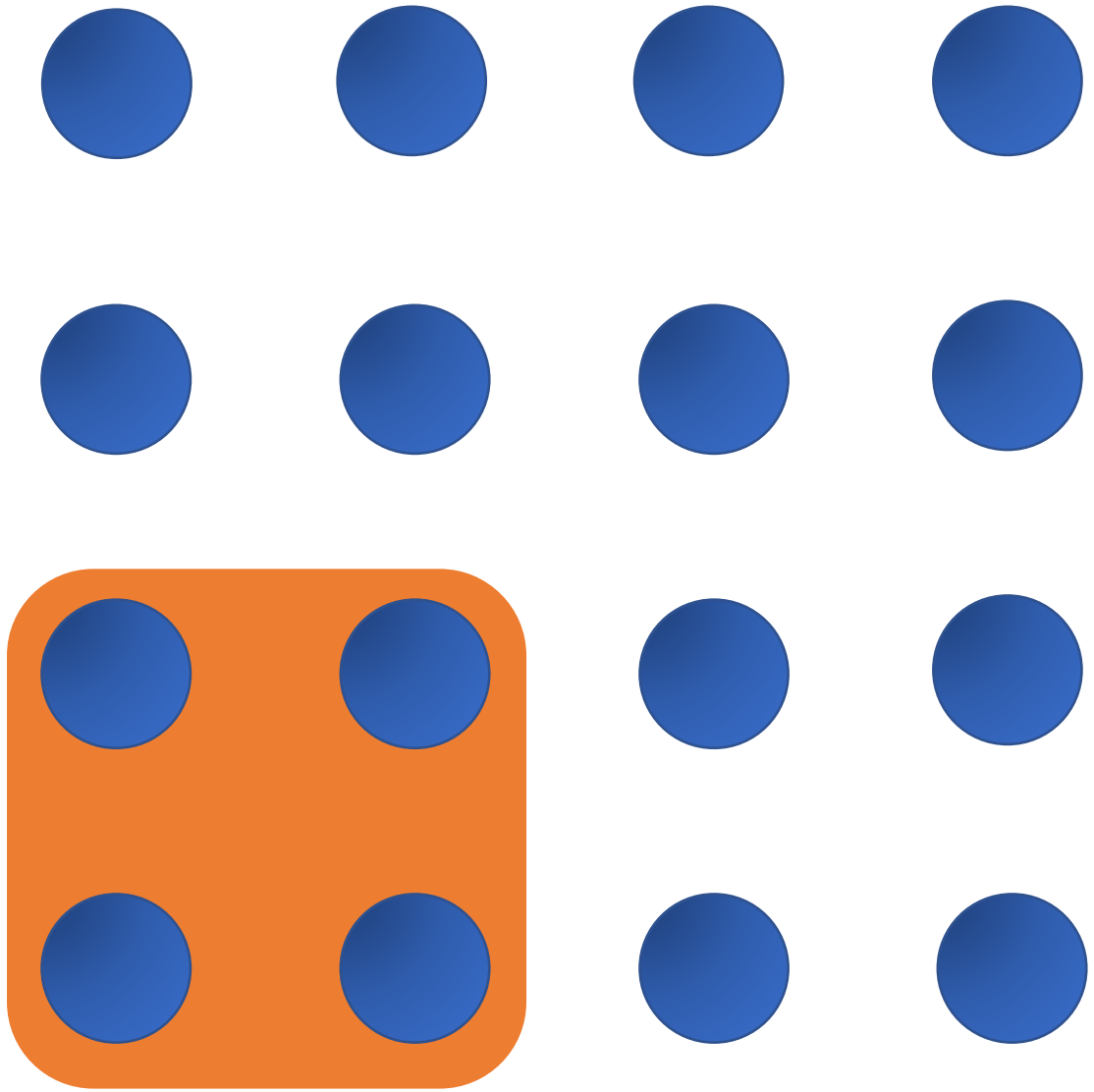
Kitaev 97

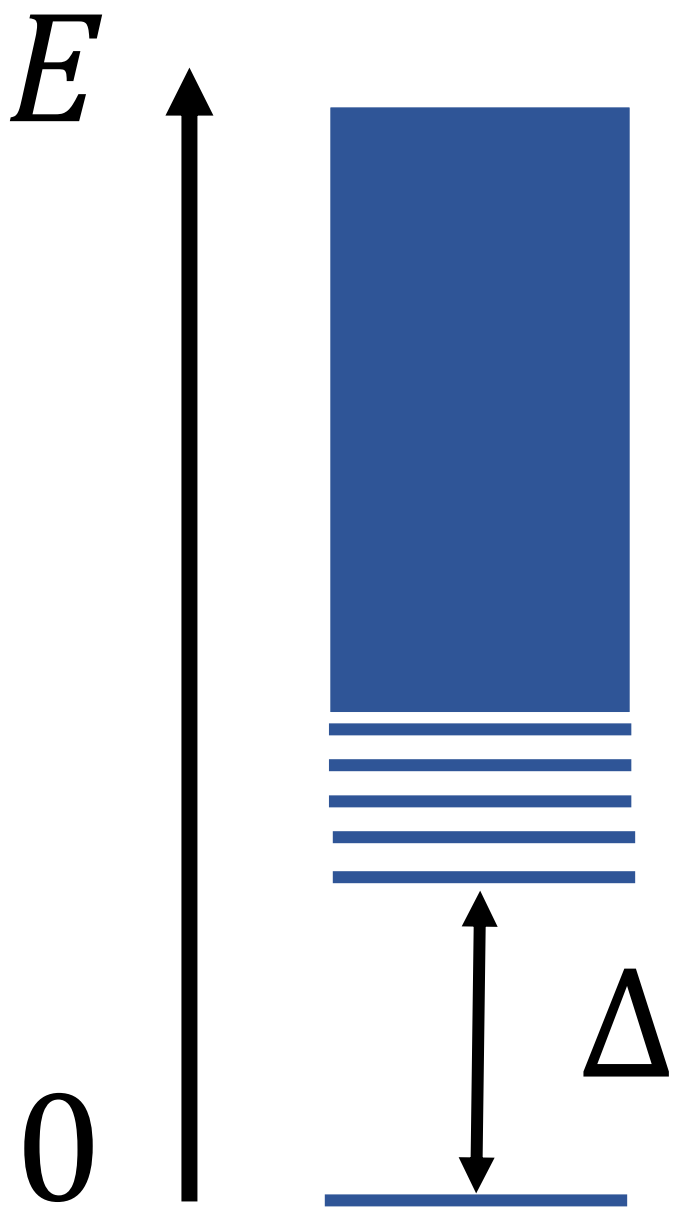
$\mathbb{C}^d$

$\mathbb{H} = \bigotimes_v \mathbb{C}^d$

h_v

$H = \sum_v h_v$




E
0
Δ

$$\Pi \mathcal{O} \Pi = c(\mathcal{O}) \Pi$$

Π-ground space projector

$\mathcal{O}$-local operator

Topological order

Quantum code

$$\Pi \mathrm{E}_\mathrm{b}^\dagger \mathrm{E}_\mathrm{a} \Pi = c_{ab} \Pi$$

Π-code space projector

E_a-error operator

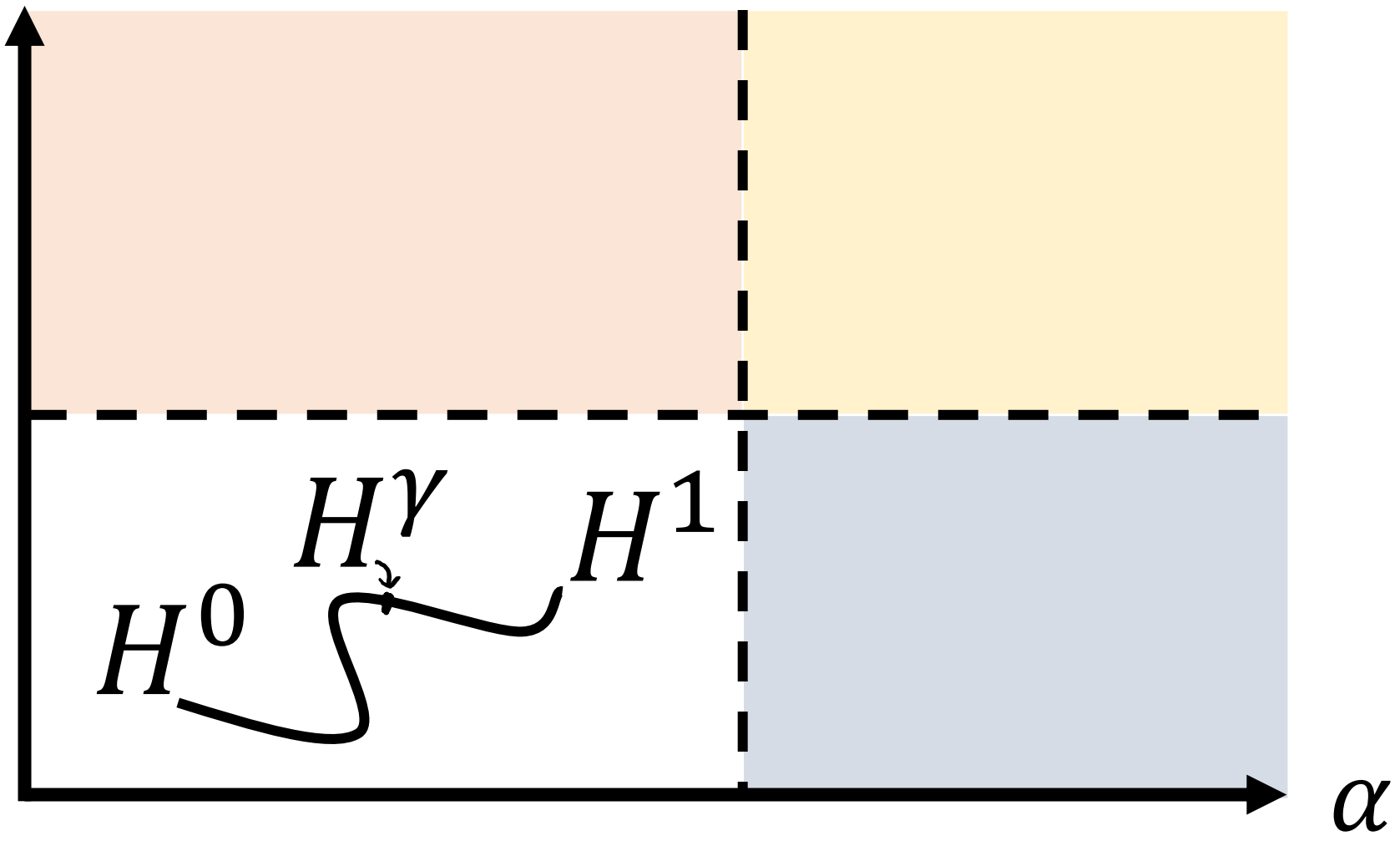
Topological phases of matter

$$H^0 \sim H^1$$

iff $\exists$

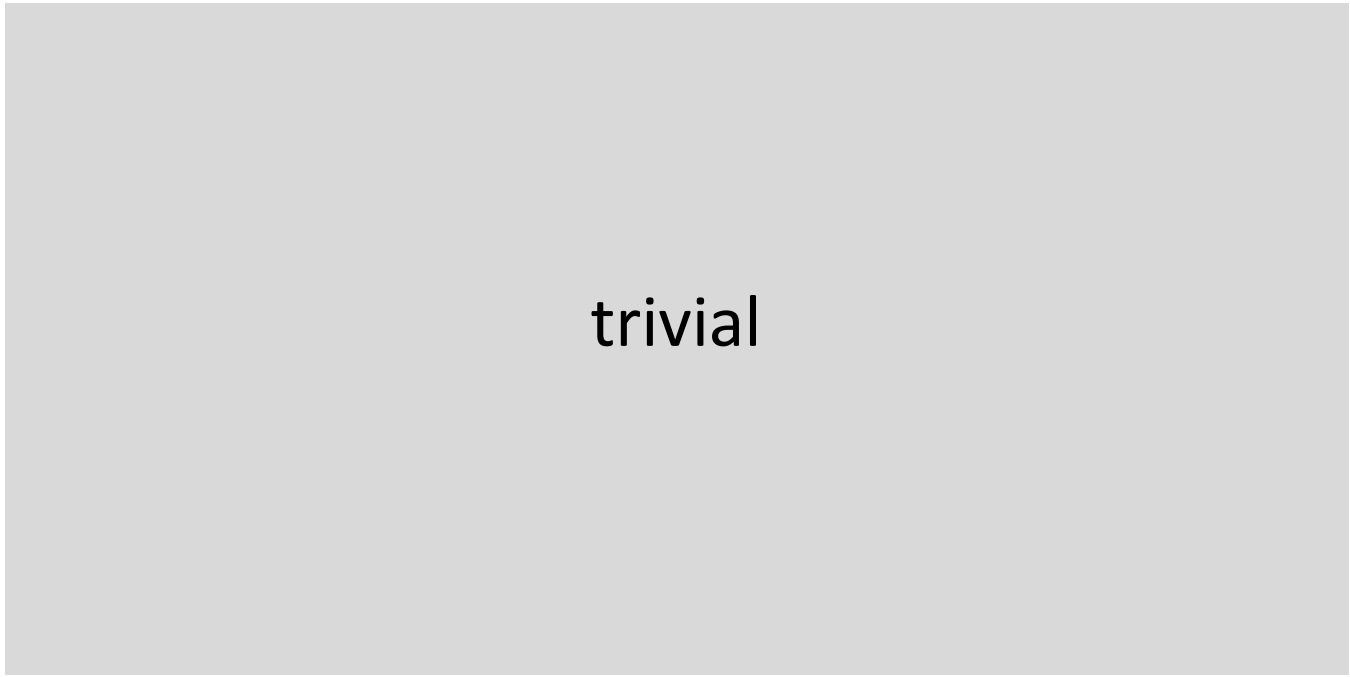
$$H^\gamma = \sum_v h_v^\gamma$$

for $\gamma \in [0,1]$



Topological phases: gotta classify `em all

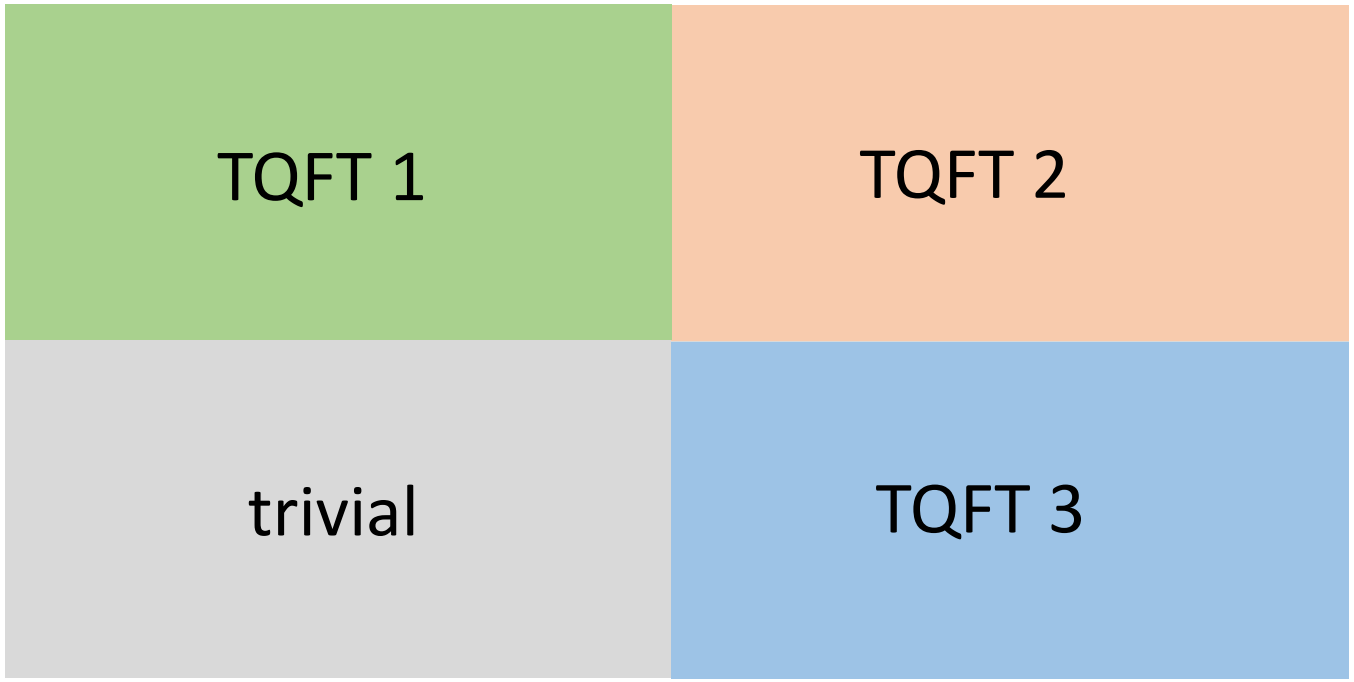
1D:



Chen, Gu, Wen 2011
Schuc, Perez-Garcia, Cirac 2011

Topological phases: gotta classify `em all

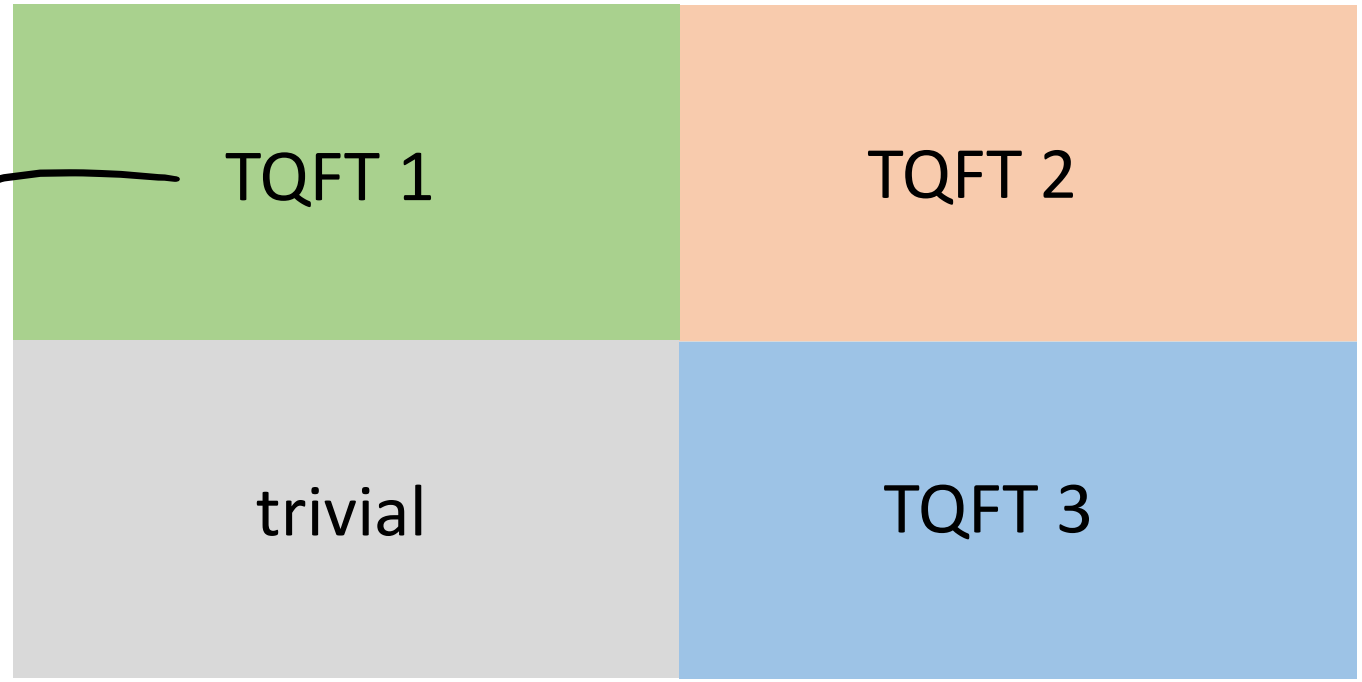
2D:



Witten 1988
Moore, Seiberg 1988
Wen 1990

Topological phases: gotta classify `em all

2D:



Mobile anyons /
String operators
⇒ no self-correction!

Witten 1988
Moore, Seiberg 1988
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Topological phases: gotta classify `em all

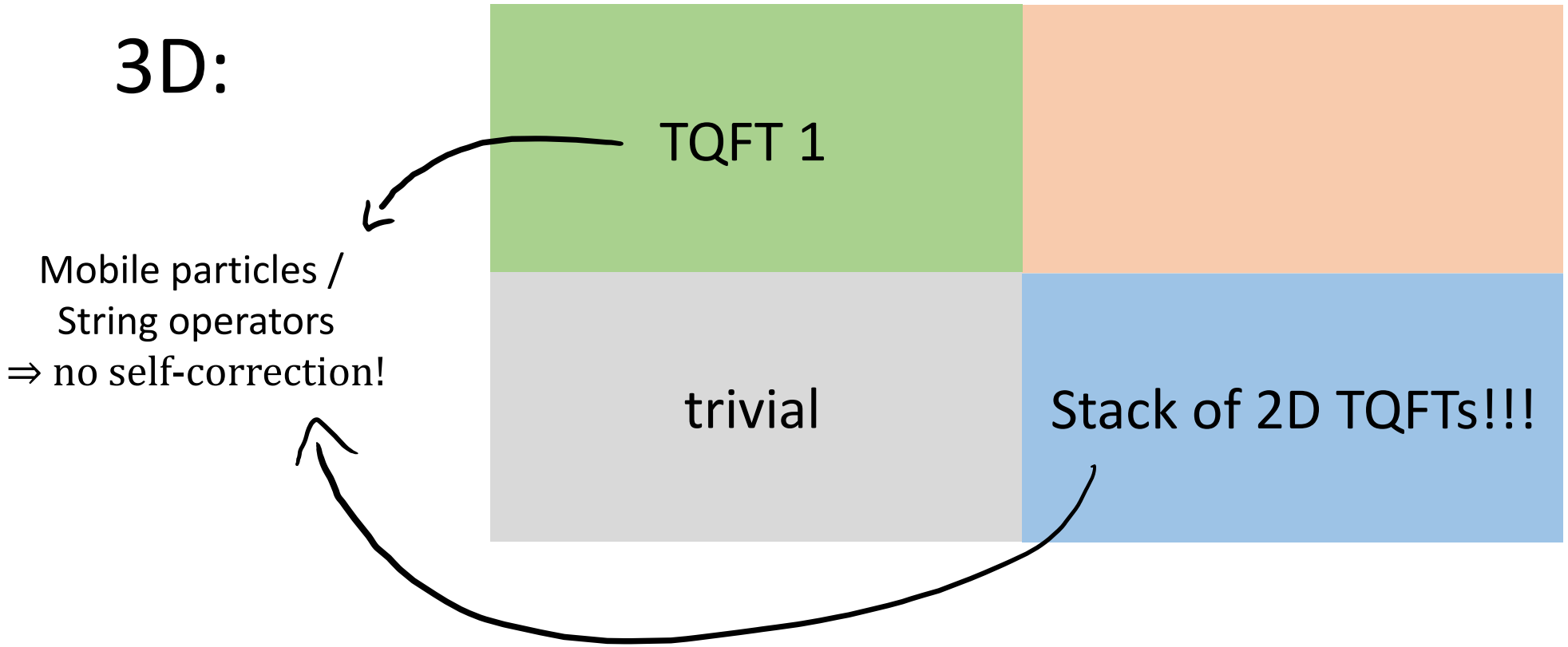
3D:

Topological phases: gotta classify `em all

3D:

Topological phases: gotta classify `em all

3D:



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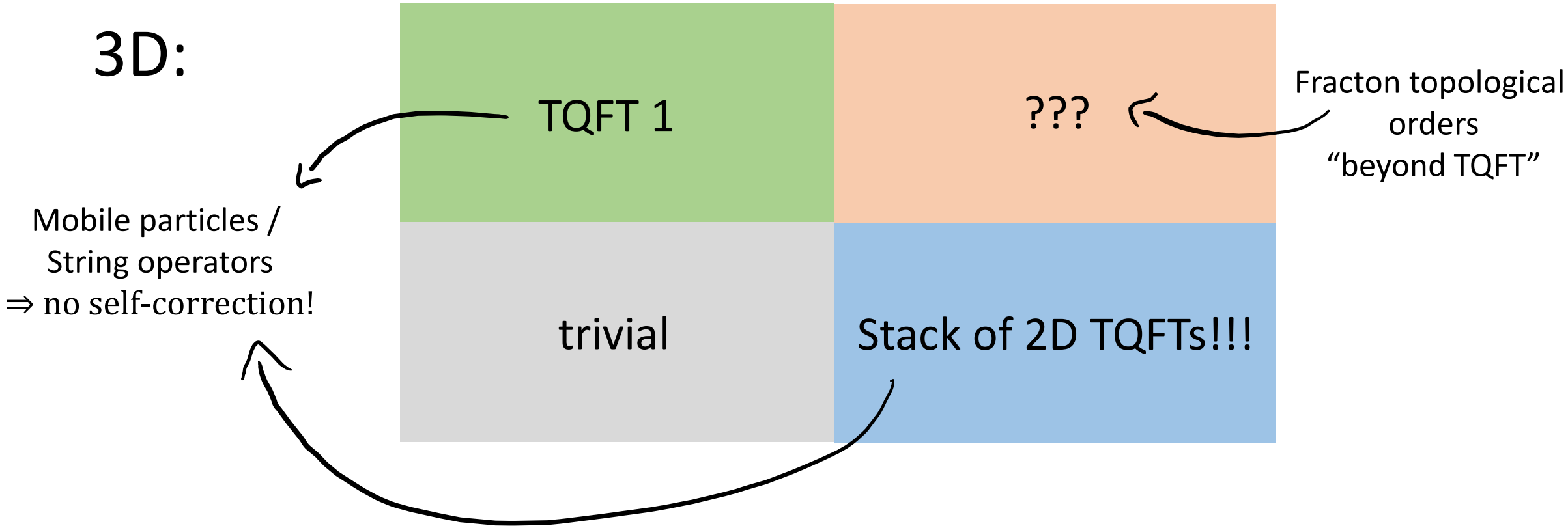
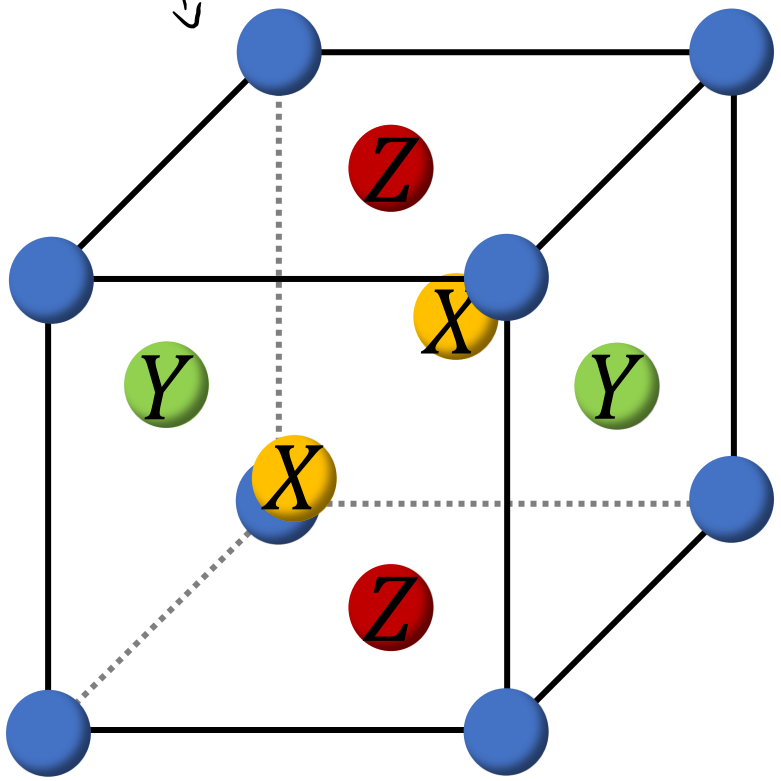


Chamon's model

Chamon 2004

Inspiration: Newman, Moore 1997
Xu, Moore 2003 …

$H = -\sum B$



Log GSD = $\gcd(L_x, L_y, L_z)$

Bravyi, Leemhuis, Terhal 2010

$X/Y/Z = \sigma^x/\sigma^y/\sigma^z$

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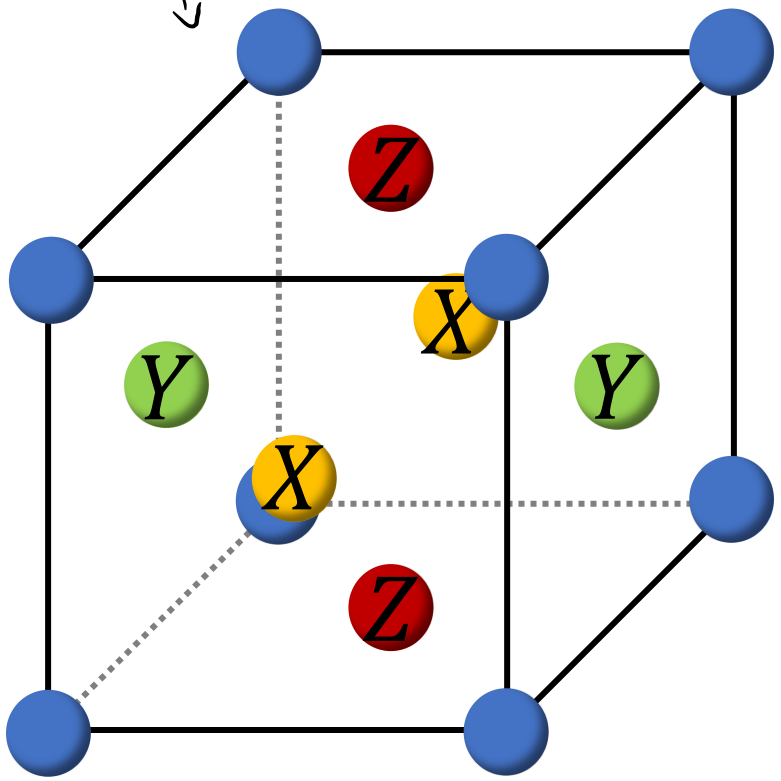

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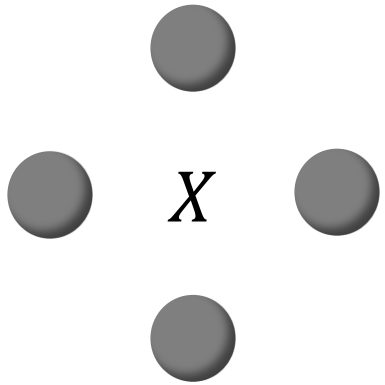

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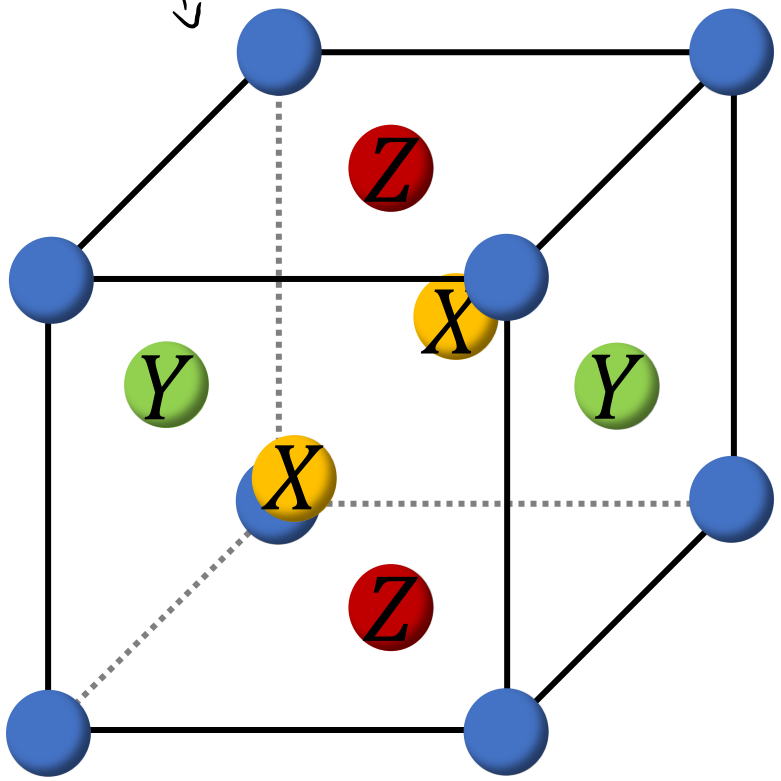

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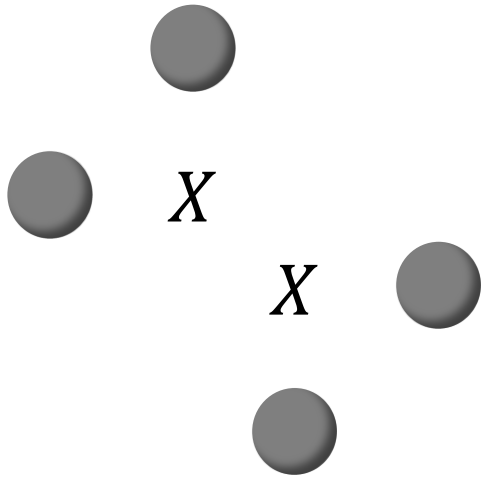

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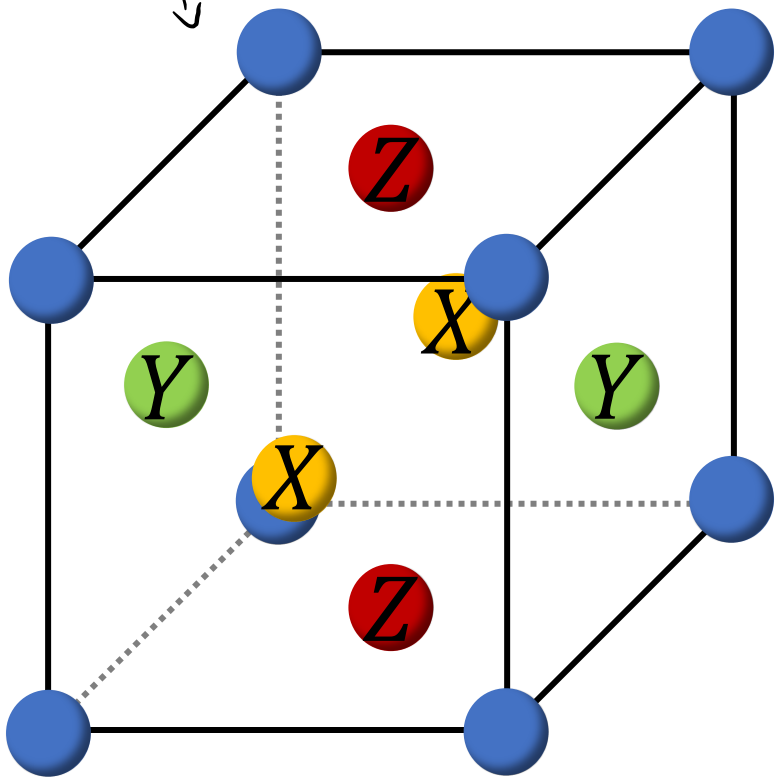

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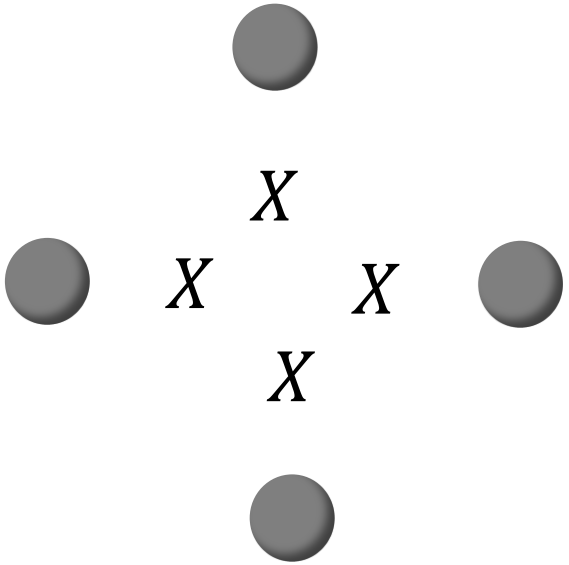

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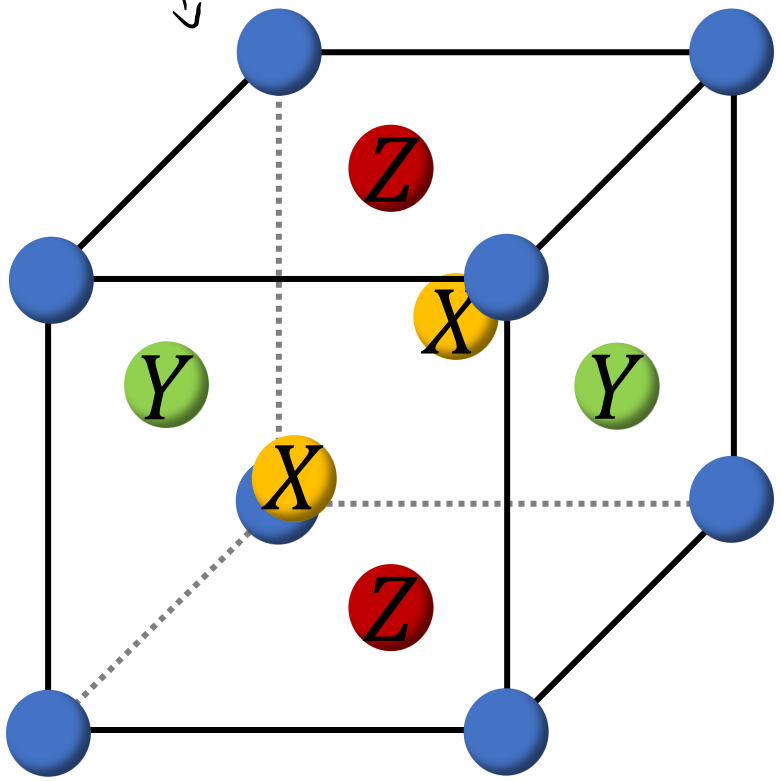

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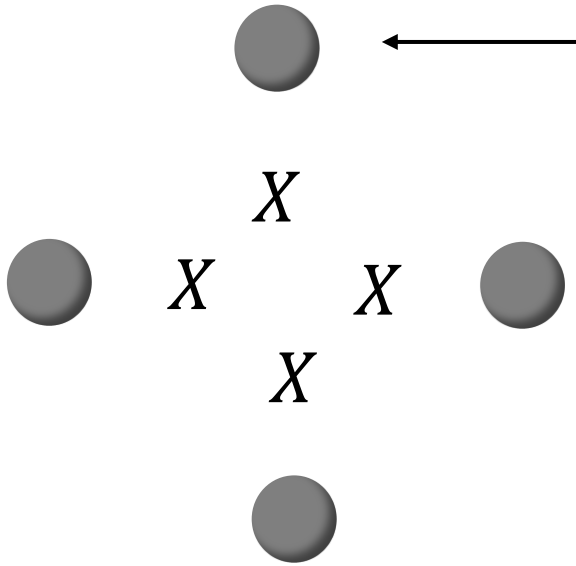

- Fracton

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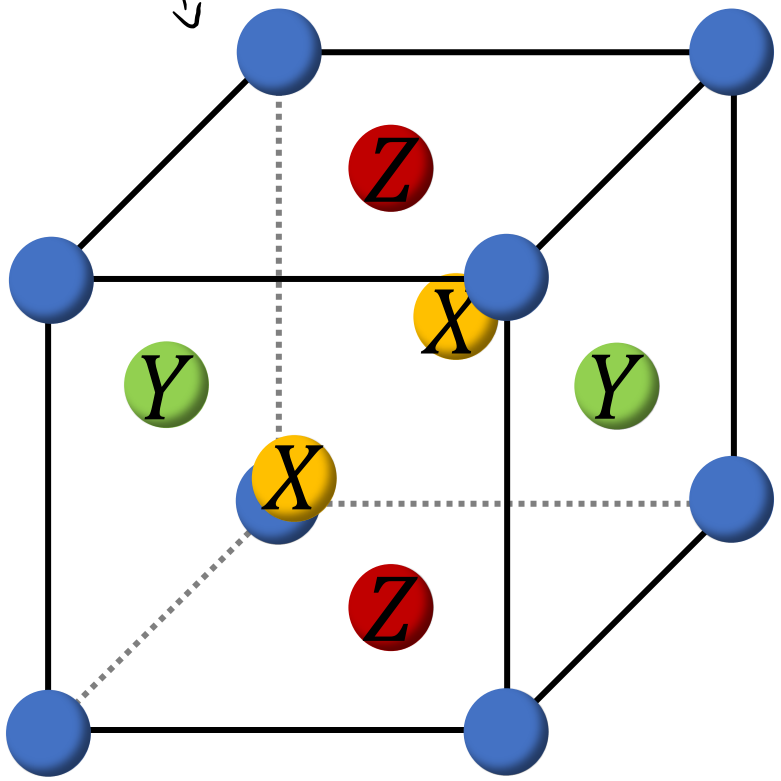

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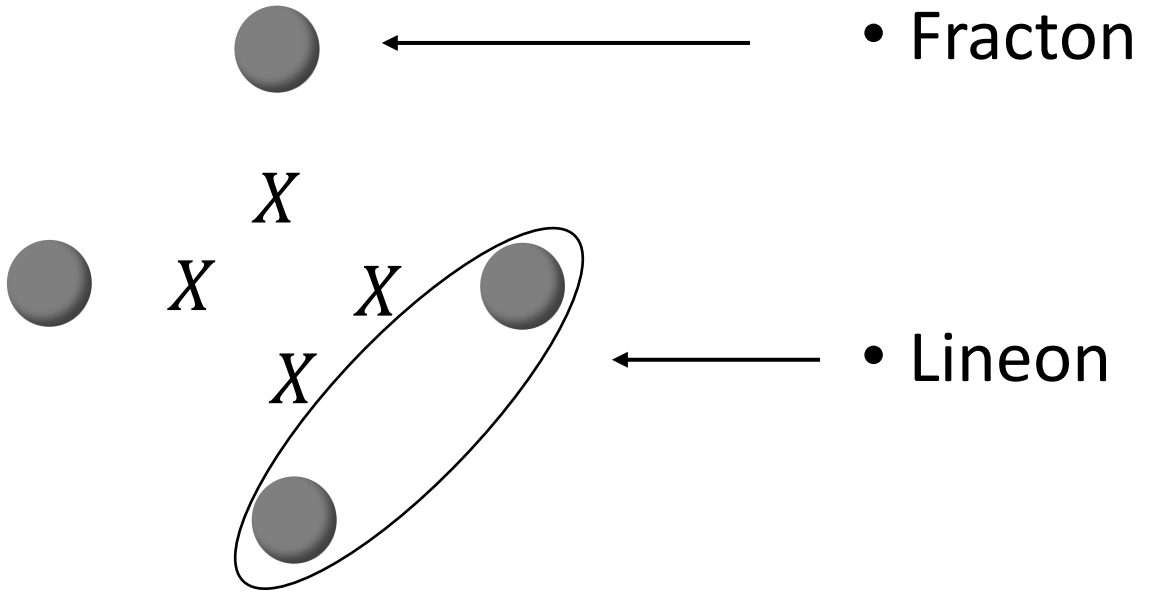

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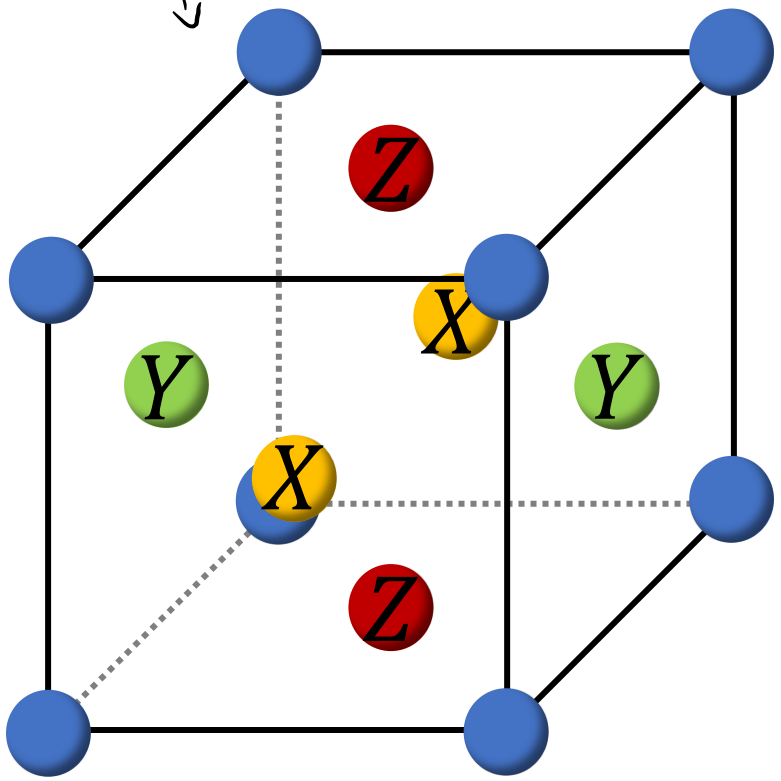

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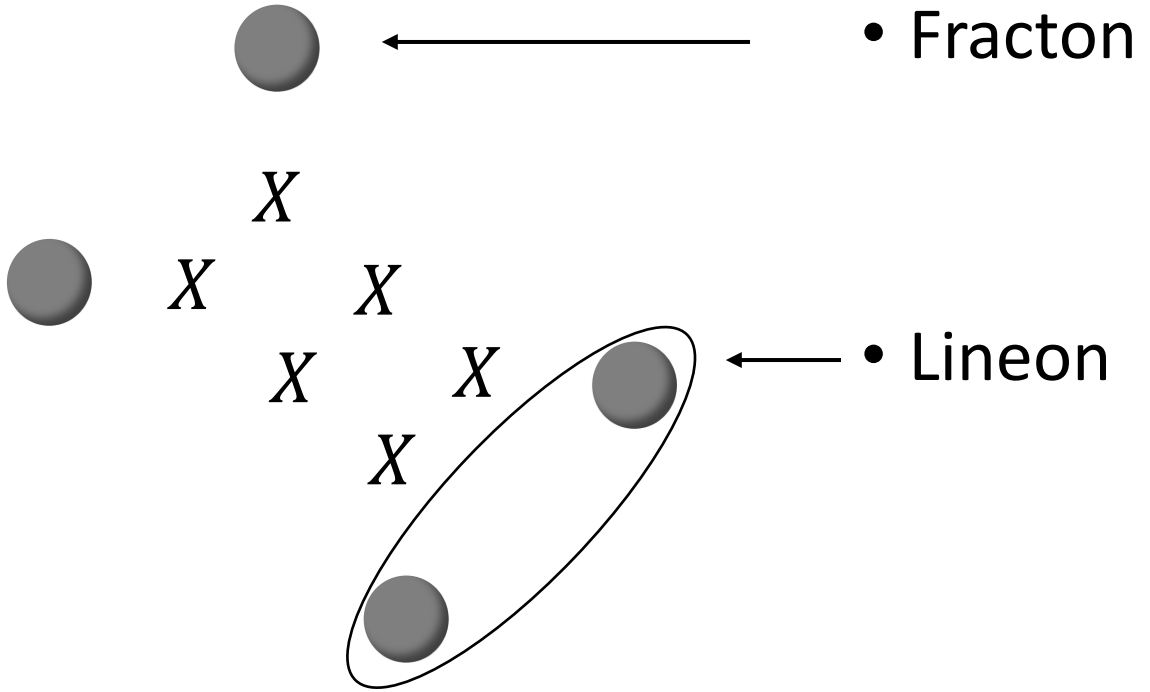

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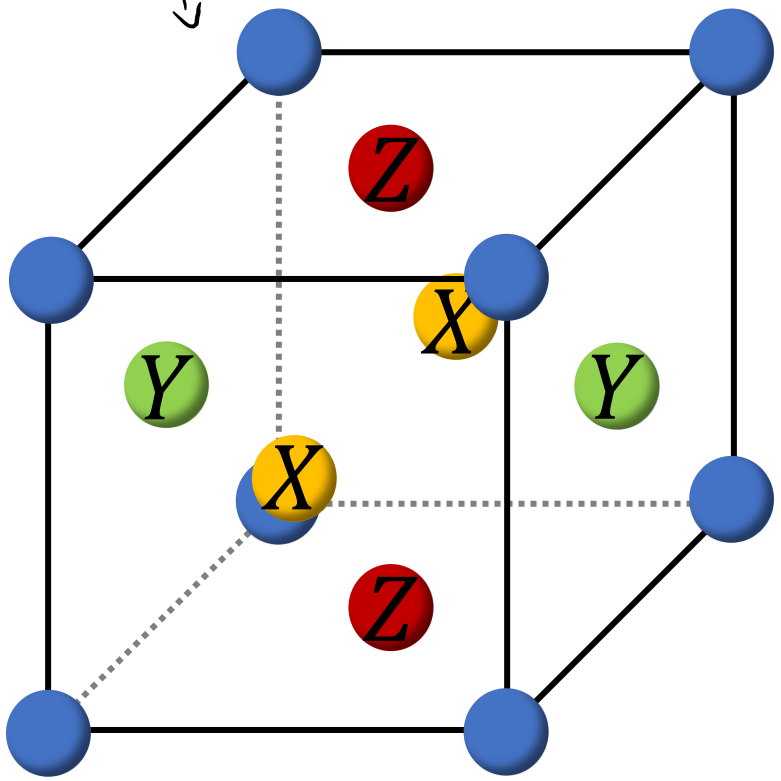

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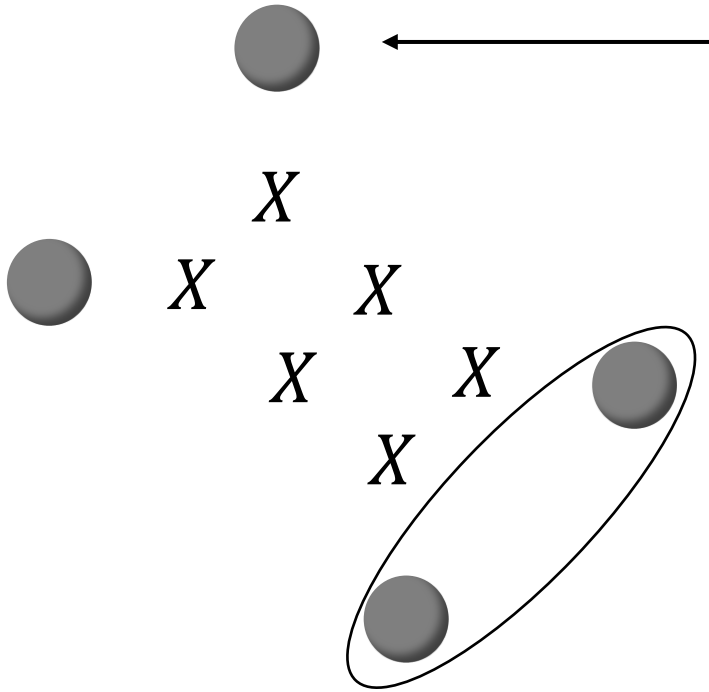

- Fracton
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Chamon's model

a type-I fracton order

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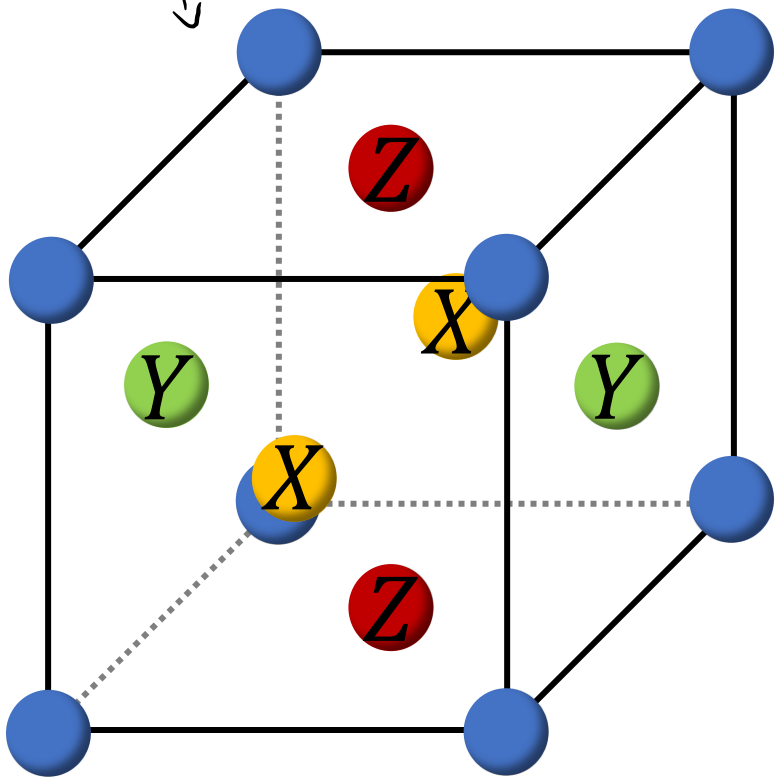

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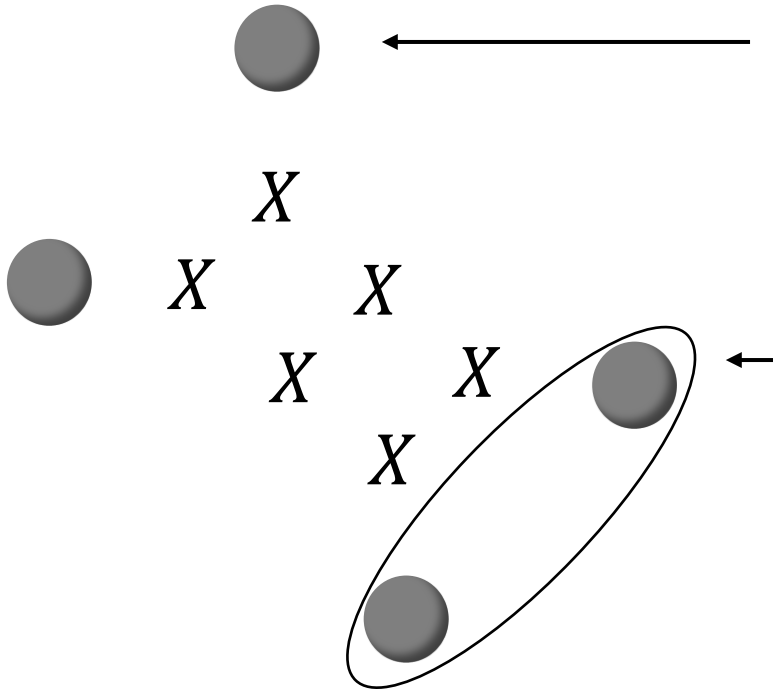
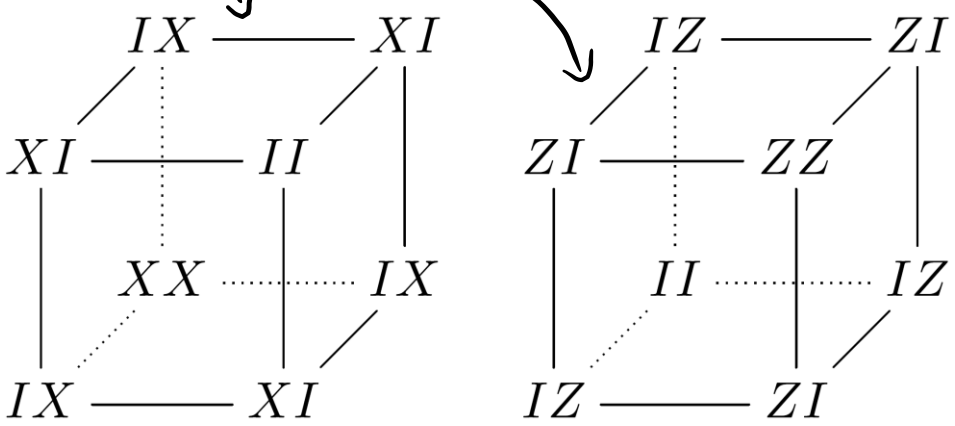


- Fracton
- Lineon
- Planons

Haah's cubic code (1 of 18)

$H = -\sum \mathrm{A_c} - \sum B_c$



Haah 2011

Haah's cubic code (1 of 18)

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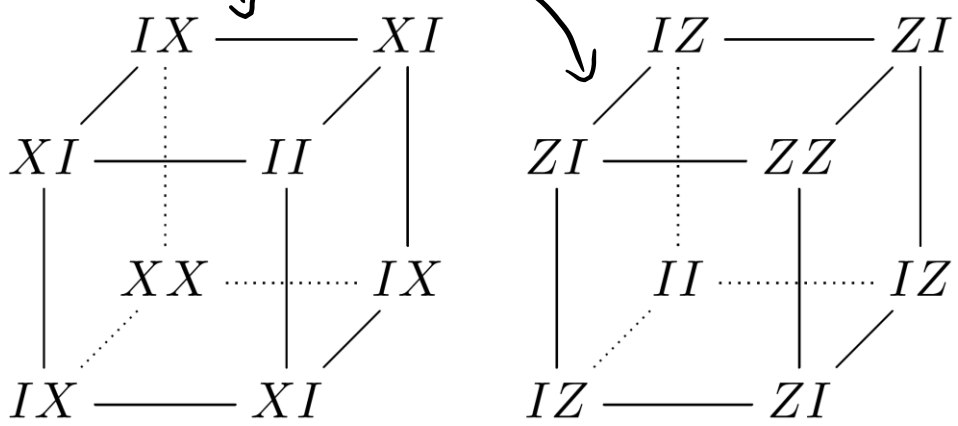

Haah 2011

- No string operators! … a type-II fracton order

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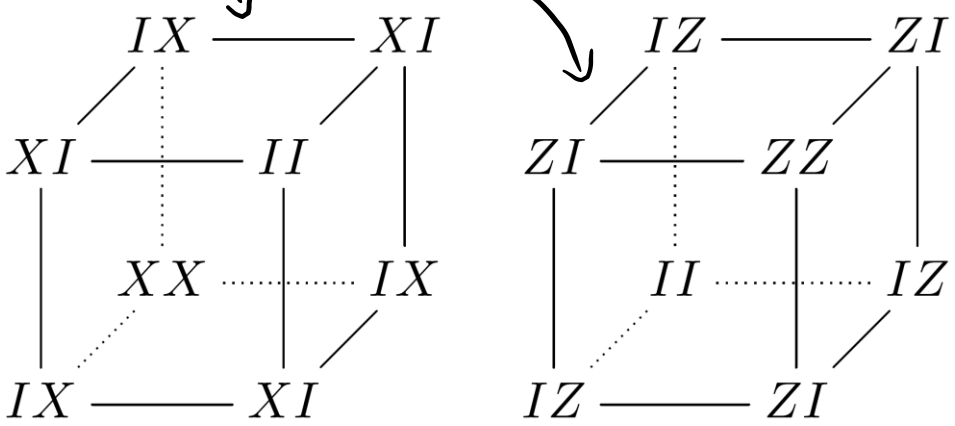

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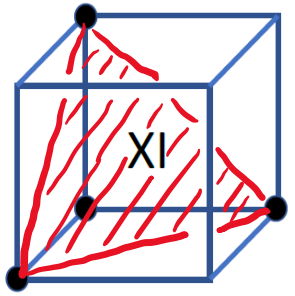
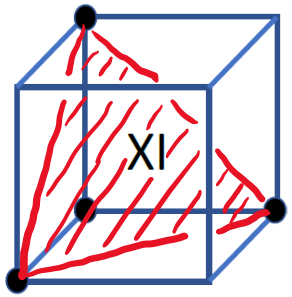
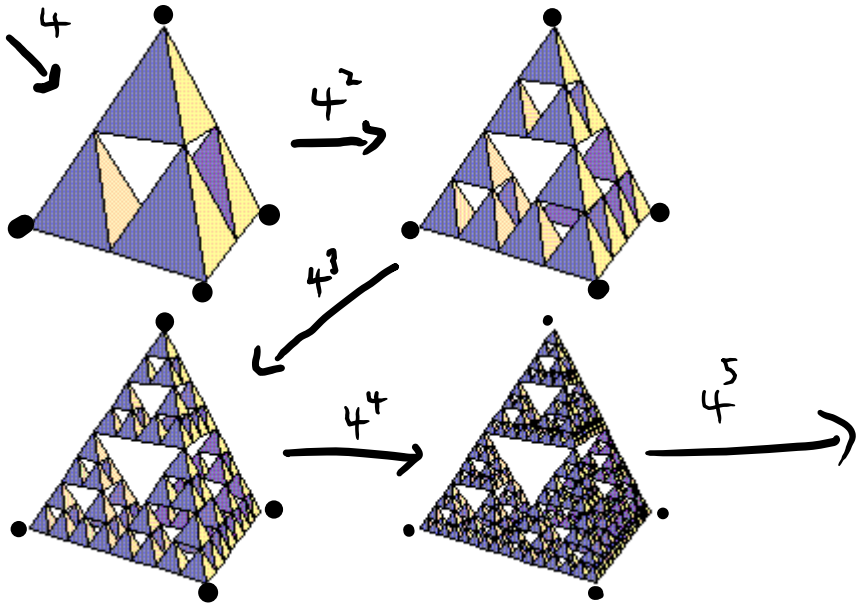
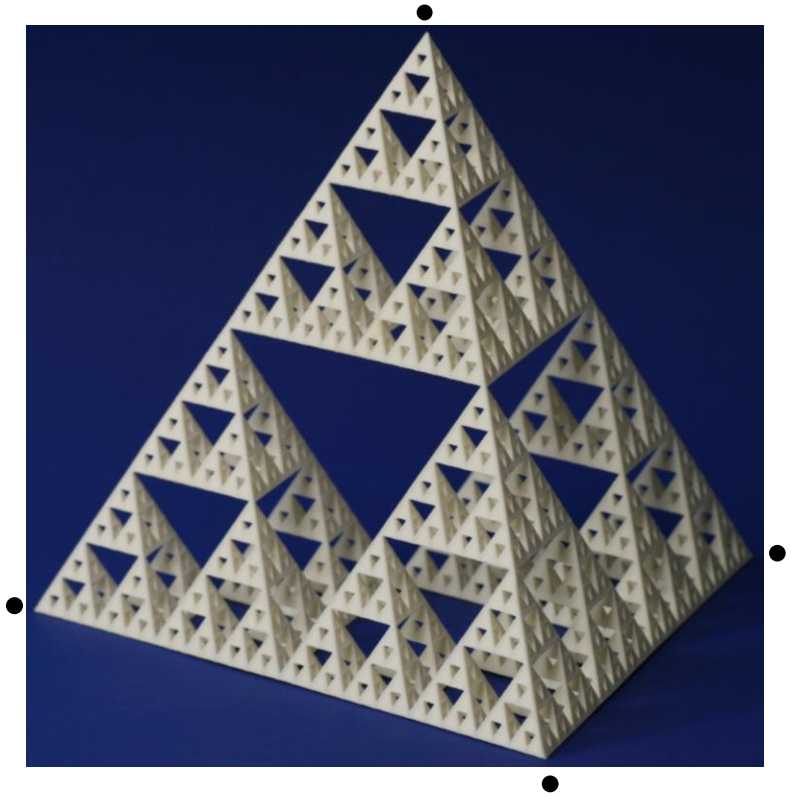


- No string operators! … a type-II fracton order

Haah's cubic code (1 of 18)



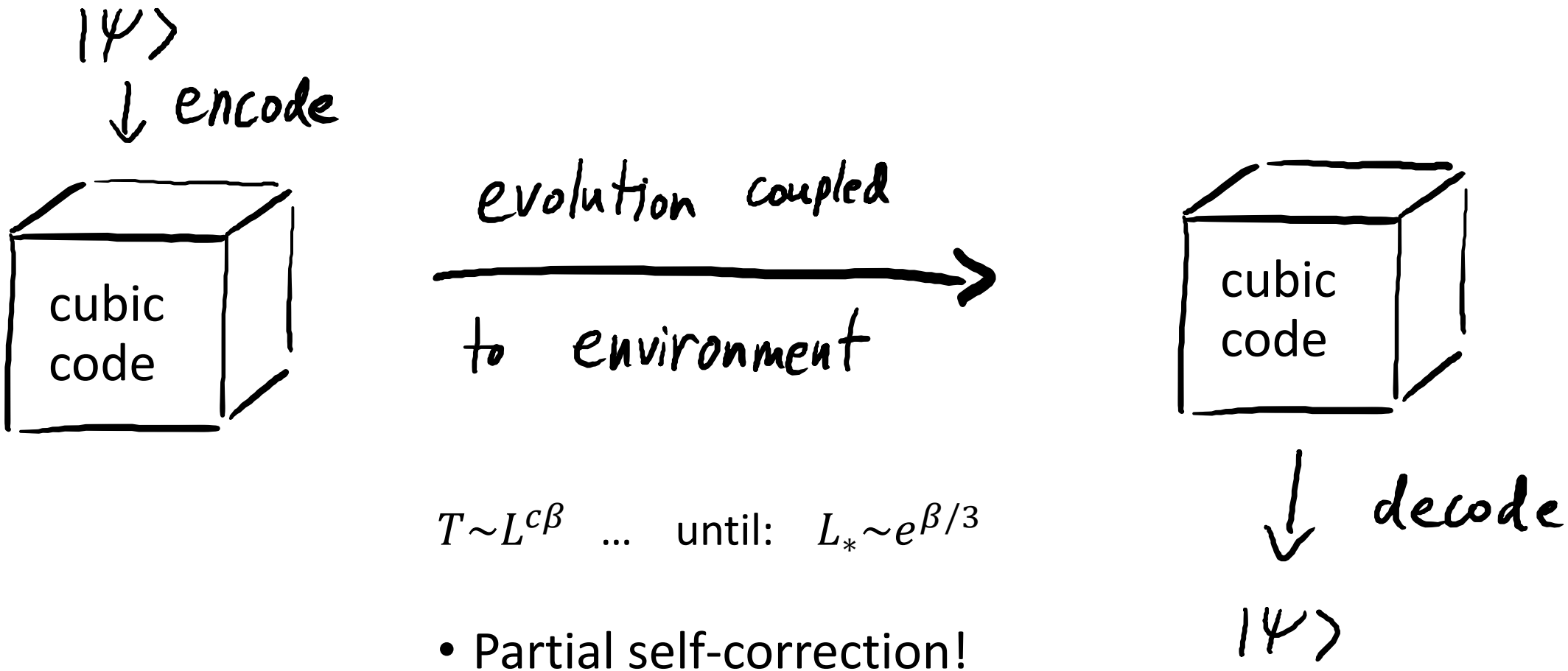




- Log(L) energy barrier
- Erratic GSD

1105.4159

Topological quantum memory



$T \sim L^{c\beta}$ … until: $L_* \sim e^{\beta/3}$

- Partial self-correction!

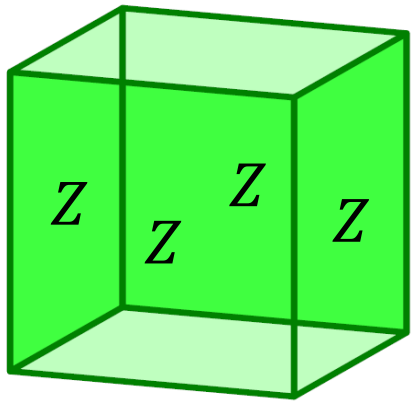
Bavyi, Haah 2013

X-cube model

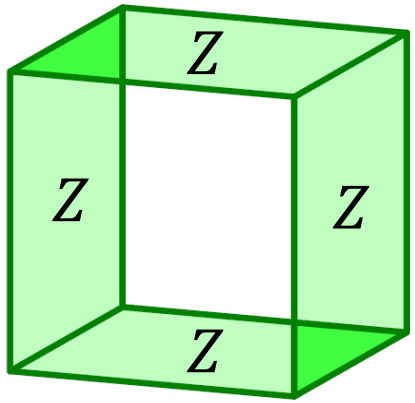
A simple type-I fracton order

Vijay, Haah, Fu 2016

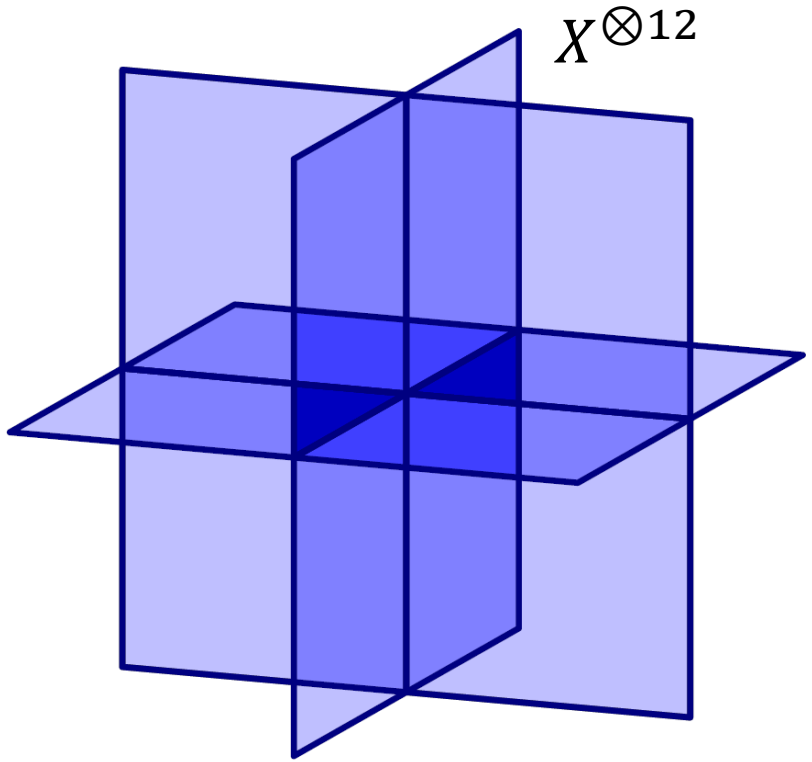
$$H = -\tilde{J}\sum(B_c^z + B_c^x) - h\sum\tilde{A}_v$$



B_c^z



B_c^x



$\tilde{A}_v$

X-cube model

A simple type-I fracton order

Log GSD = $2(L_x + L_y + L_z) - 3$

Vijay, Haah, Fu 2016

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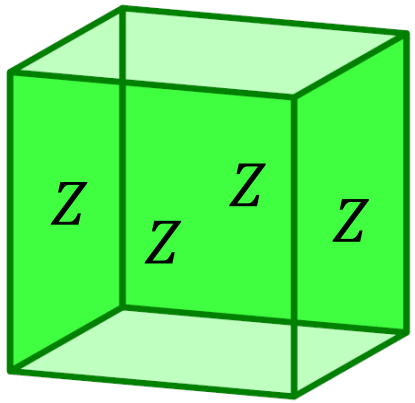
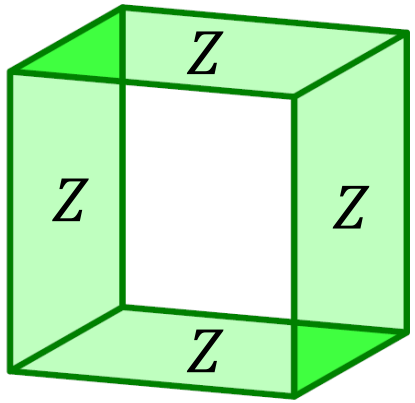
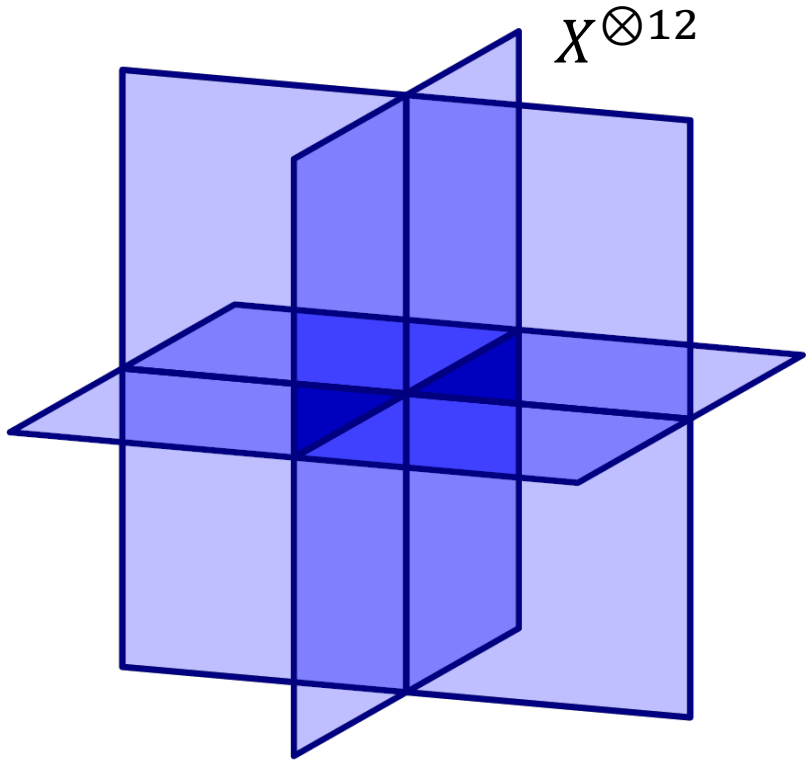


B_c^z

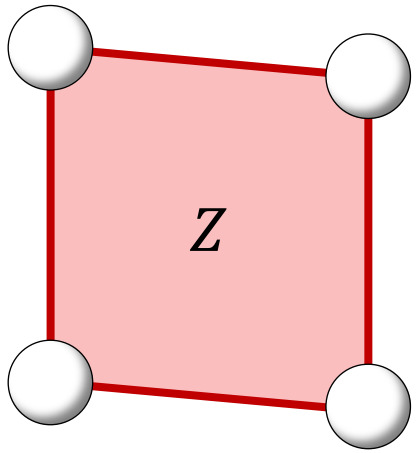


B_c^x

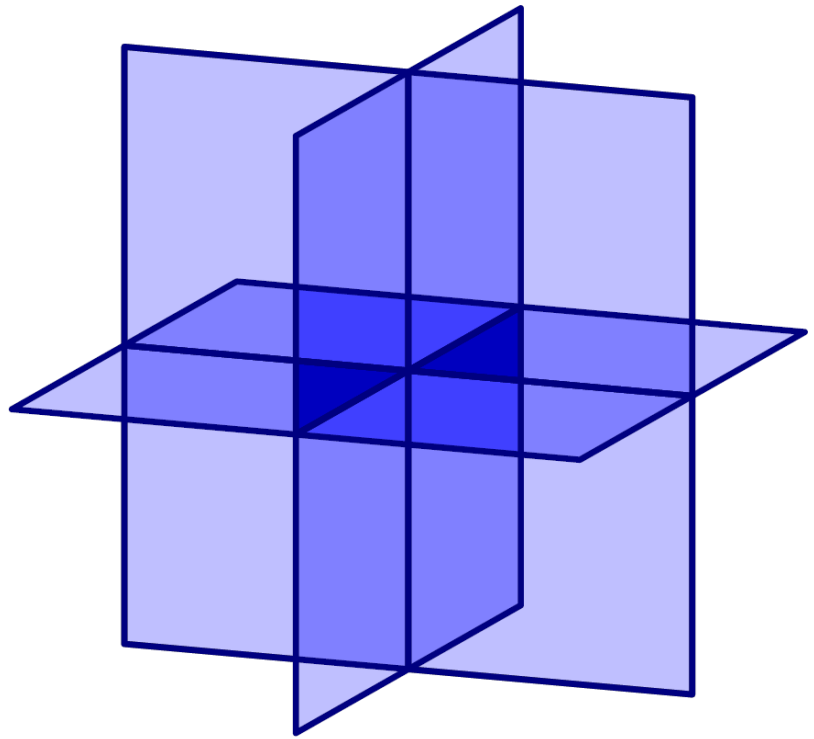


$\tilde{A}_v$

X-cube model fracton sector



fracton



$\tilde{A}_v$

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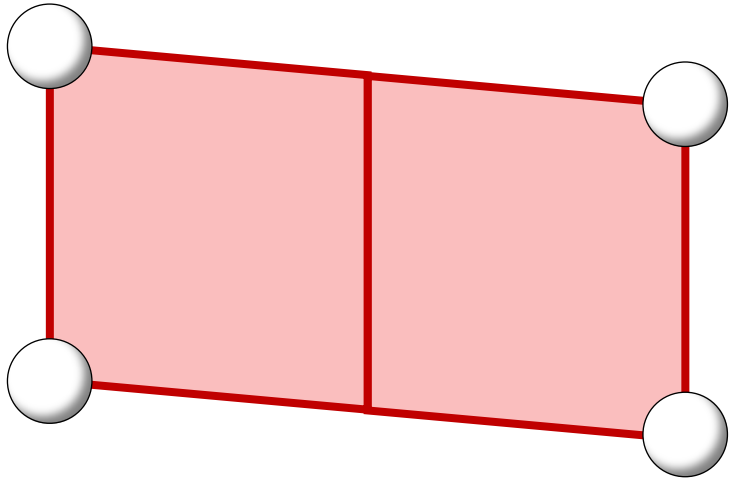

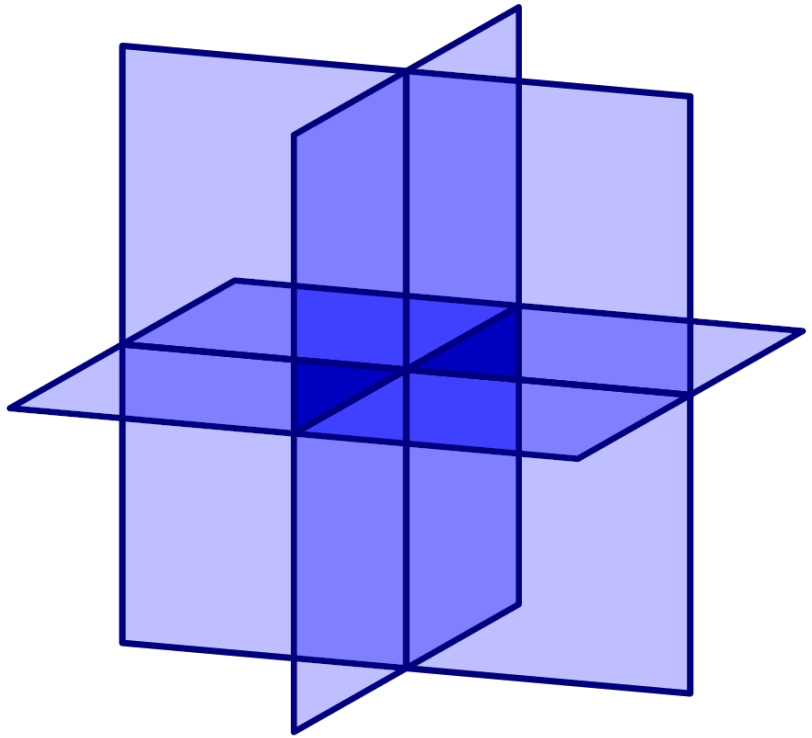

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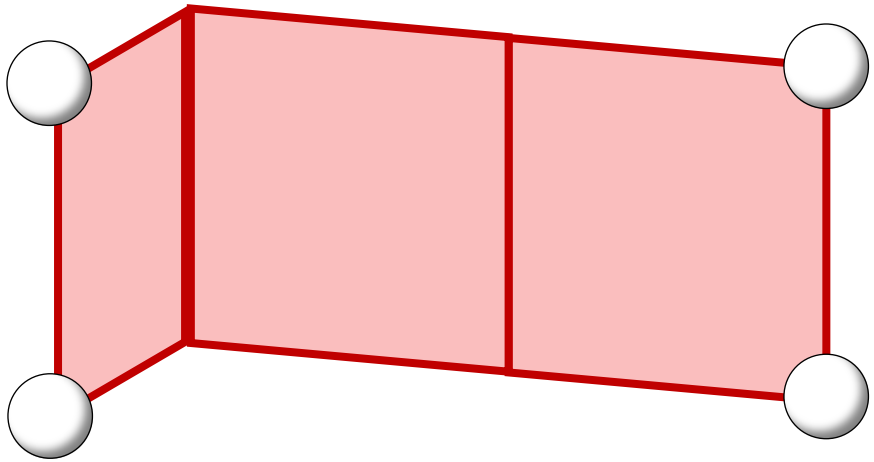
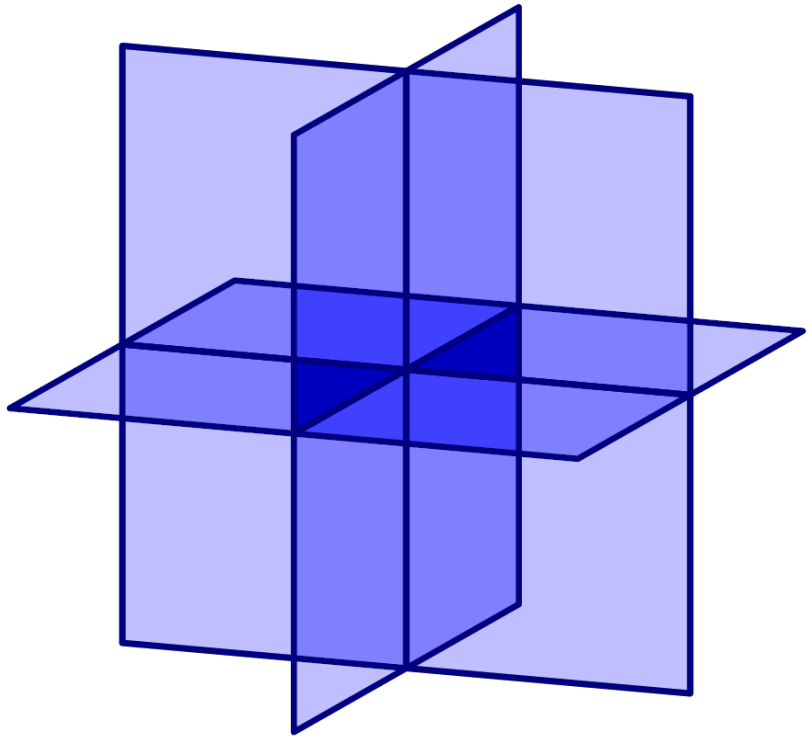
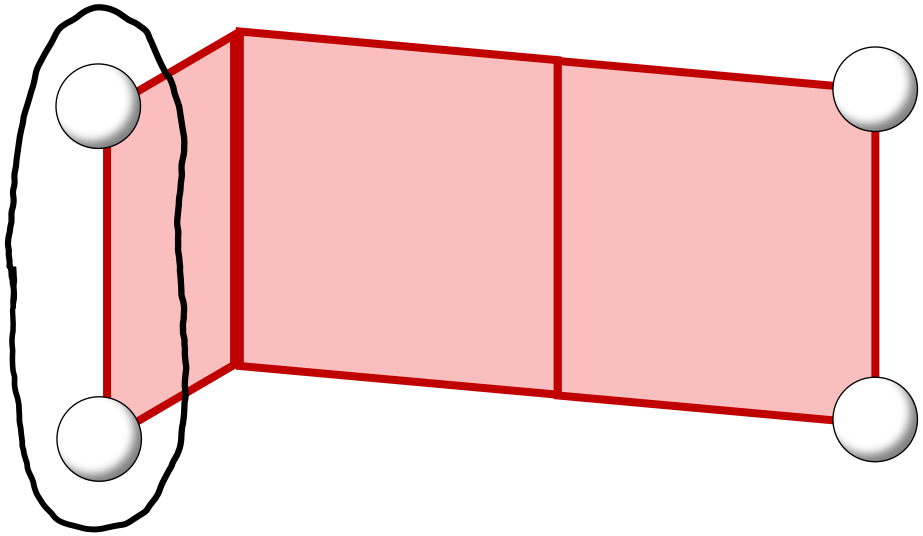
X-cube model fracton sector



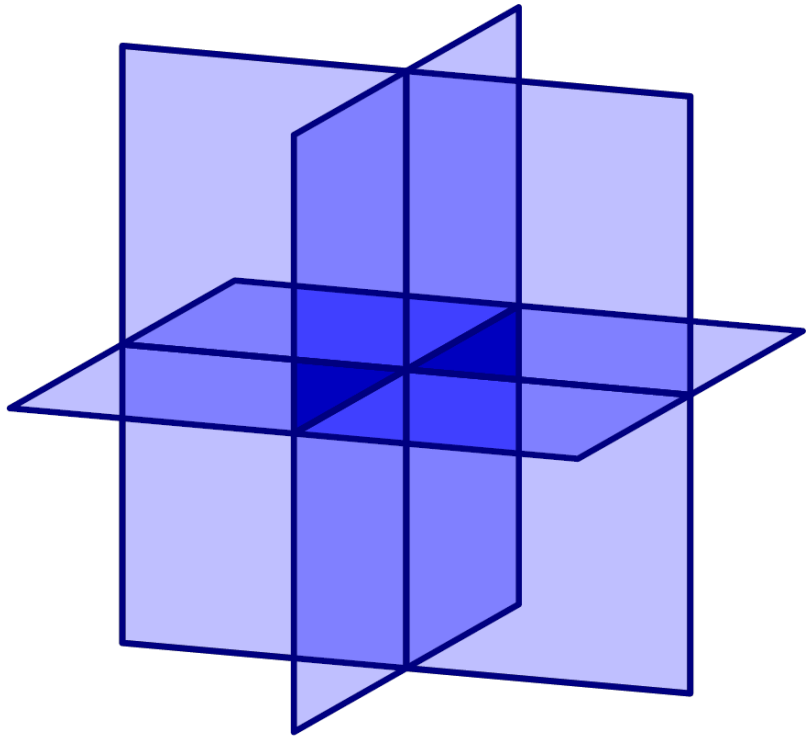


$\tilde{A}_v$

X-cube model fracton sector

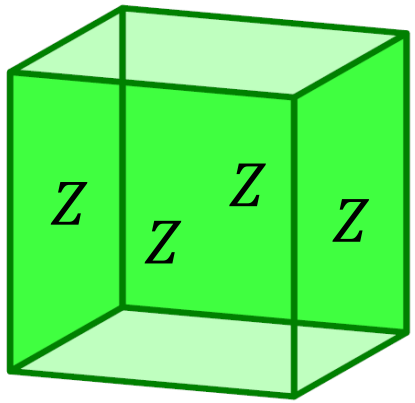


planon

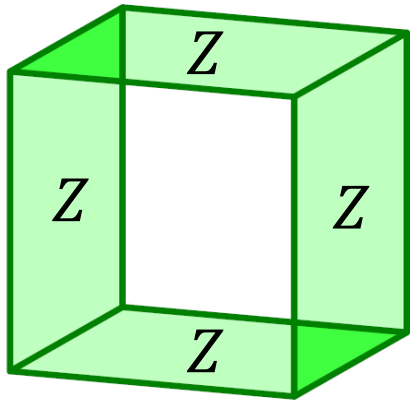


$\tilde{A}_v$

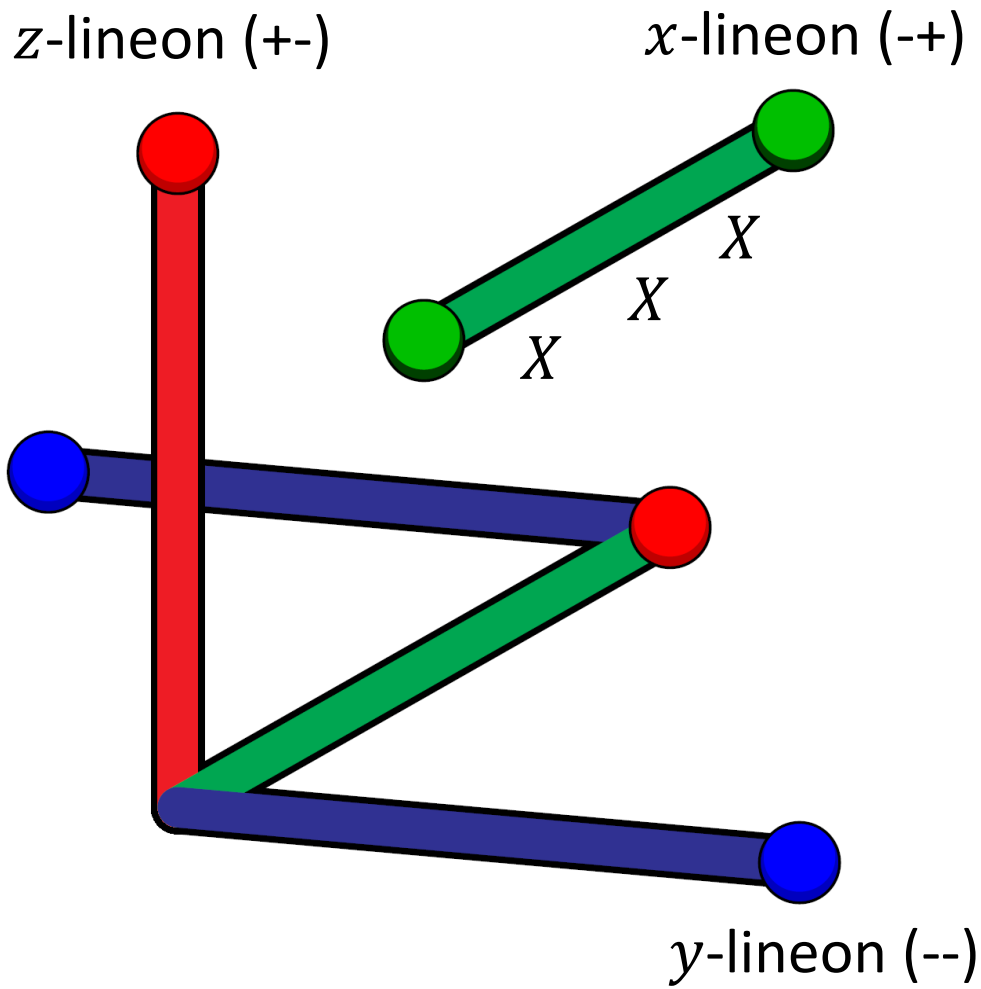
X-cube model lineon sector



B_c^z

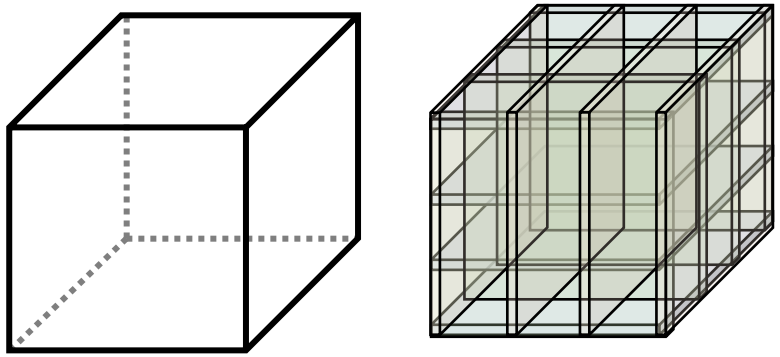


B_c^x



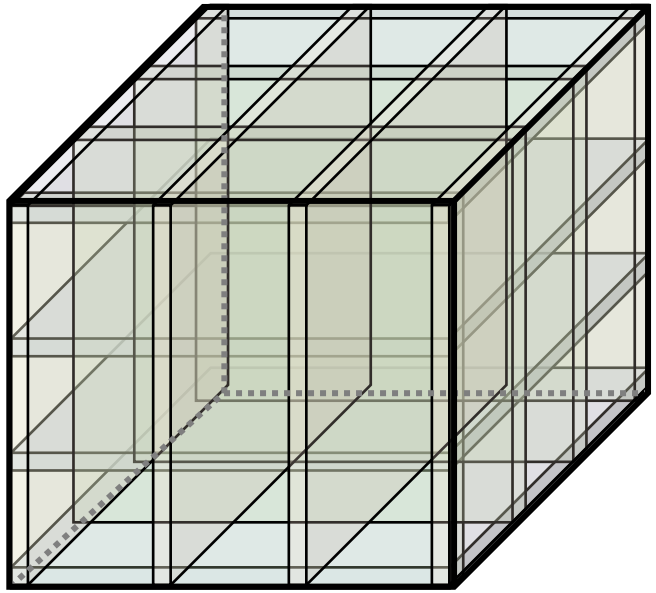
More complicated (nonabelian) models

Yoshida 2013
Vijay, Fu 2017
Prem, Huang, Song, Hermele 2018
Song, Prem, Huang, Delgado 2018
Bulmash, Barkeshli 2019
Prem, W 2019
Slagle, Aasen, W 2019
W, Cheng 2020



3D TQFT + 2D TQFT layers

Couple interpenetrating layers in a nontrivial way



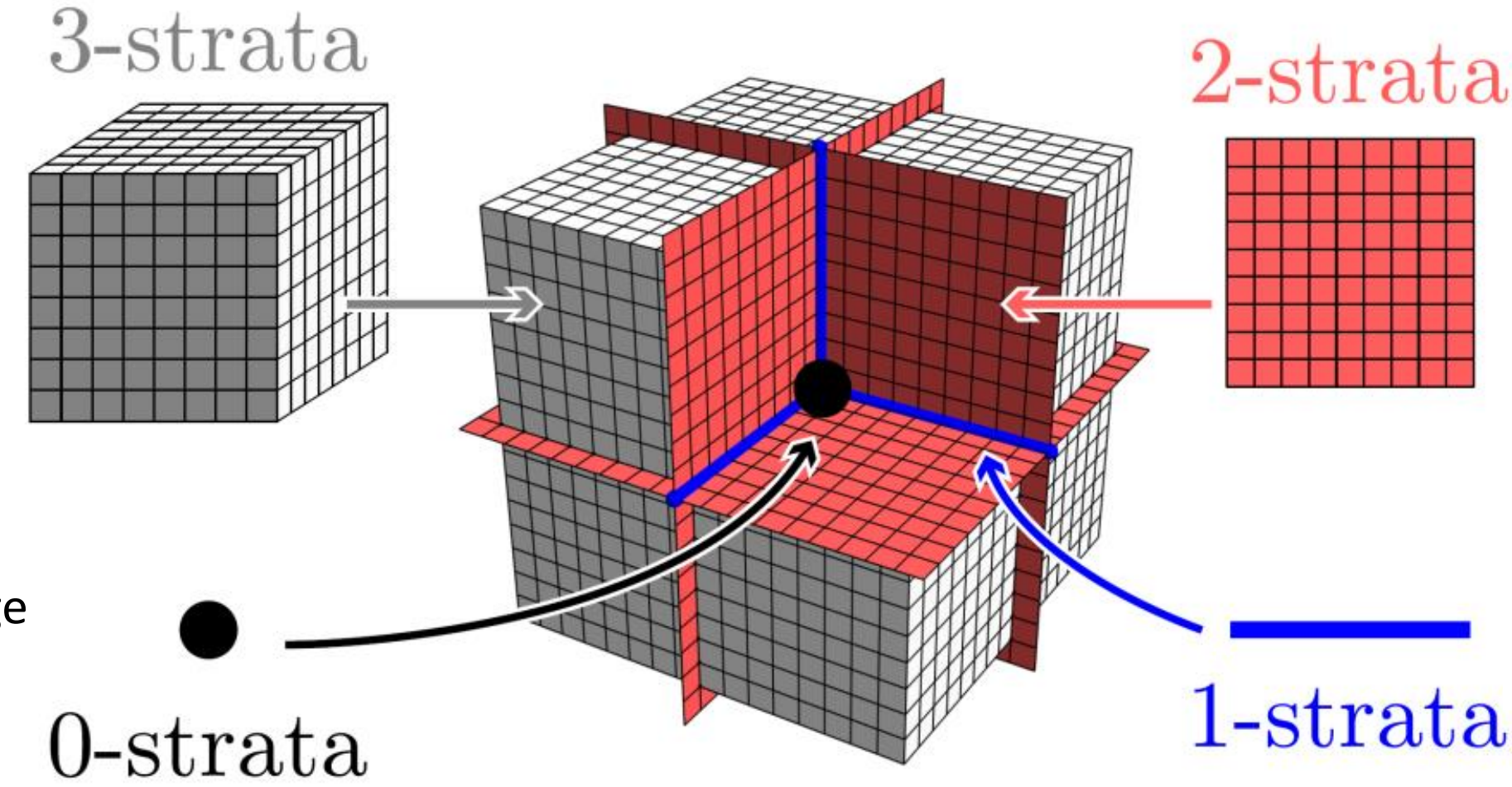
General fracton model (can be nonabelian)

Topological defect networks

Aasen, Bulmash, Prem, Slagle, W 2020

Inspired by: Else, Thorngren 2019

Related works: Wen 2020
Wang 2020



3D TQFT

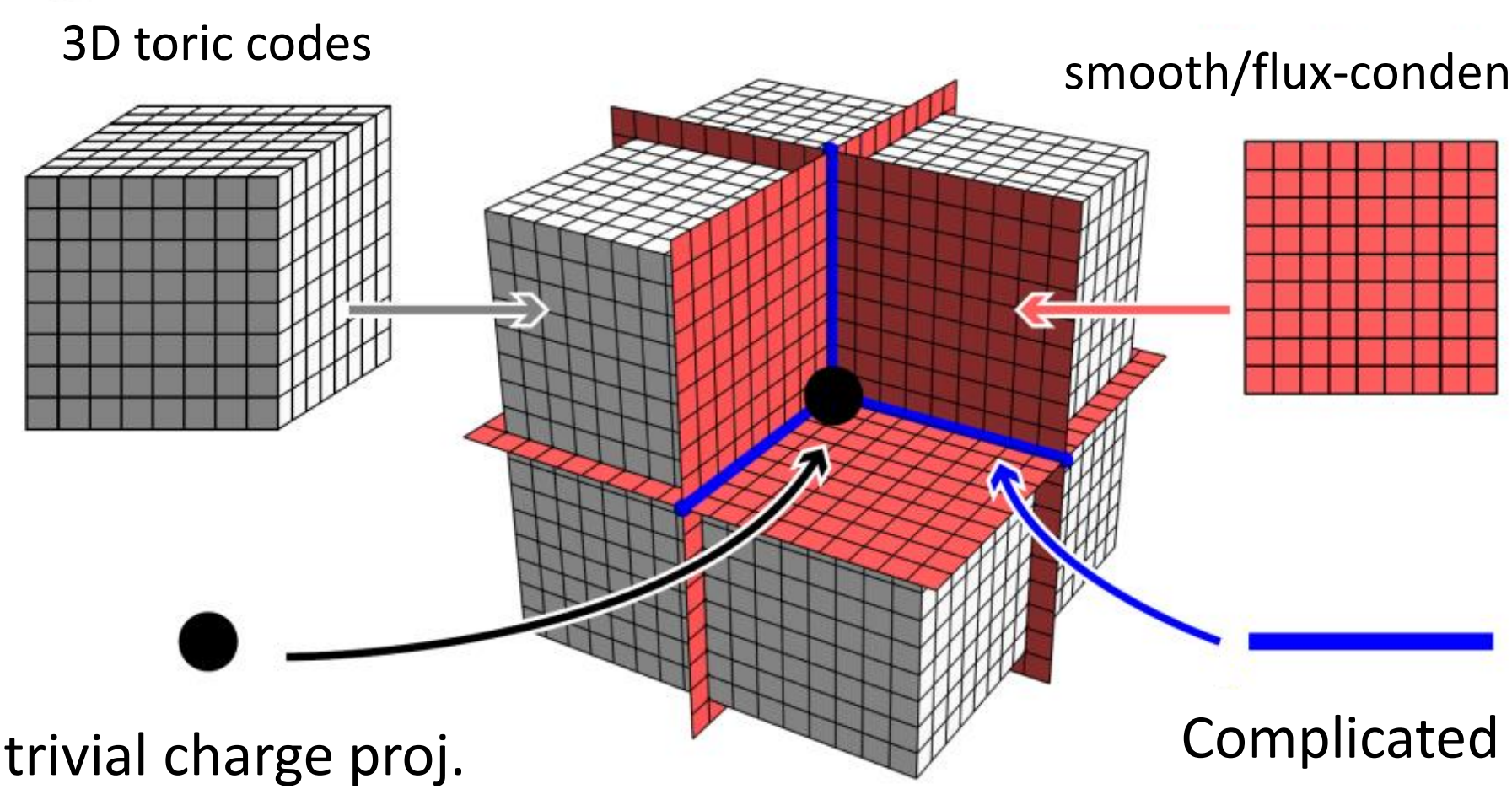
2D top. defect/DW

Top. charge proj.

1D top. defect

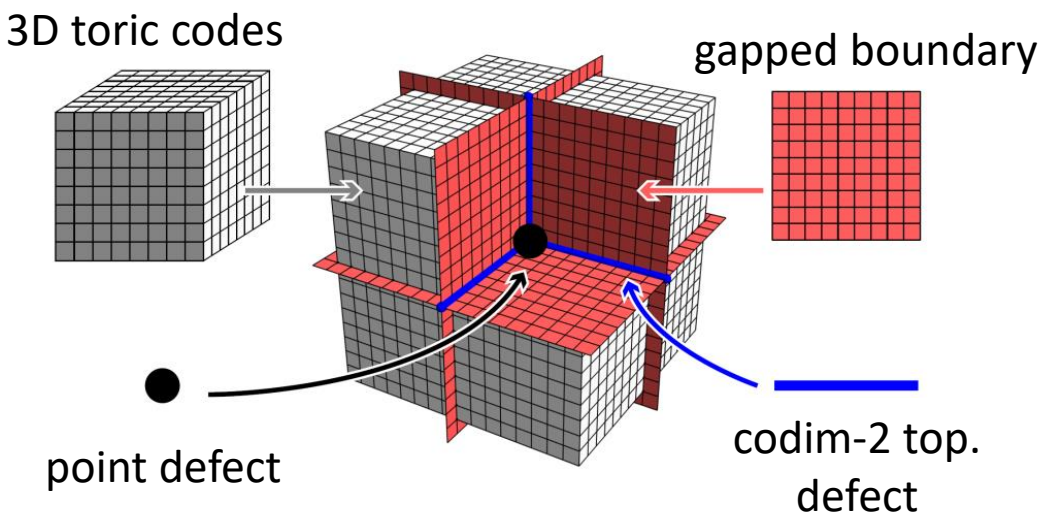
Topological defect networks e.g.

Aasen, Bulmash, Prem, Slagle, W 2020



Why topological defect networks?

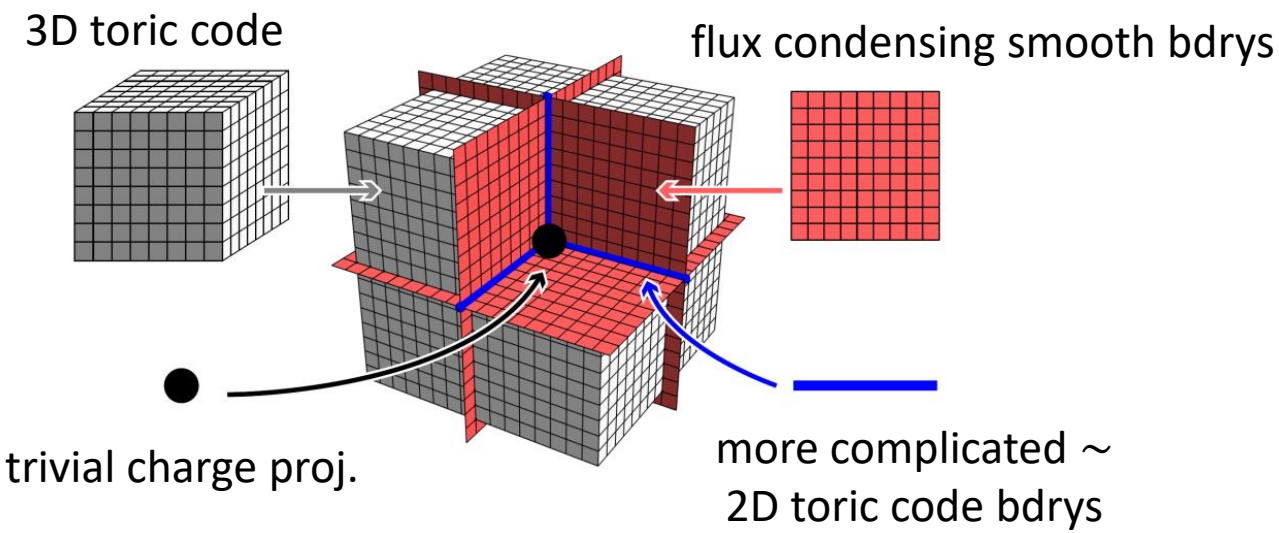
Aasen, Bulmash, Prem, Slagle, W 2020



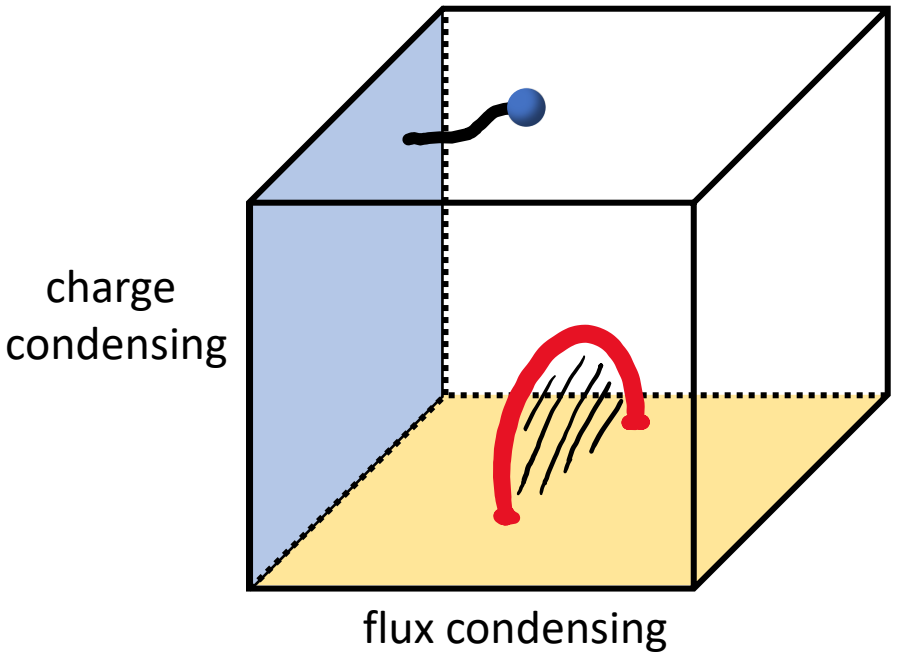
- Separates lattice scale from excitation mobility scale allowing lattice continuum limit to be taken
- Divides complicated topological states into tractable pieces
- Can be used to construct, classify and constrain possible 3D topological orders

X-cube defect network e.g.

Aasen, Bulmash, Prem, Slagle, W 2020

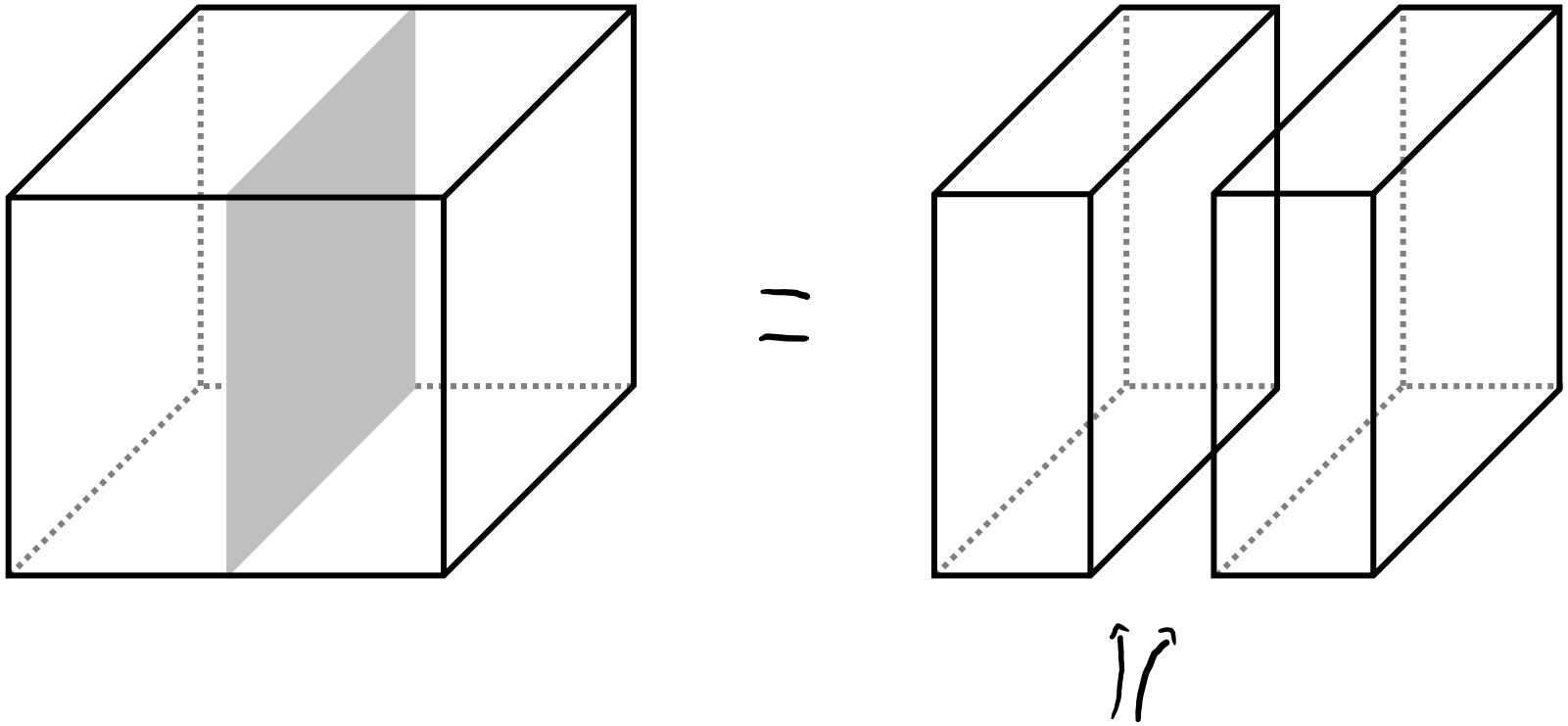


Topological boundaries in 3D toric code



3D toric code

Topological domain wall defect in 3D TC

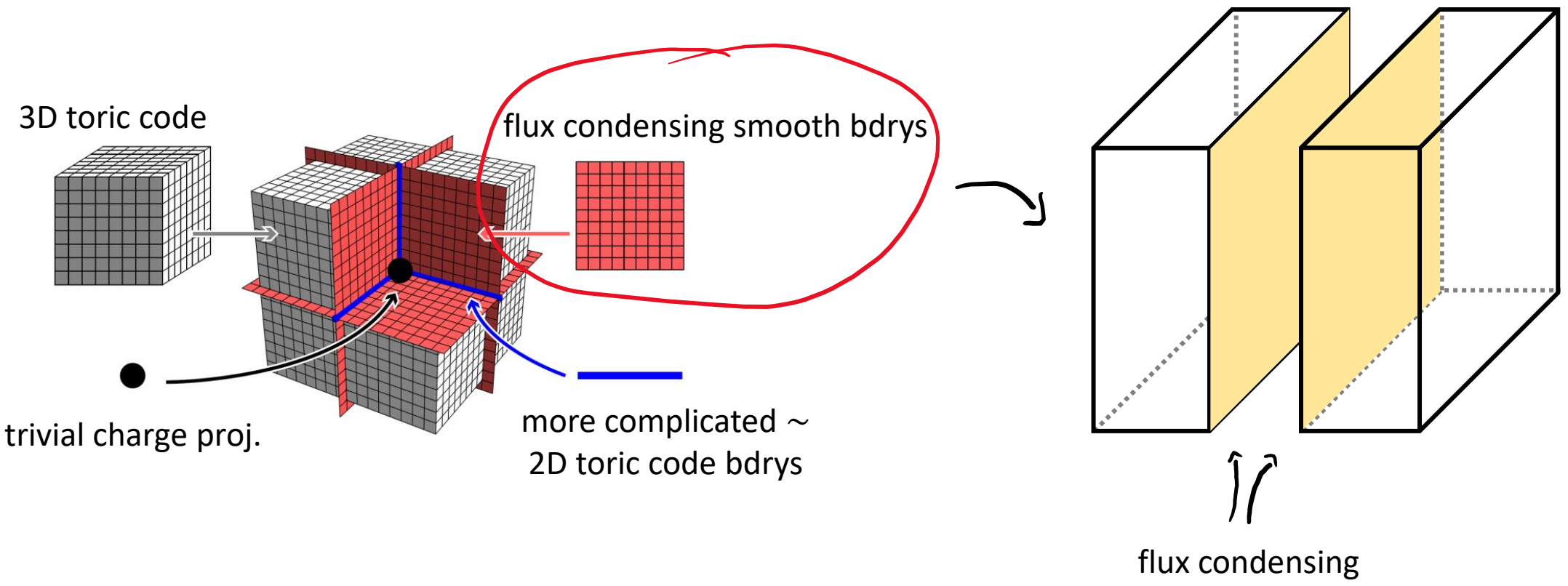


3D toric code

charge or flux condensing
(up to attaching surface
anyons)

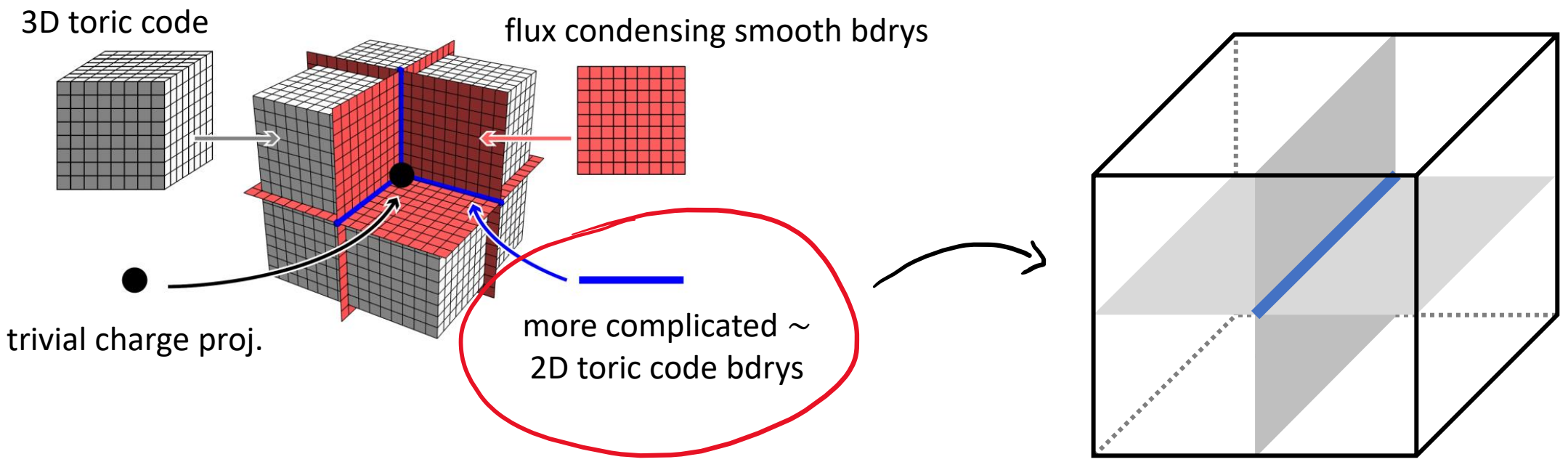
X-cube defect network e.g.

Aasen, Bulmash, Prem, Slagle, W 2020



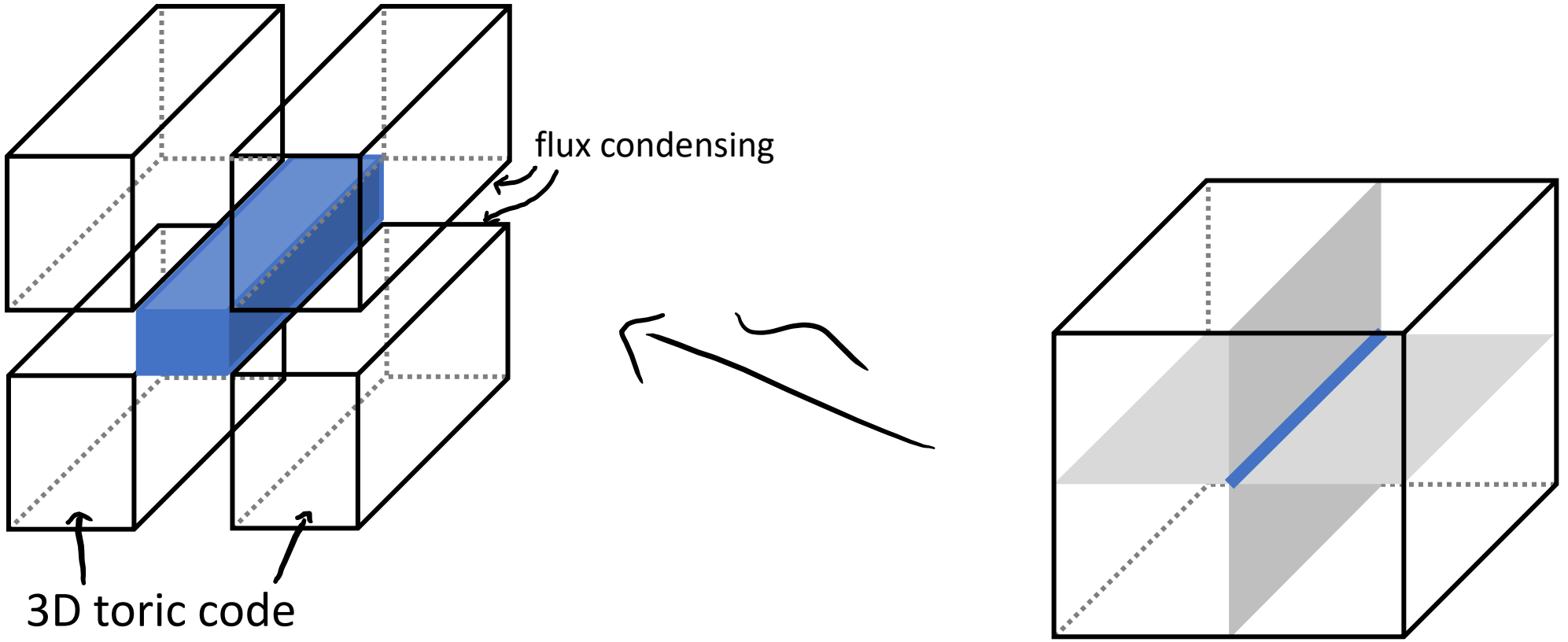
X-cube defect network e.g.

Aasen, Bulmash, Prem, Slagle, W 2020



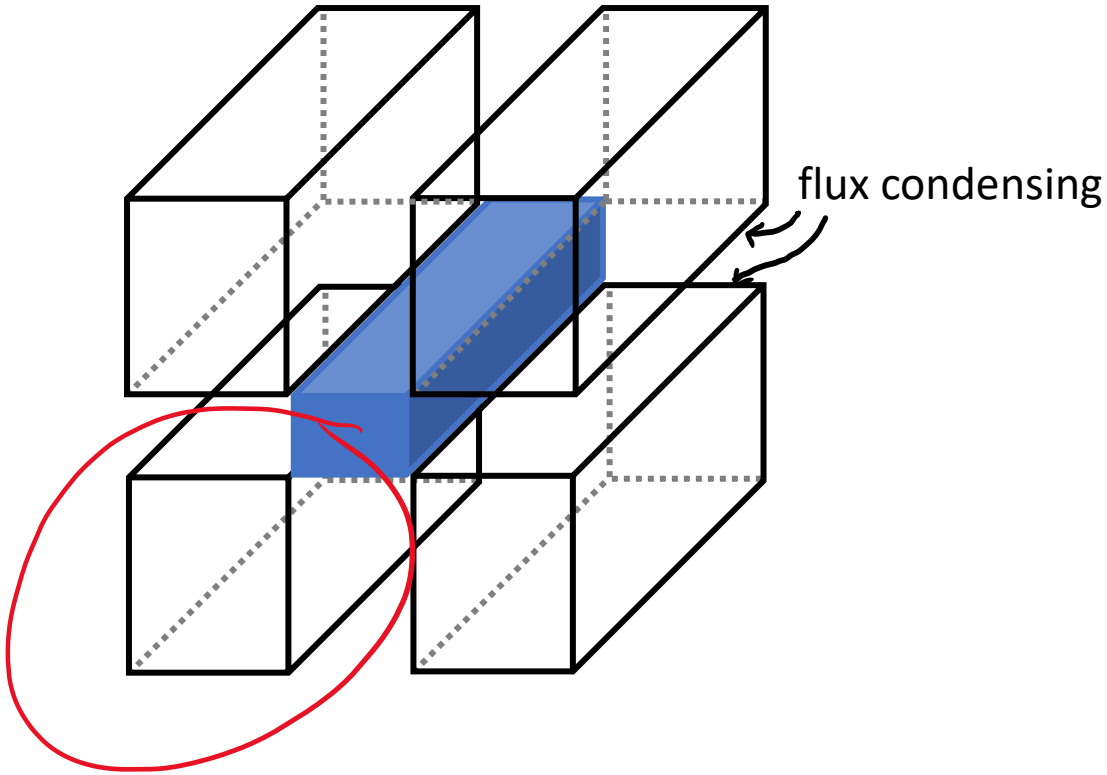
X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020



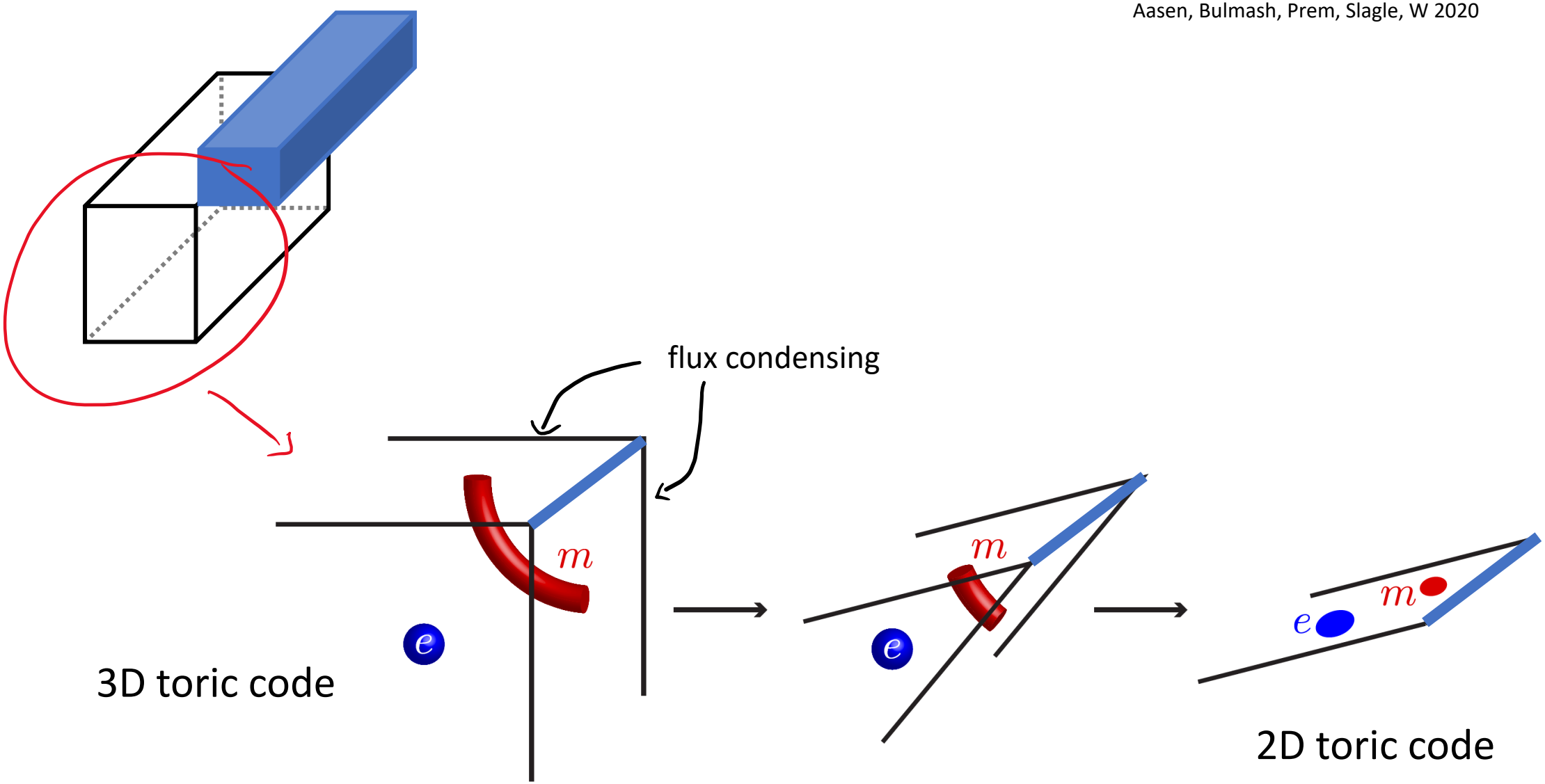
X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020



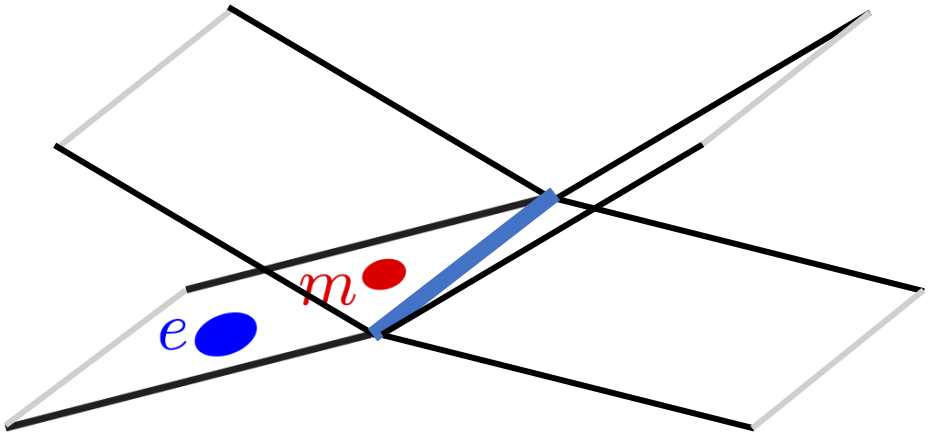
X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020

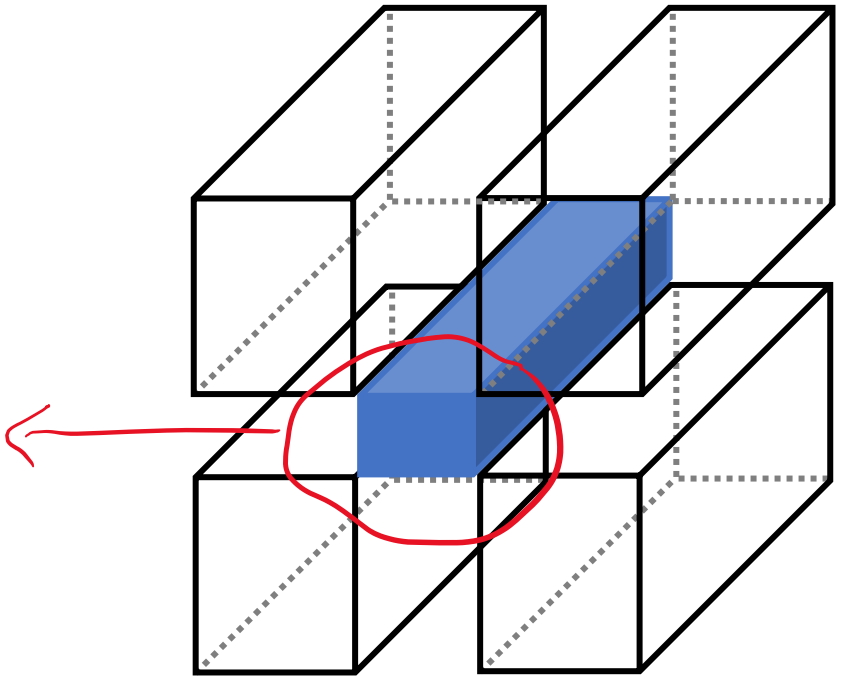


X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020

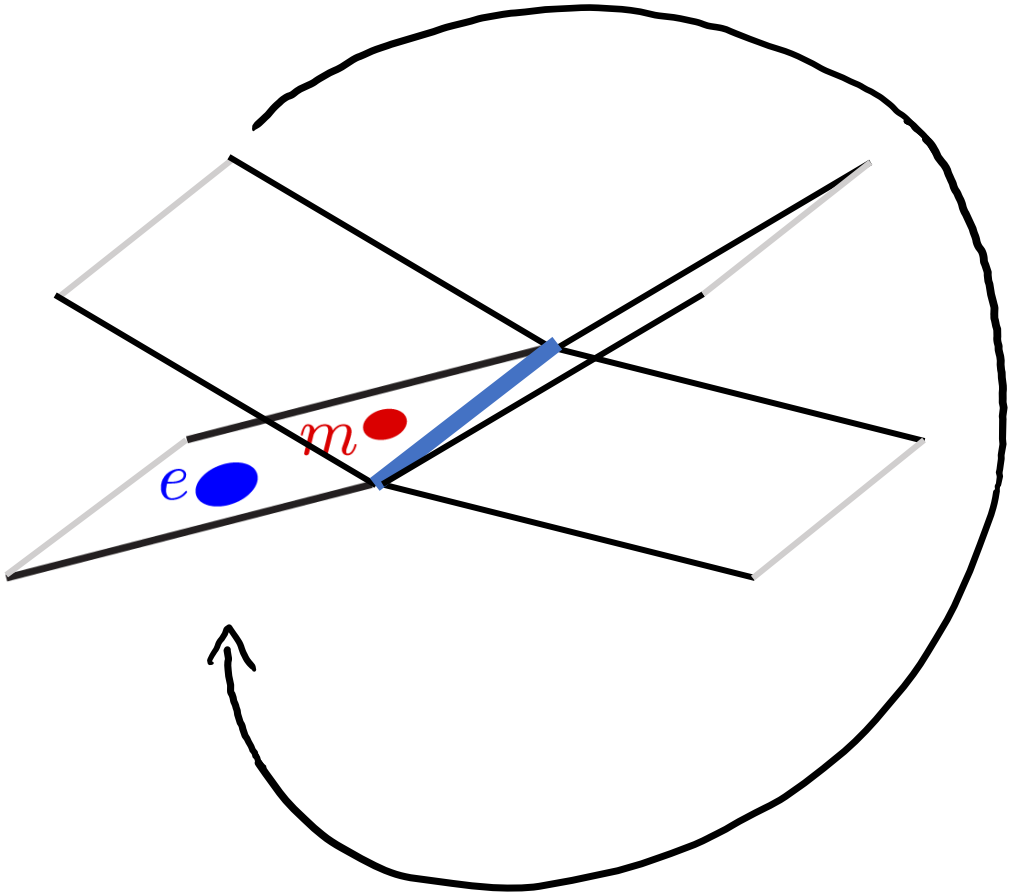


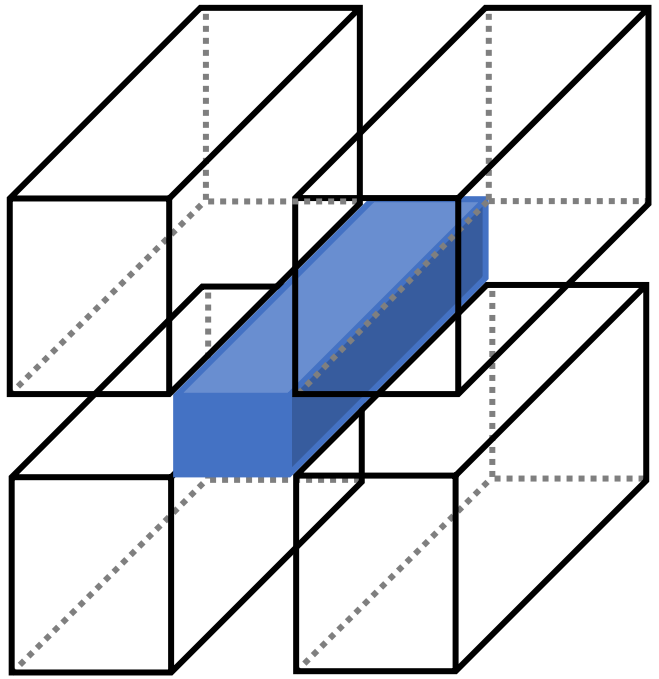
Four 2D toric codes



X-cube defect network

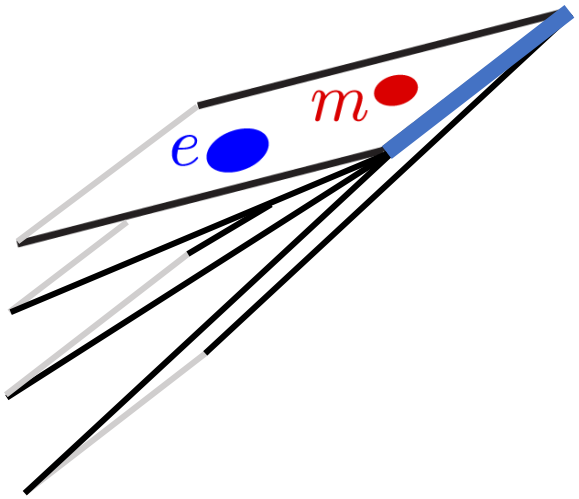
Aasen, Bulmash, Prem, Slagle, W 2020

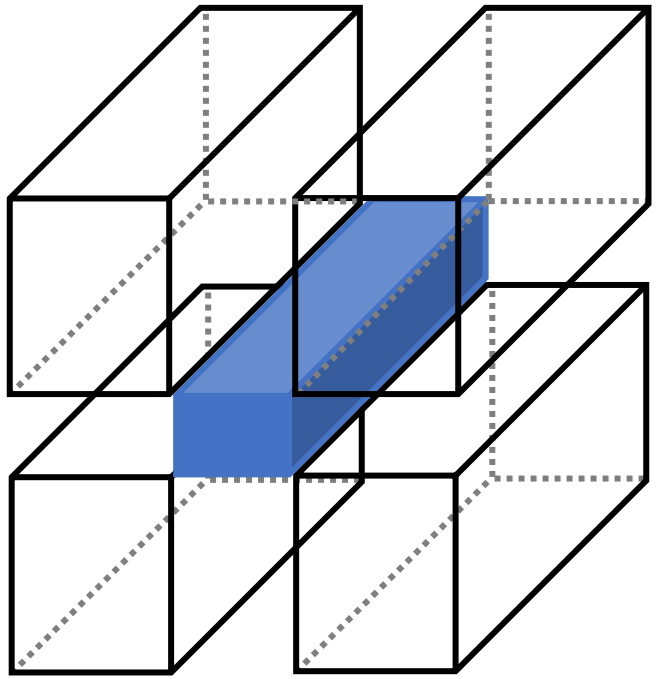




X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020



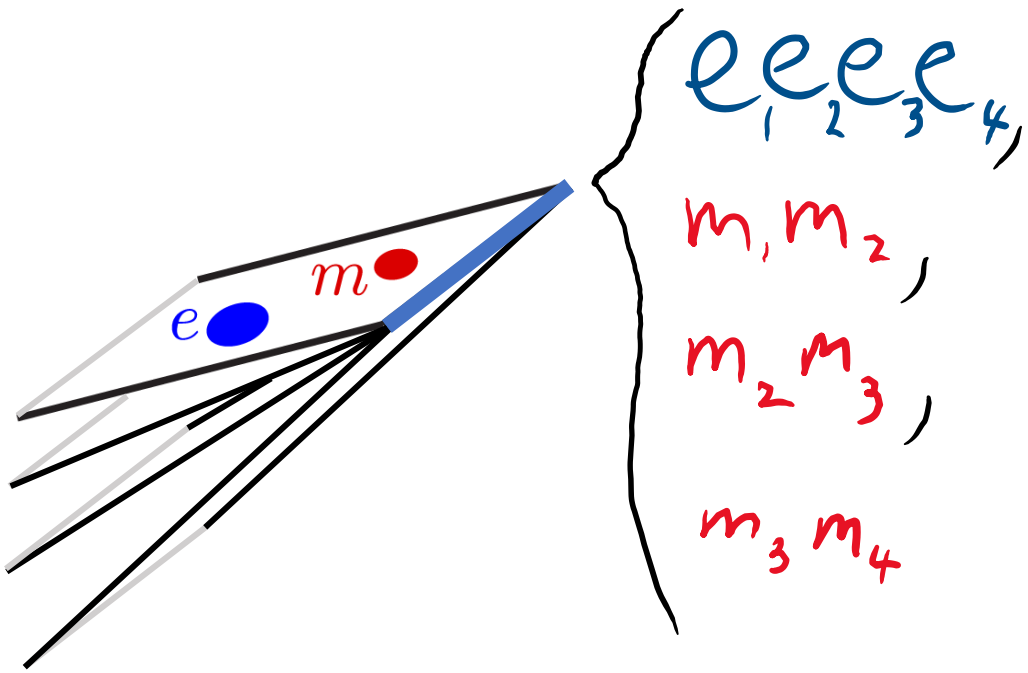


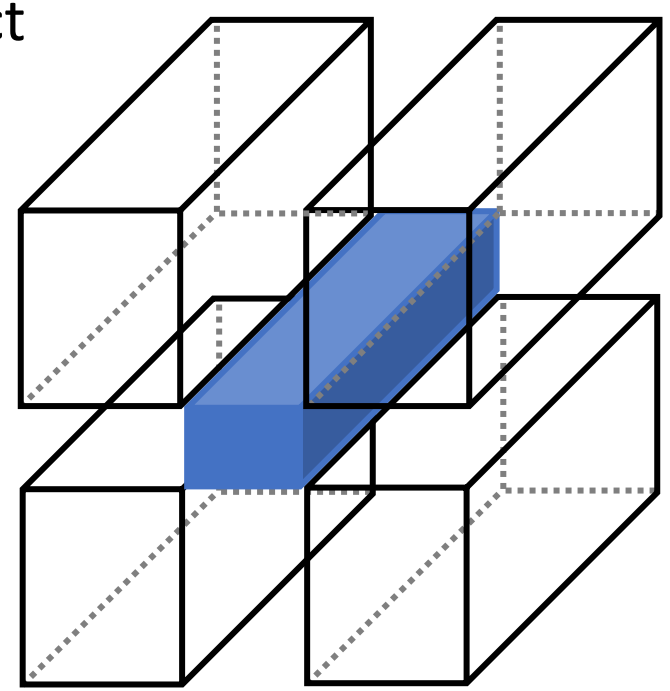
Four 2D toric codes

X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020

1D gapped bdry / defect

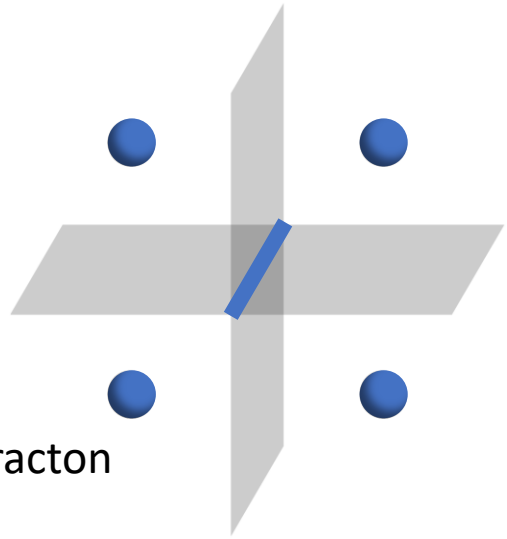




Four 2D toric codes

X-cube defect network

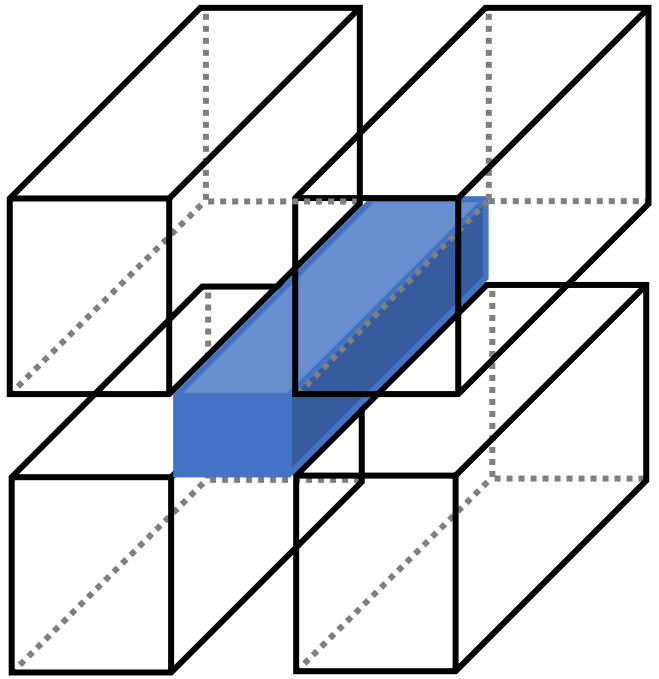
Aasen, Bulmash, Prem, Slagle, W 2020



fracton

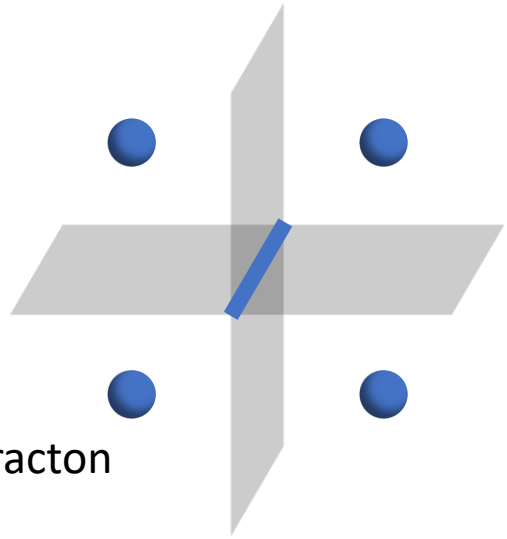
$e_1 e_2 e_3 e_4$

unfold
reinflate



X-cube defect network

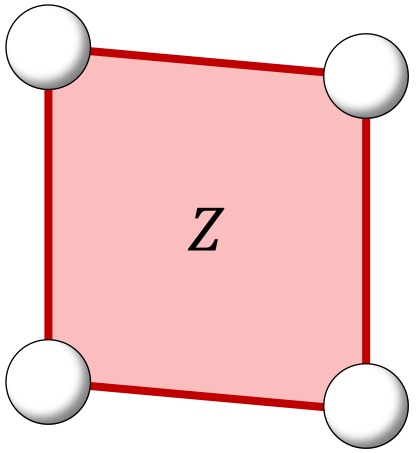
Aasen, Bulmash, Prem, Slagle, W 2020



fracton

$e_1 e_2 e_3 e_4$

$\sim$

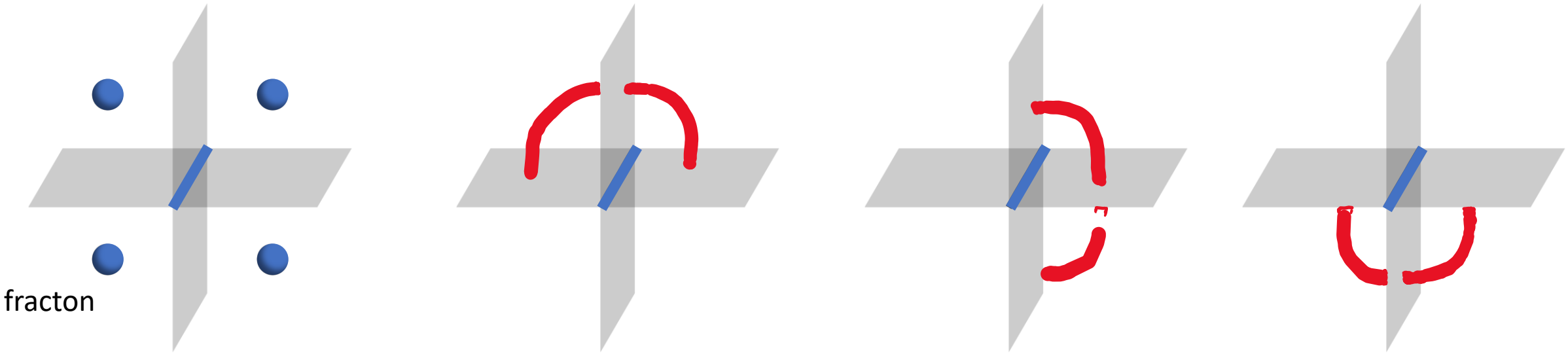


Fracton cluster
in X-cube

X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020

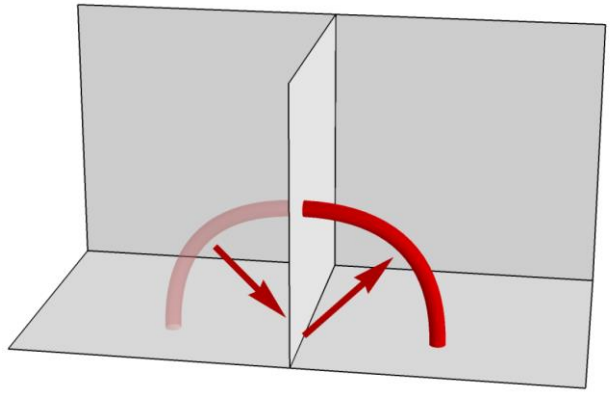
lineons



X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020

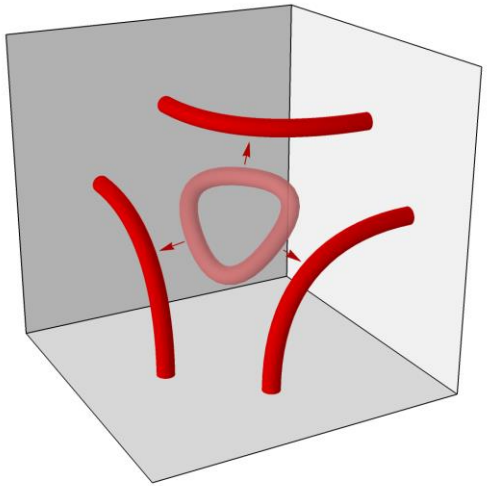
lineons

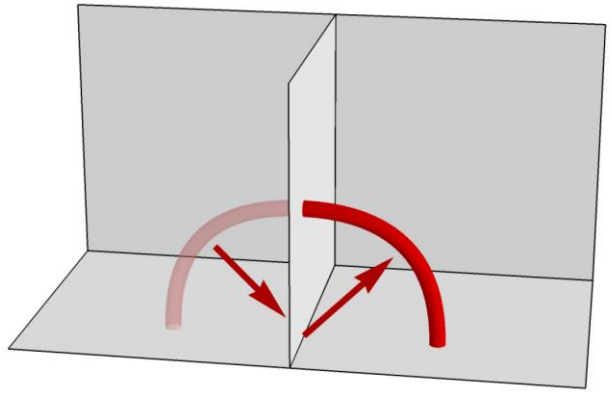


X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020

lineons

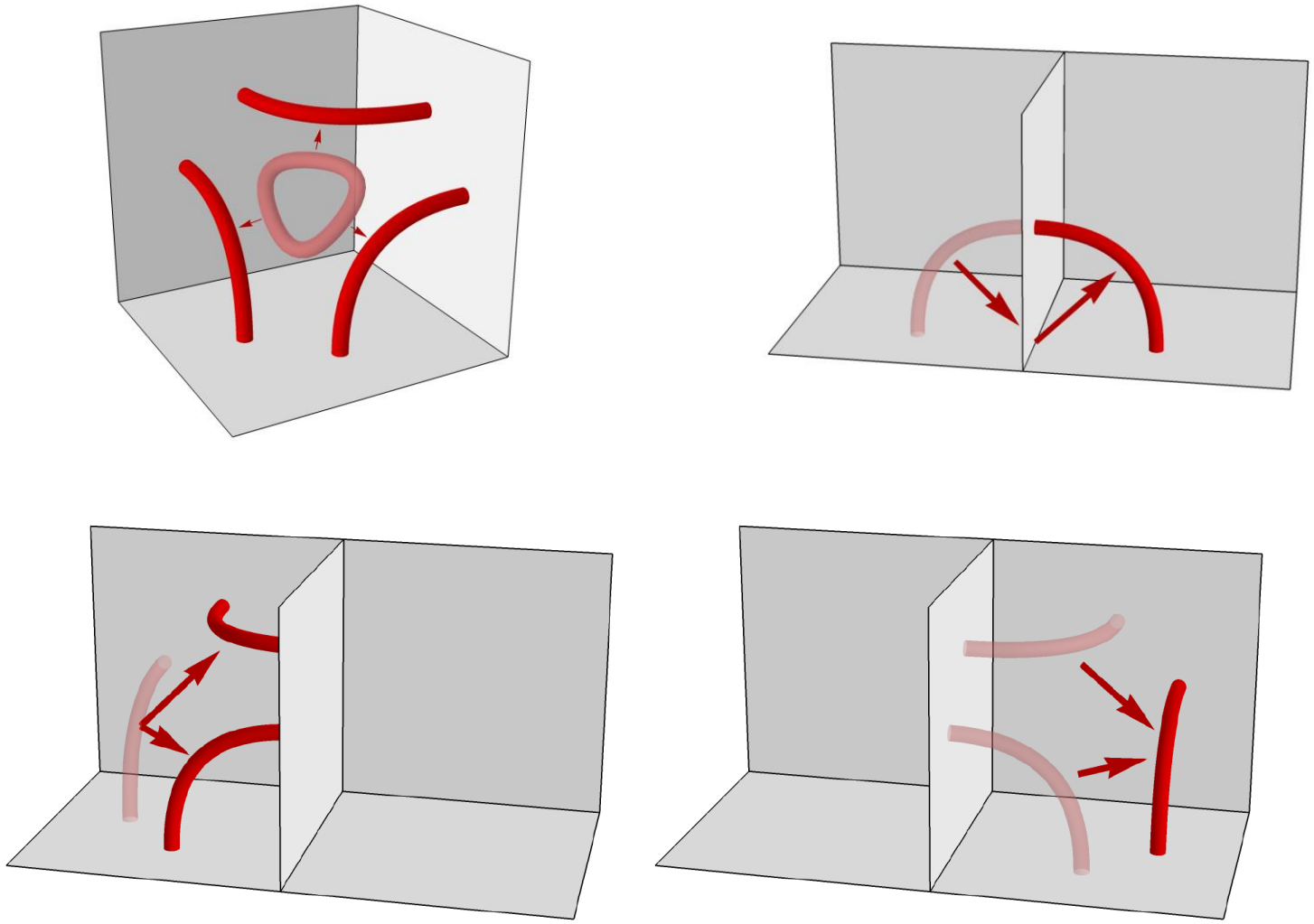




X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020

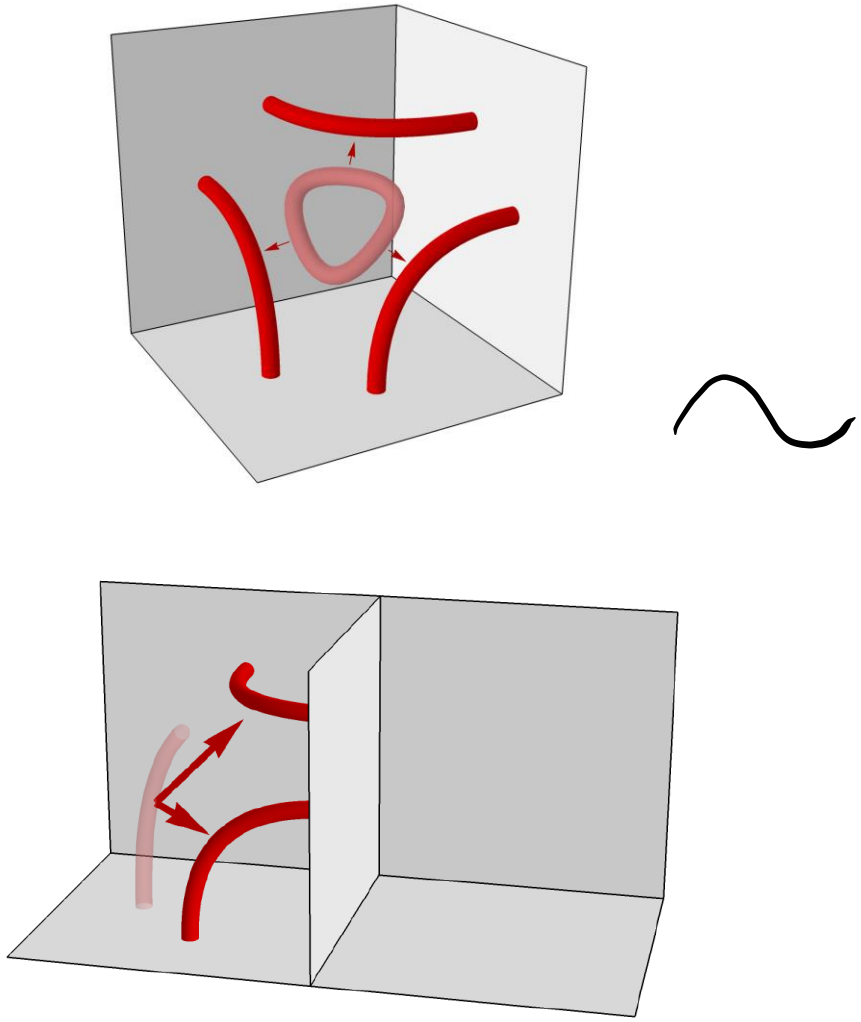
lineons



X-cube defect network

Aasen, Bulmash, Prem, Slagle, W 2020

lineons



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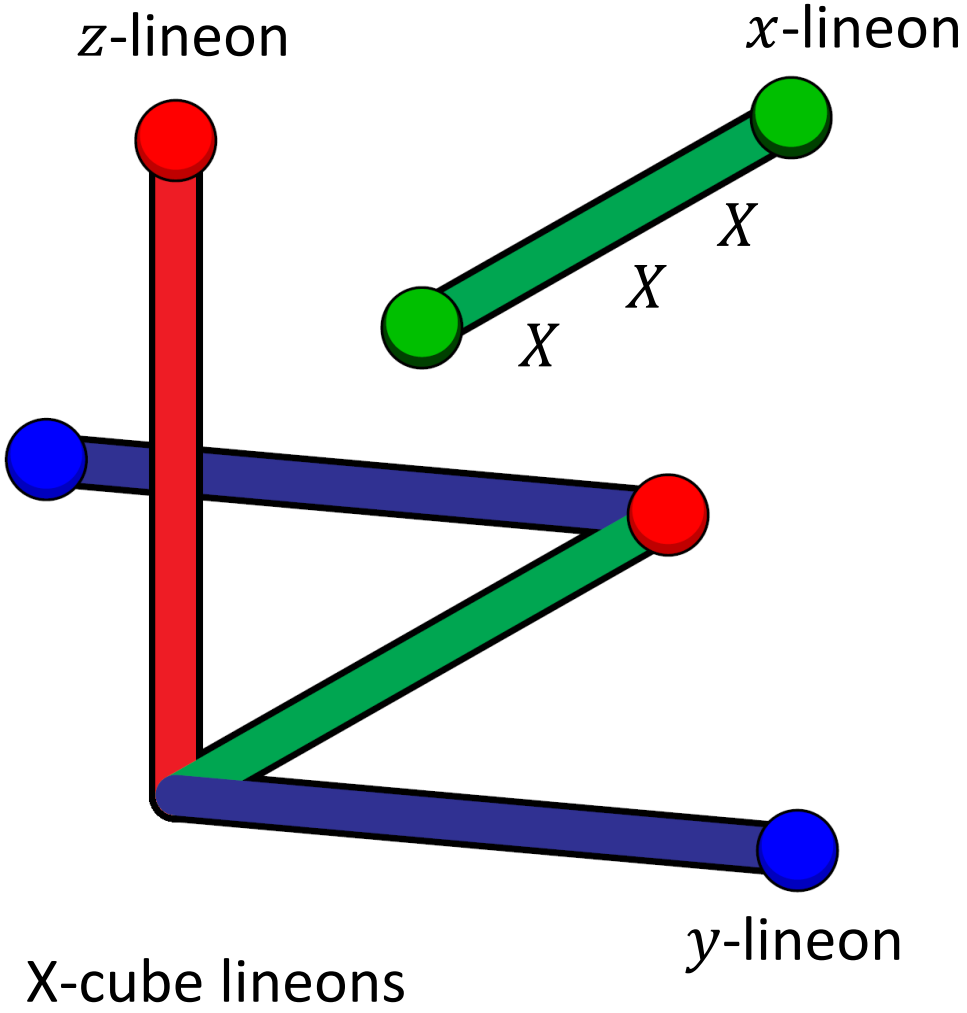
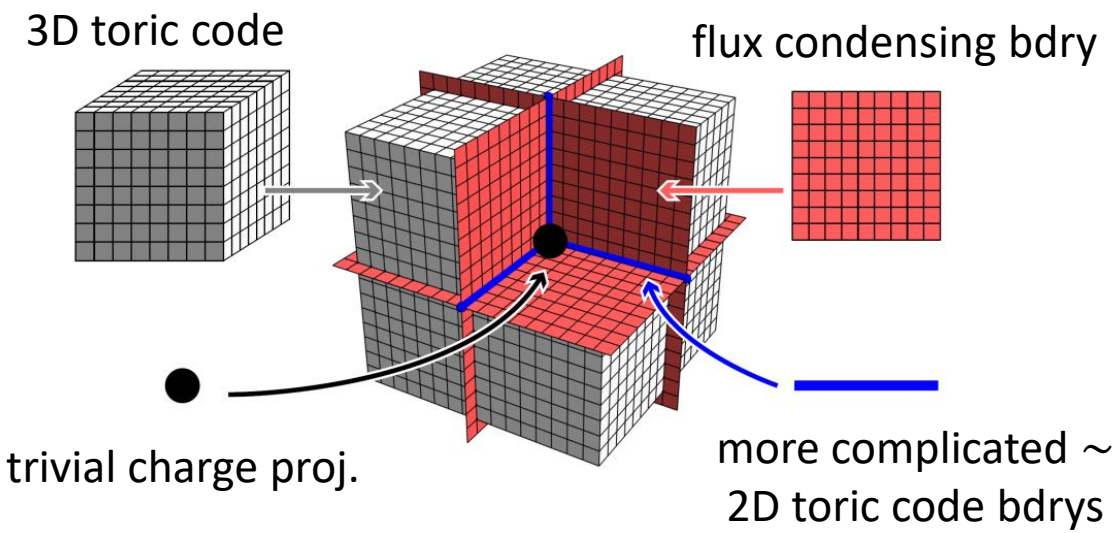


X-cube lineons

X-cube defect network

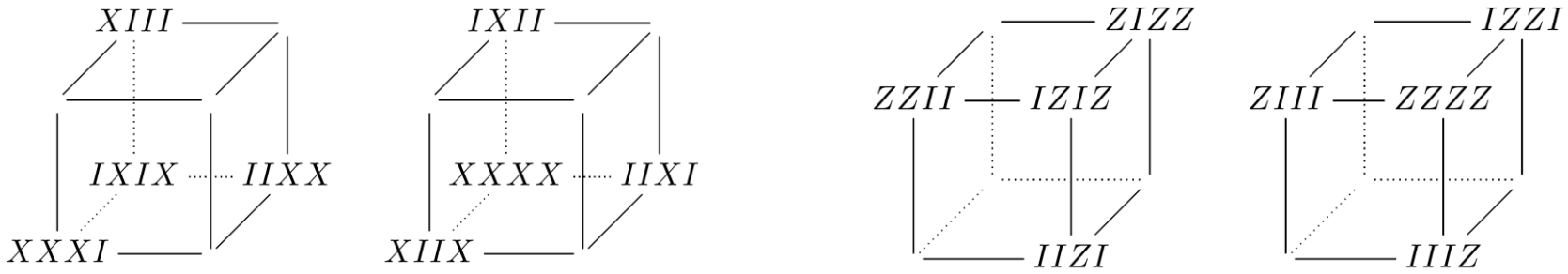
Aasen, Bulmash, Prem, Slagle, W 2020



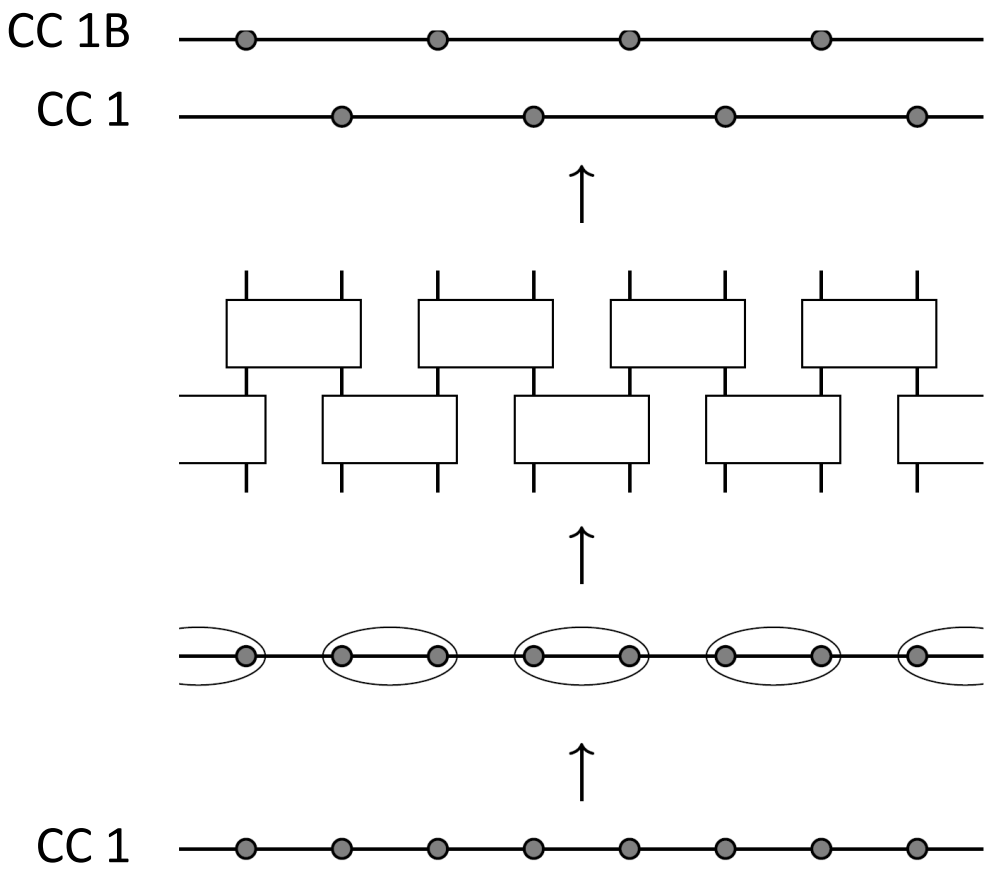
- Charge patterns match X-cube
- Admits a lattice Hamiltonian
- Provably in the X-cube phase

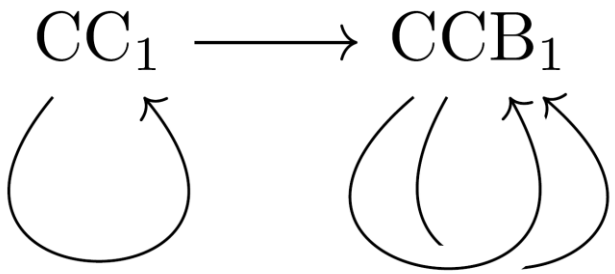
Cubic code 1 B defect network

Aasen, Bulmash, Prem, Slagle, W 2020



Fracton phase equivalence & renormalization

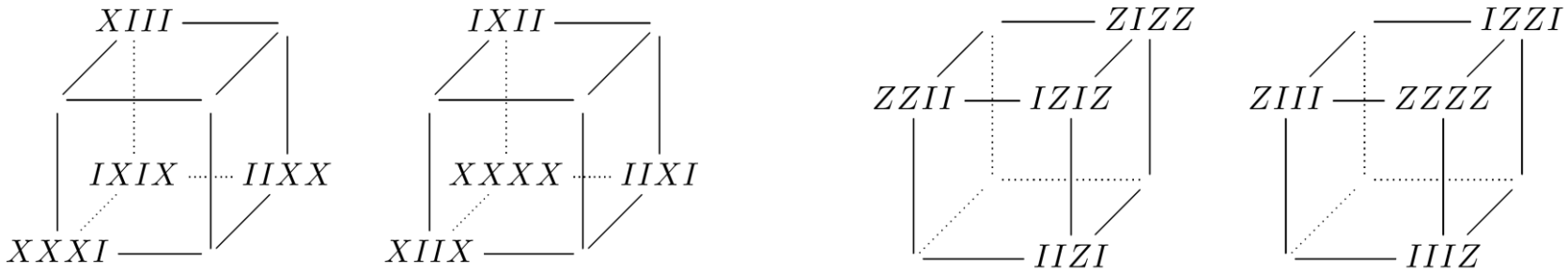




Haah 2013

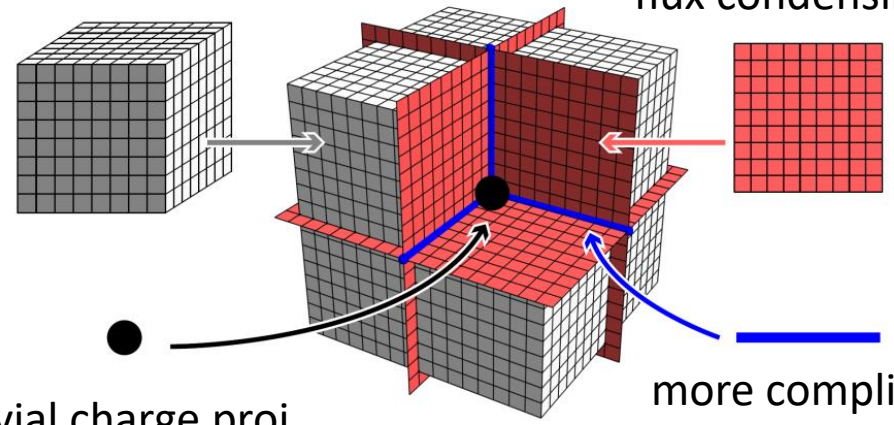
Cubic code 1 B defect network

Aasen, Bulmash, Prem, Slagle, W 2020



pair of 3D toric codes

flux condensing bdry

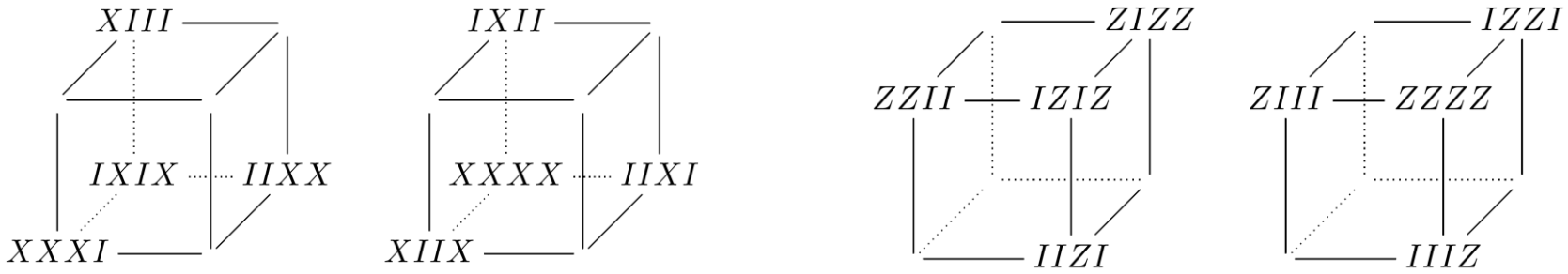


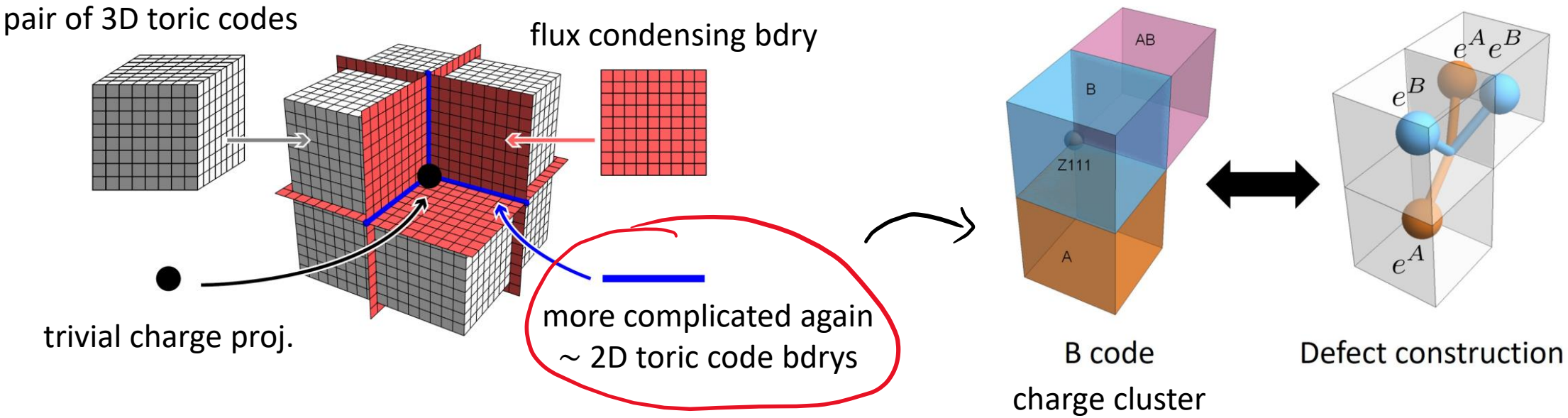
trivial charge proj.

more complicated again
~ 2D toric code bdrys

Cubic code 1 B defect network

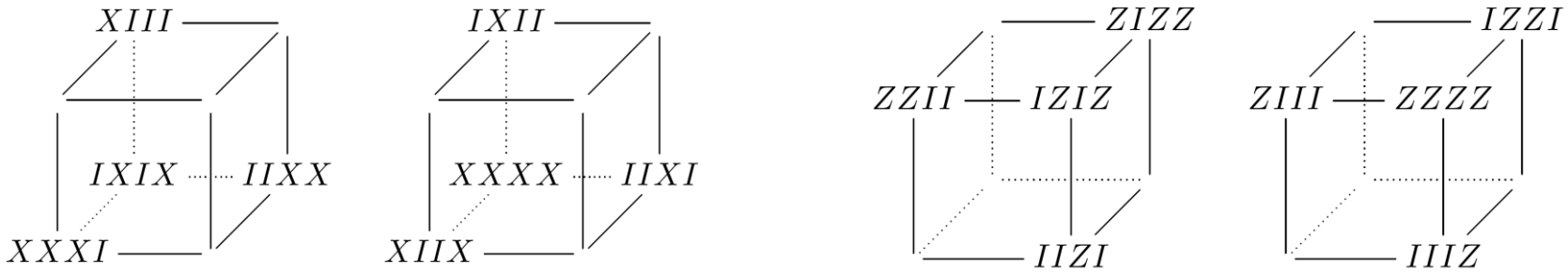
Aasen, Bulmash, Prem, Slagle, W 2020

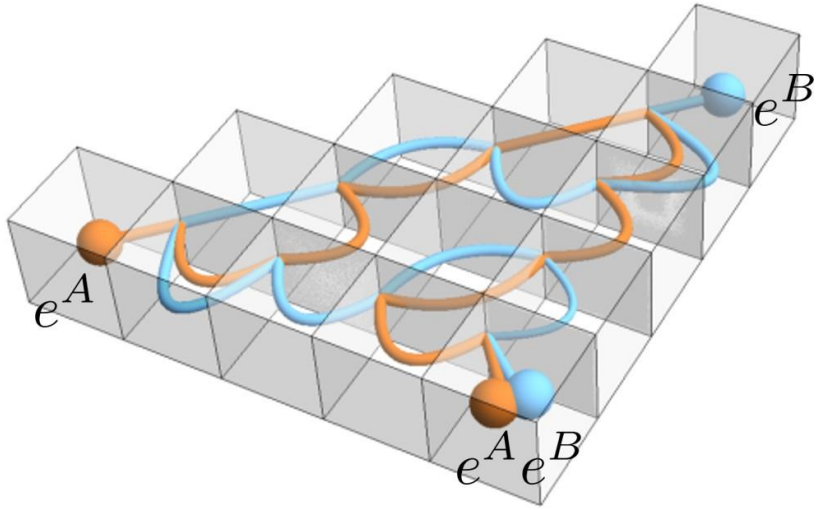


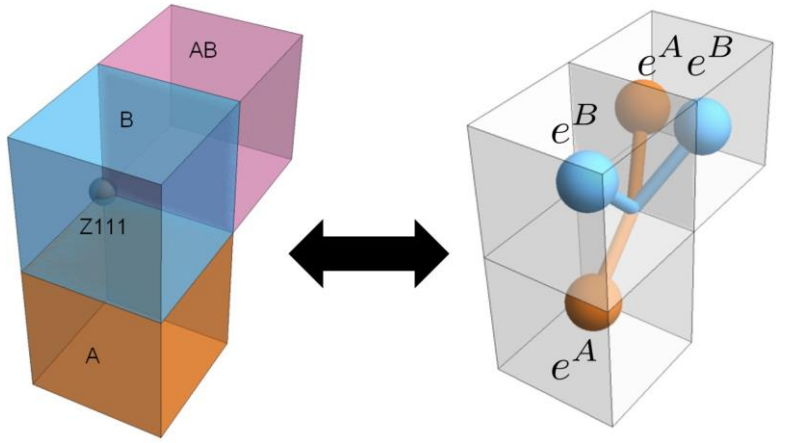


Cubic code 1 B defect network

Aasen, Bulmash, Prem, Slagle, W 2020





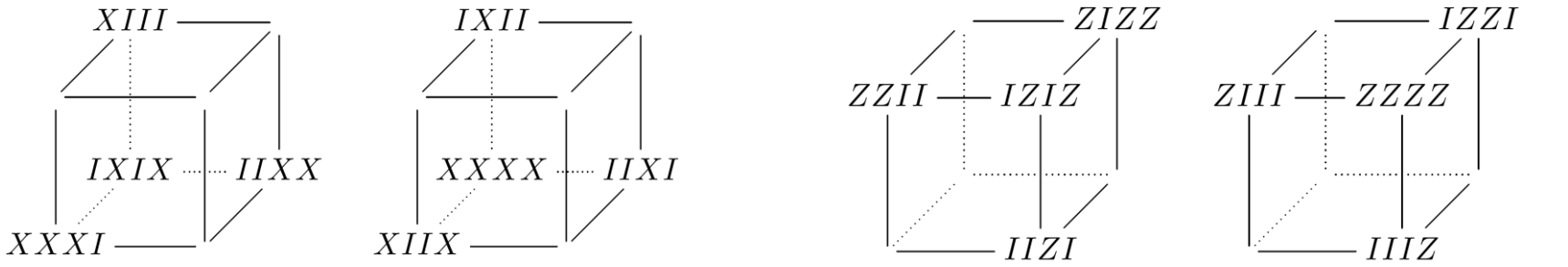


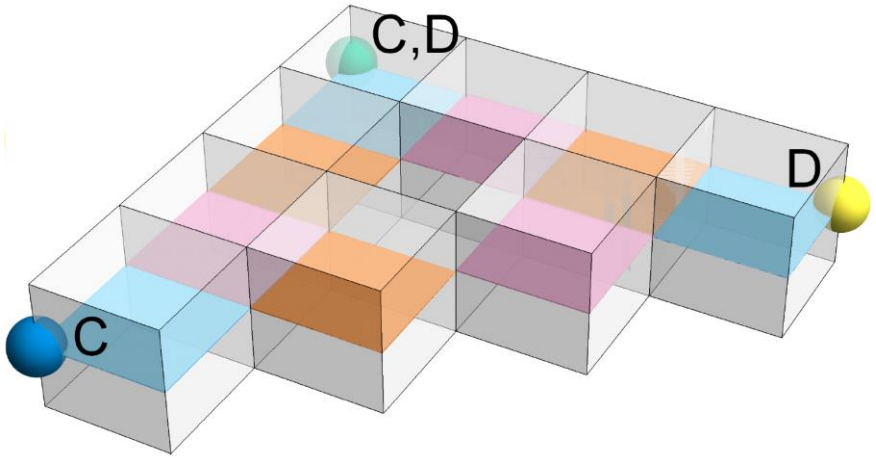
B code charge cluster

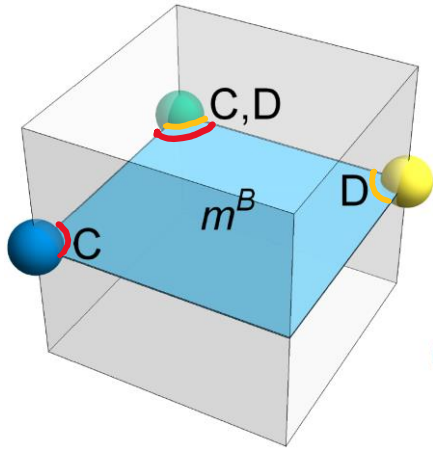
Defect construction

Cubic code 1 B defect network

Aasen, Bulmash, Prem, Slagle, W 2020







Topological defect networks:

- A whole world of weird fracton lattice models
- Fracton phases can be described by TQFT + network of defects
- A family of states with topological order that we have a handle on

The future for topological defect networks:

- Prove that all gapped phases can be described by TDNs
- Utilize the TDN class of states to construct and classify all phases
- Derive universal properties of, and bounds on, TDN states

Questions?