Noise and the frontier of quantum supremacy

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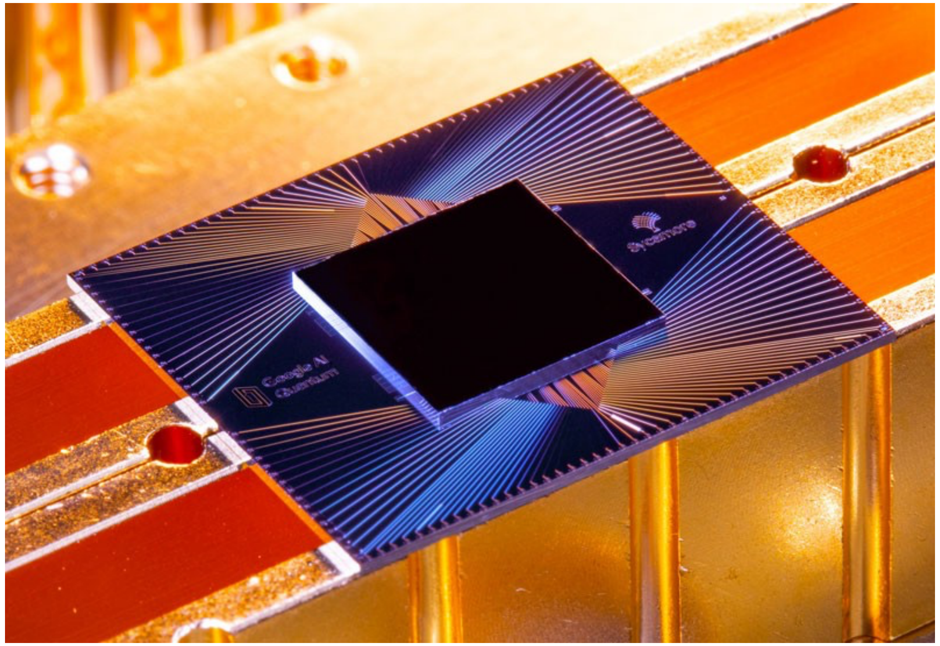
joint work with Adam Bouland (UC Berkeley), Bill Fefferman (U of Chicago), Zeph Landau (UC Berkeley)

Quantum supremacy experiments

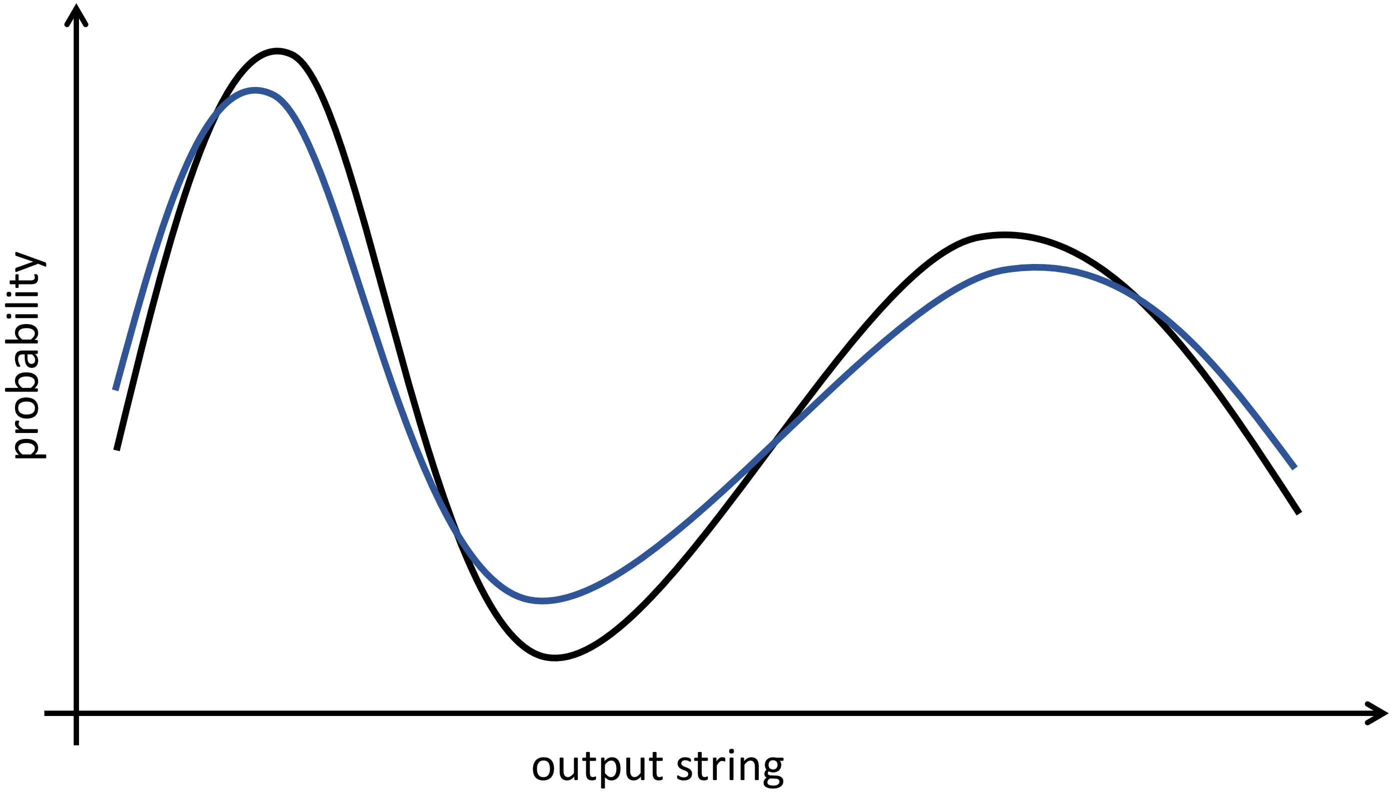


Random Circuit Sampling (Google Sycamore)

BosonSampling (USTC Jiuzhang)

This talk: improved complexity-theoretic evidence that these tasks are hard for classical computers

Evidence of hardness for quantum supremacy experiments

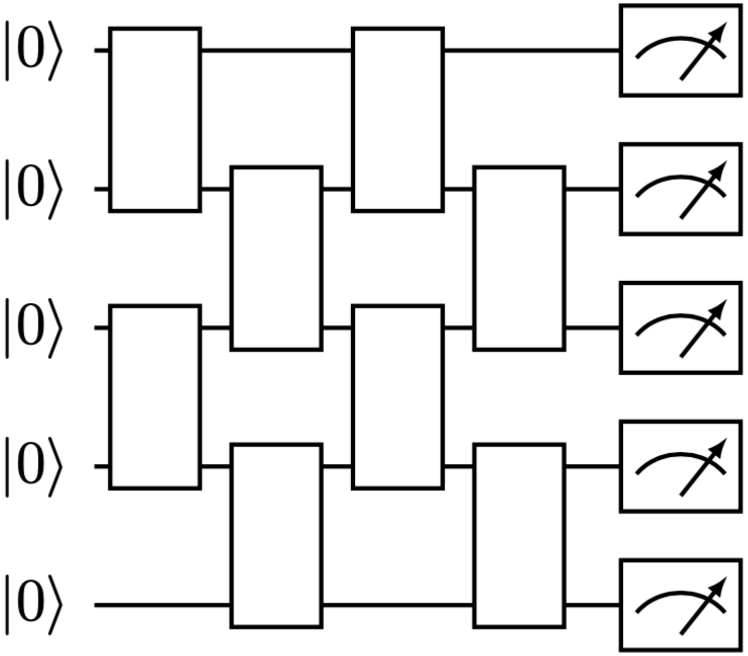


- Model: prove hardness of sampling with high fidelity as system size scales
- *Low noise regime*

- Limitations: cannot prove hardness even in this idealized setting due to insufficient robustness

First result: significantly improve the robustness of prior hardness results

Computational model: low-noise regime



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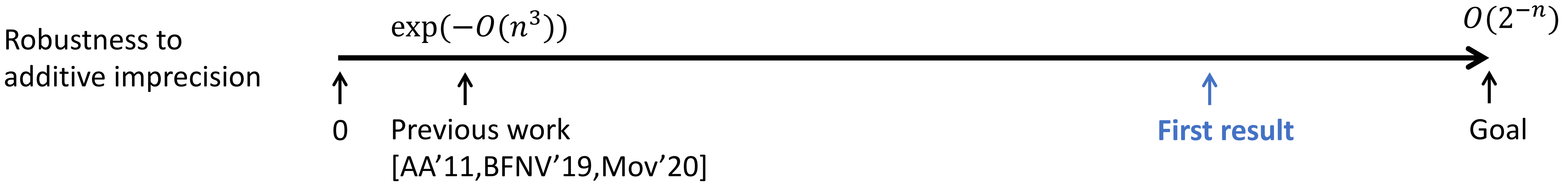
Low-noise regime:

Goal: Prove it is hard to sample from a distribution that is very close to the ideal distribution

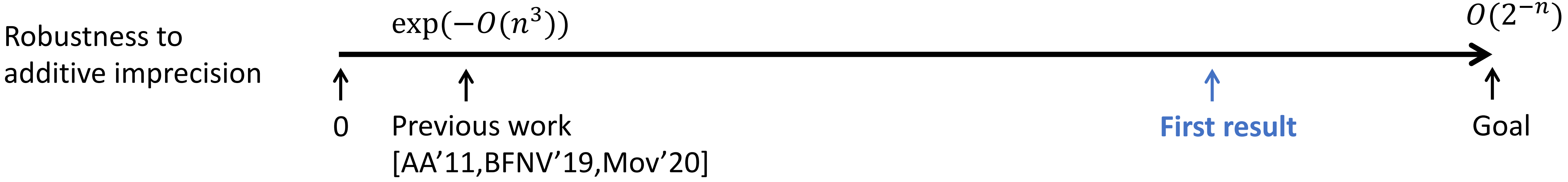
Evidence for hardness of sampling

- To prove hardness of sampling, it suffices to prove robust hardness results for computing the output probability [Stockmeyer'85, AA'11]



Evidence for hardness of sampling

Goal: prove hardness of approximating the output probability of random cirucuits (linear optical networks)
to additive imprecision on the order of the average output probability



Evidence for hardness of sampling

Goal: #P hardness of computing $|\langle 0|C|0\rangle|^2 \pm O(2^{-n})$

Robustness to additive imprecision

$\exp(-O(n^3))$

$O(2^{-n})$

0

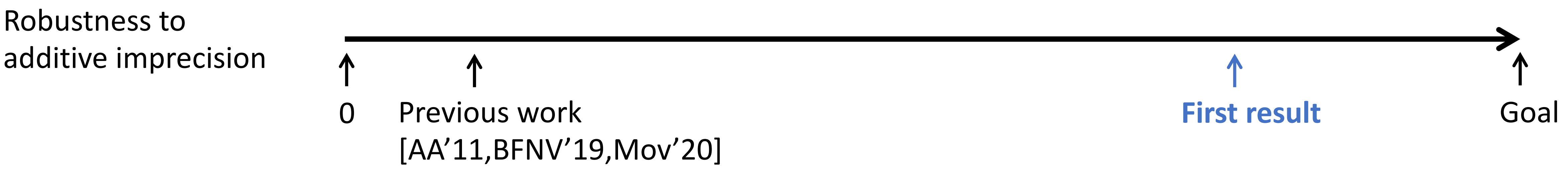
Previous work [AA'11,BFNV'19,Mov'20]

First result

Goal

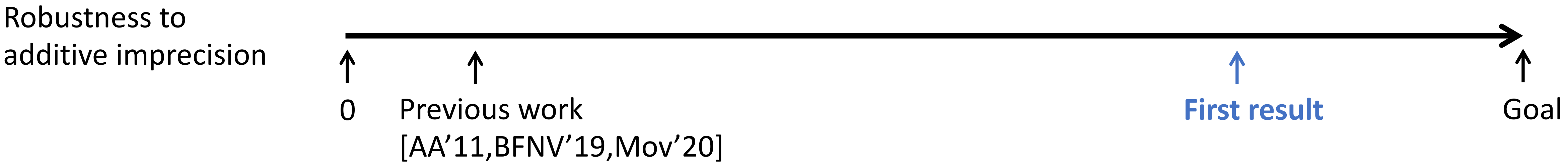
First result: improved robustness in the low-noise regime

Task	Previous result	Our result	Goal	Remark
Random circuit sampling (n qubits, m gates)	$\exp(-O(m^3))$ [BFNV'19, Mov'20]	$\exp\left(-O(m \log m)\right)$	$O(2^{-n})$	



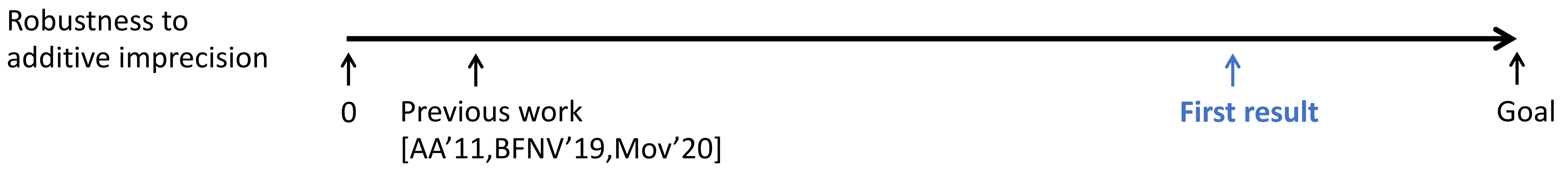
First result: improved robustness in the low-noise regime

Task	Previous result	Our result	Goal	Remark
Random circuit sampling (n qubits, constant depth)	$\exp(-O(n^3))$ [BFNV'19, Mov'20]	$\exp\left(-O(n\log n)\right)$	$O(2^{-n})$	For constant depth circuits, tight up to $O(\log n)$ factor in the exponent

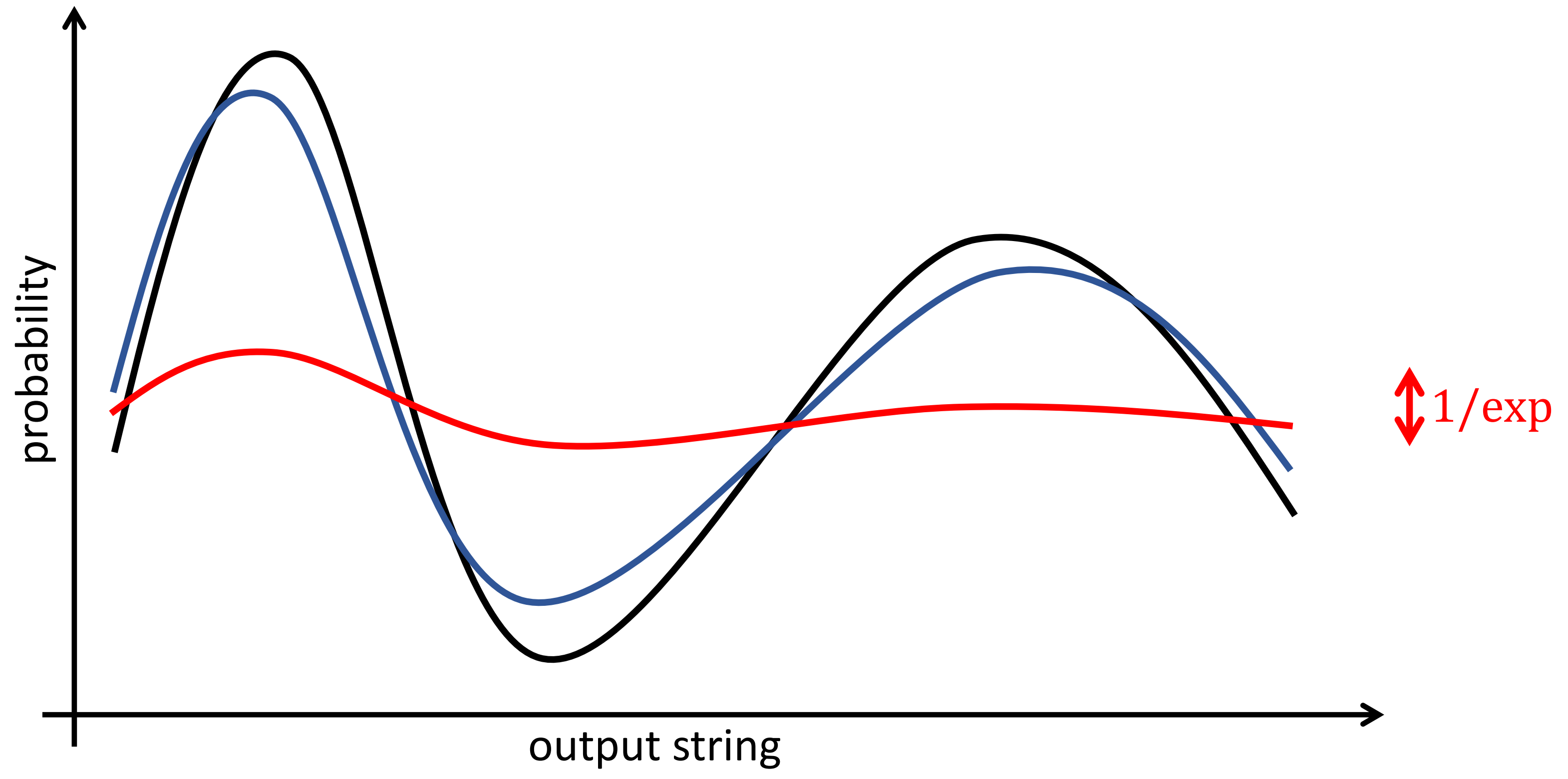


First result: improved robustness in the low-noise regime

Task	Previous result	Our result	Goal	Remark
Random circuit sampling (n qubits, constant depth)	$\exp(-O(n^3))$ [BFNV'19, Mov'20]	$\exp\left(-O(n \log n)\right)$	$O(2^{-n})$	For constant depth circuits, tight up to $O(\log n)$ factor in the exponent
BosonSampling (n photons, n^2 detectors)	$\exp(-O(n^4))$ [AA'11]	$\exp(-6n \log n)$	$\exp(-n \log n)$	Tight up to constant factor in the exponent

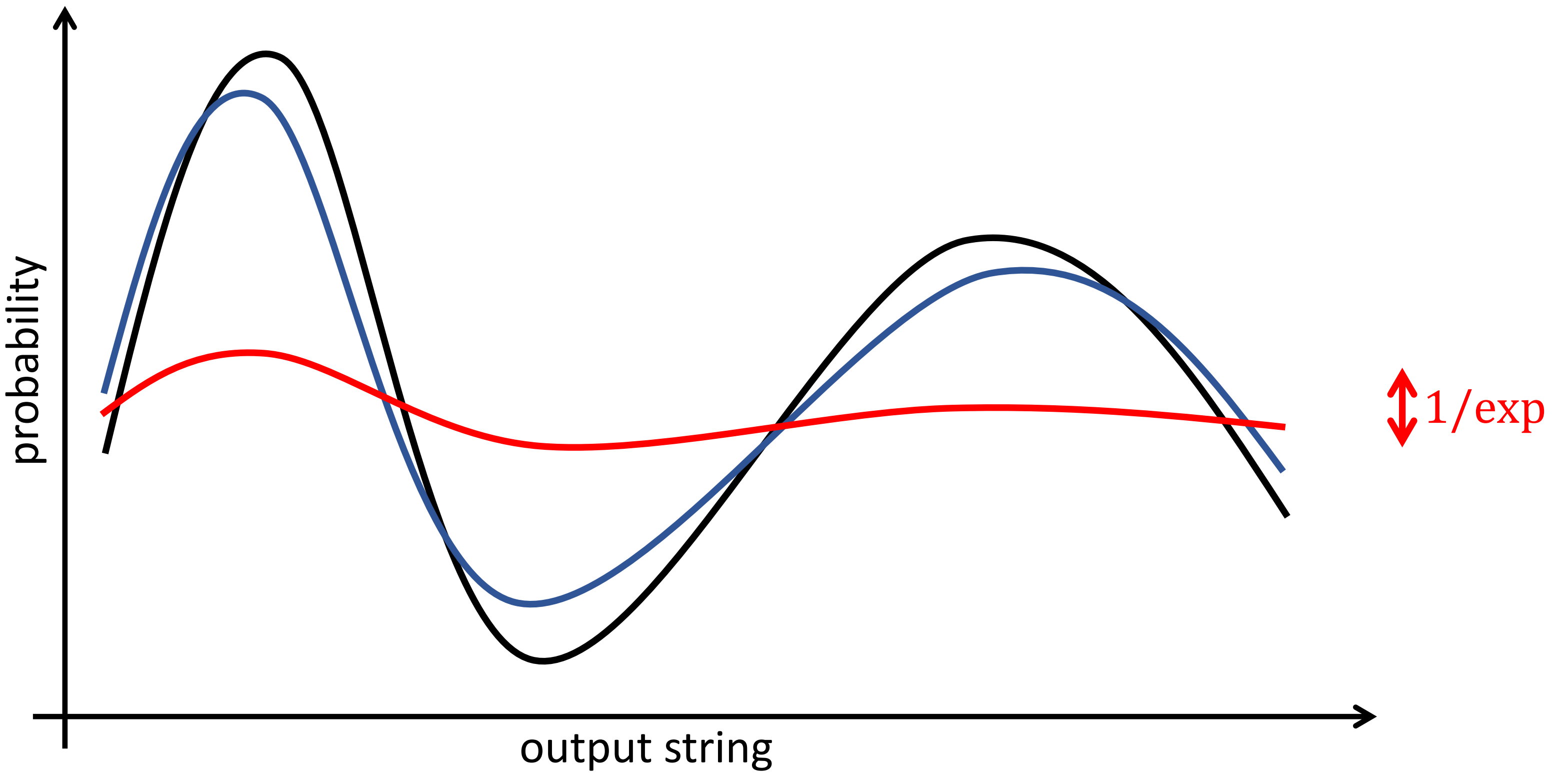


Theory vs. Experiment



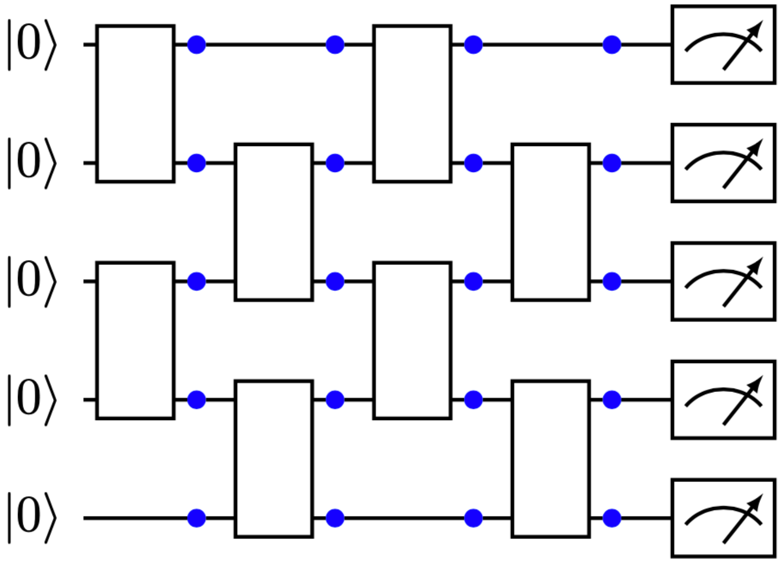
Low-noise regime: Goal is to prove hardness of sampling from a distribution that is very close to the ideal distribution

Theory vs. Experiment



High noise regime: in experiments we only observe a tiny deviation from uniform along the correct direction

New model: high-noise regime



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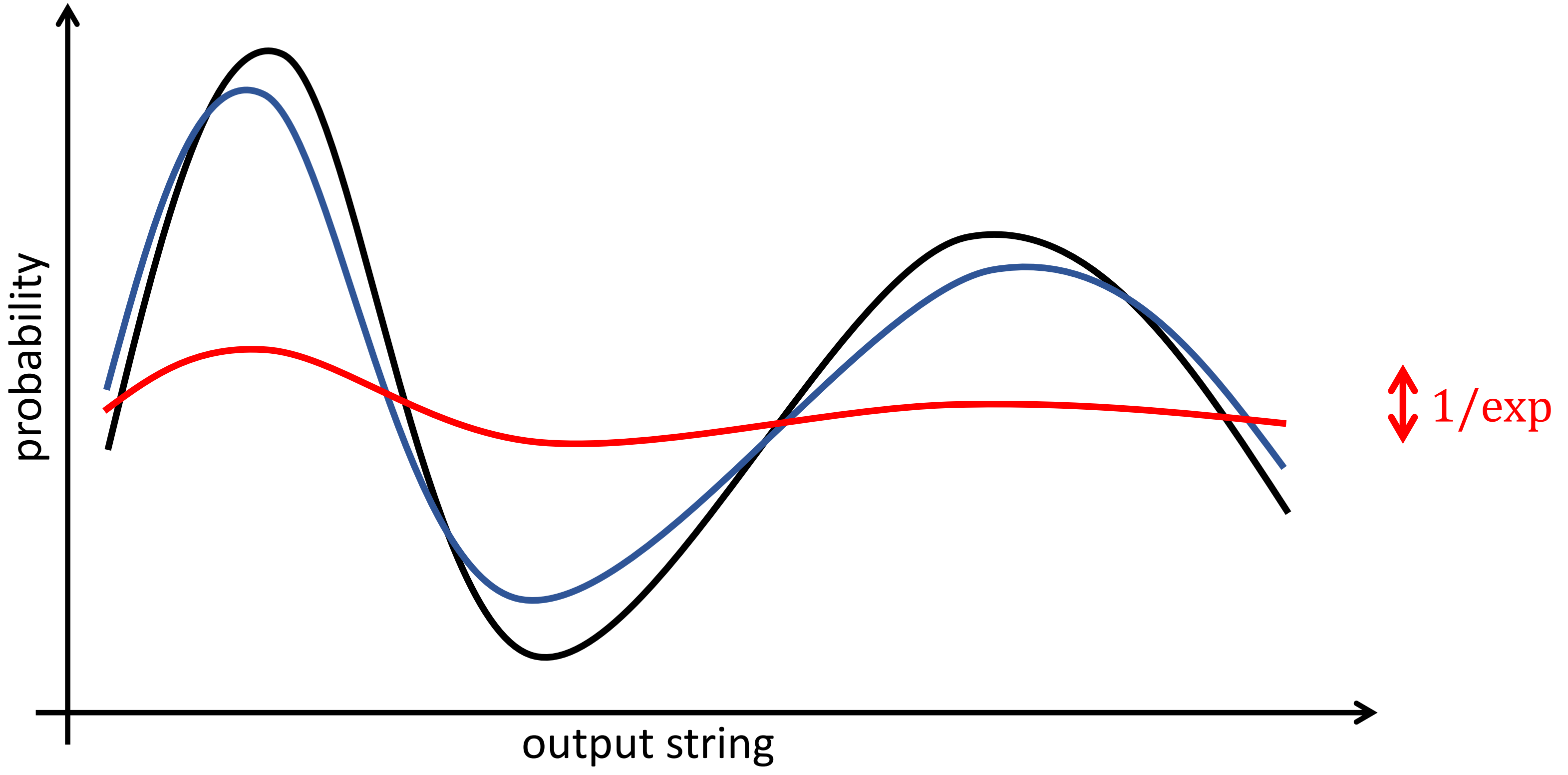
000100010110101000101100100001010000001101000010100 10...

High-noise regime:

Given a random circuit, a fixed noise model (constant noise rate),
sample from the (exact) output distribution of the noisy circuit

New model: high-noise regime

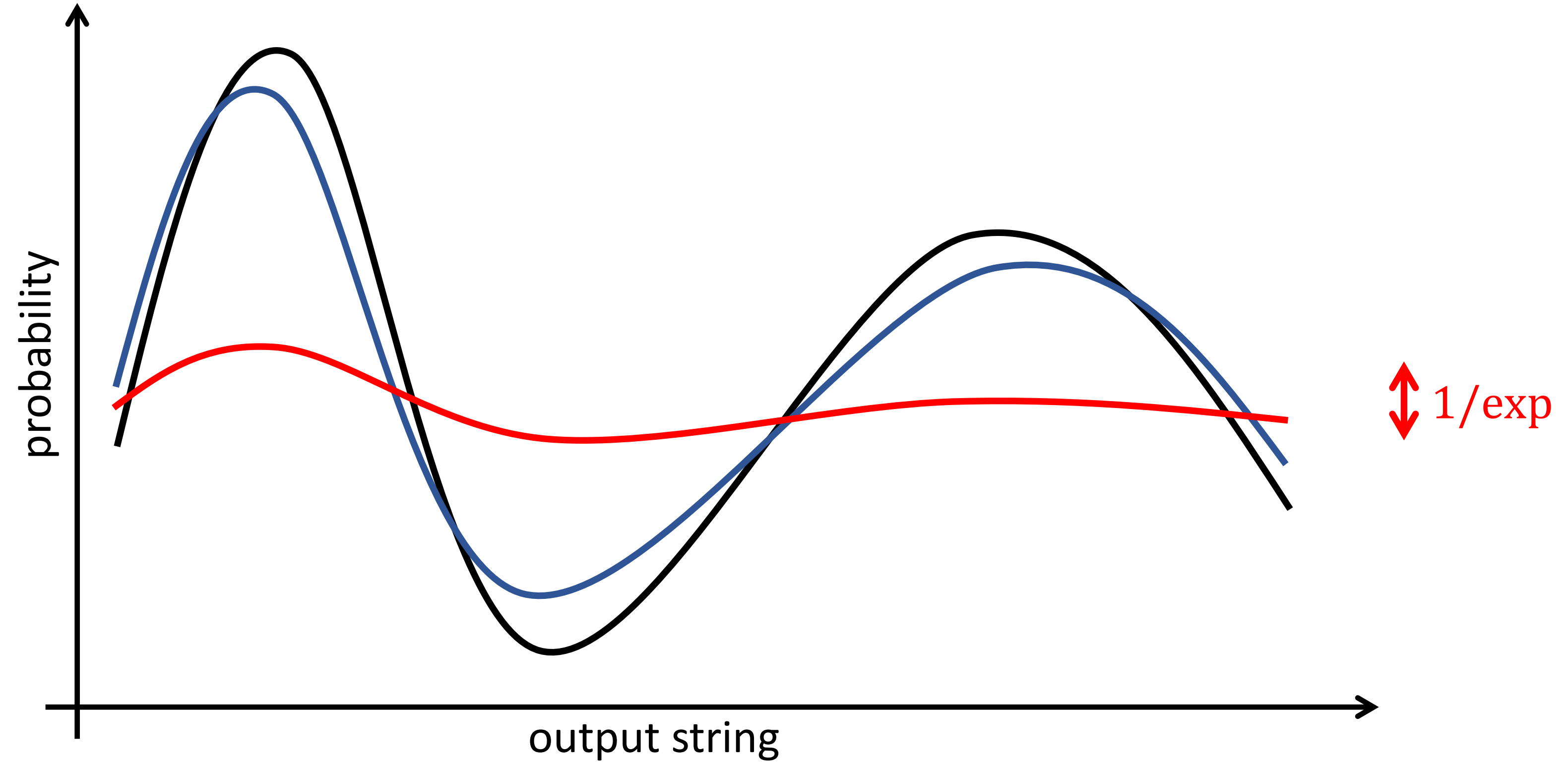


noisy random circuits converge to uniform exponentially quickly

[ABOIN’96, GD’18]

Second result: these tiny signals remain hard to *compute*

New model: high-noise regime

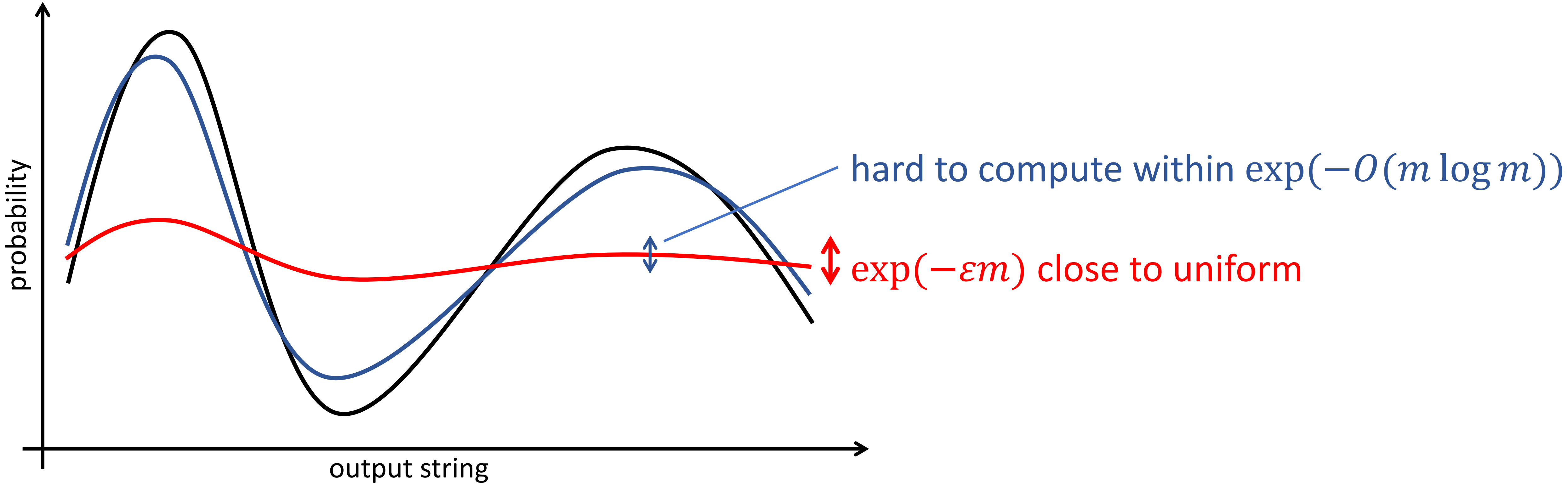


noisy random circuits converge to uniform with speed $\exp(-\varepsilon m)$

[Boixo et al’18]

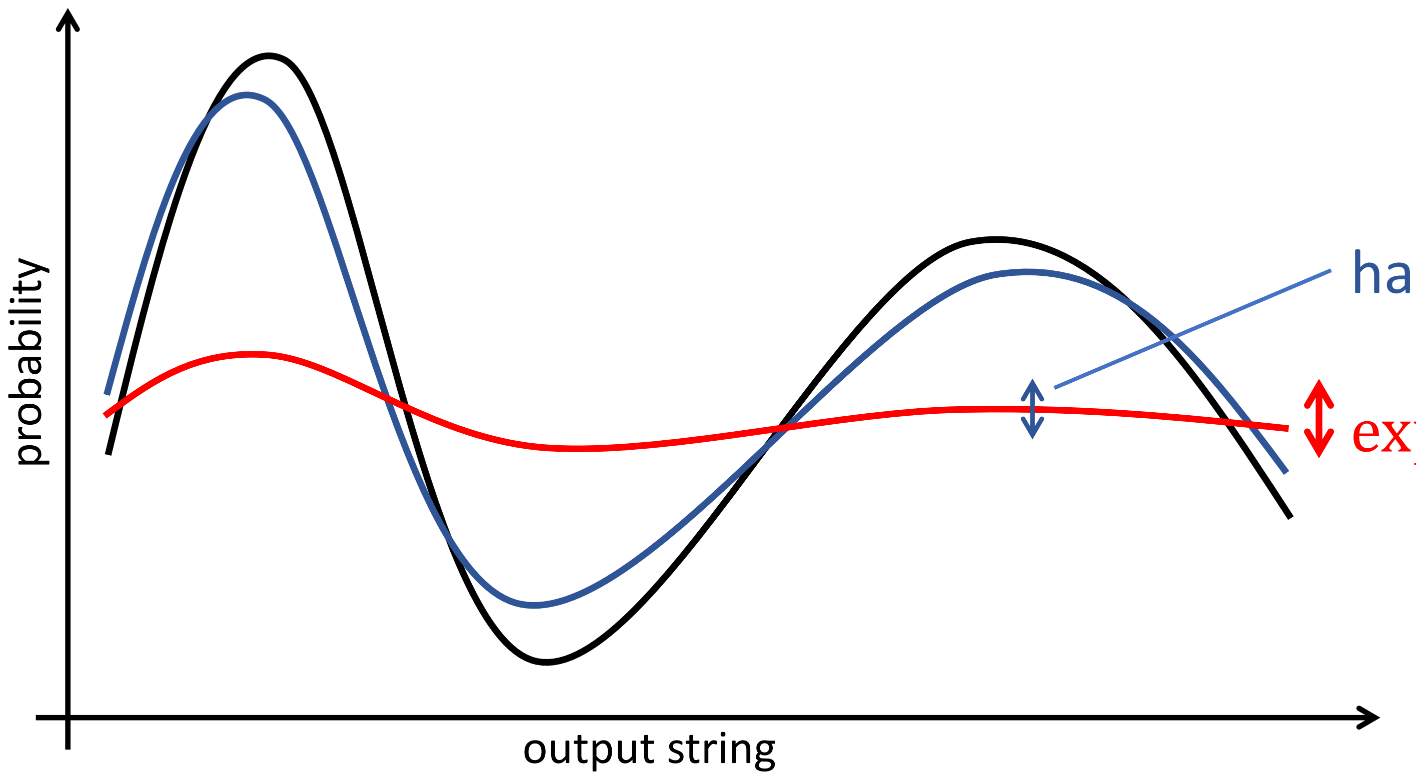
Second result: these tiny signals remain hard to *compute*

Second result: evidence of hardness in the high-noise regime



Second result: these tiny signals remain hard to *compute*

Second result: evidence of hardness in the high-noise regime



hard to compute within $\exp(-O(m \log m))$

$\exp(-\varepsilon m)$ close to uniform

Cannot improve our high-noise result much further, due to the exponential convergence to uniform

Second result: these tiny signals remain hard to *compute*

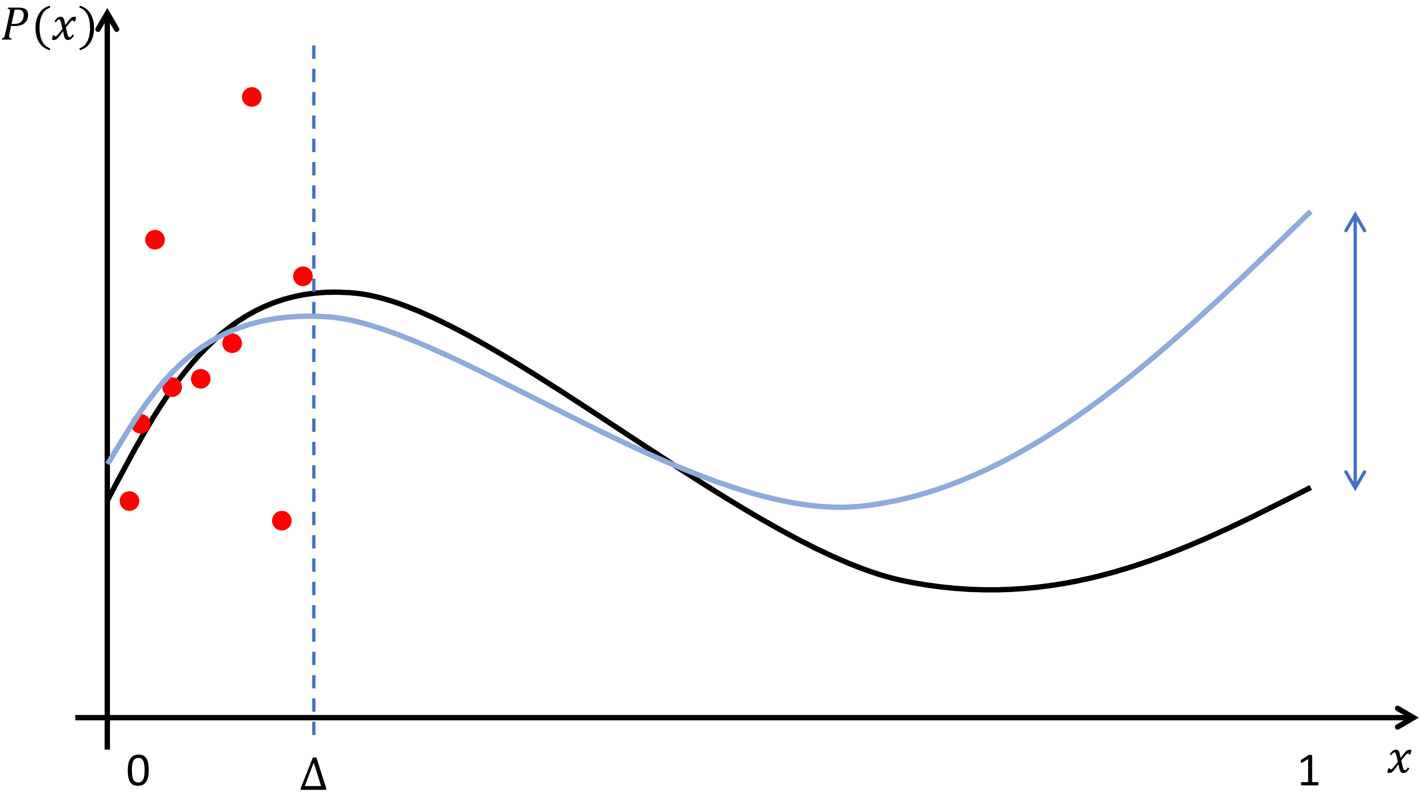
Proof sketch

An algorithm for computing the output probability of random circuits

↓ Polynomial structure [AA'11, BFNV'19, Mov'20]

An algorithm for computing the output probability of any circuit

Proof techniques: first result



- The problem reduces to polynomial interpolation on noisy data points [AA'11, BFNV'19, Mov'20]
- We develop a robust Berlekamp-Welch argument that
  - simplifies the proof
  - tolerates more errors
  - reduces the extrapolation error

Proof techniques: second result

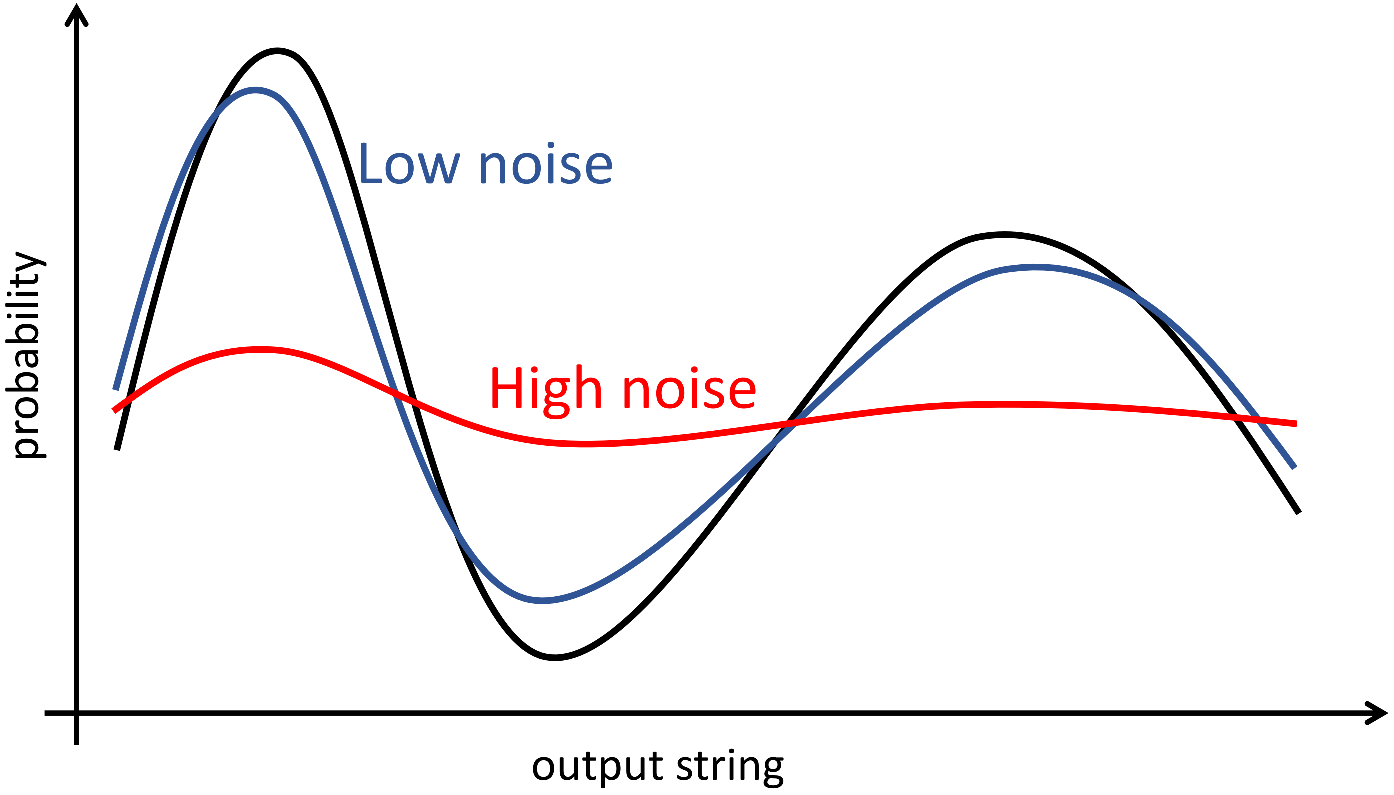
Low noise result → High noise result

Linearity of noise channels

Preserves the polynomial structure

- The same worst to average case reduction techniques also apply to the high noise setting
- *Q: what about worst case hardness?*
- A: error detection [Fujii'16]

Summary of our results



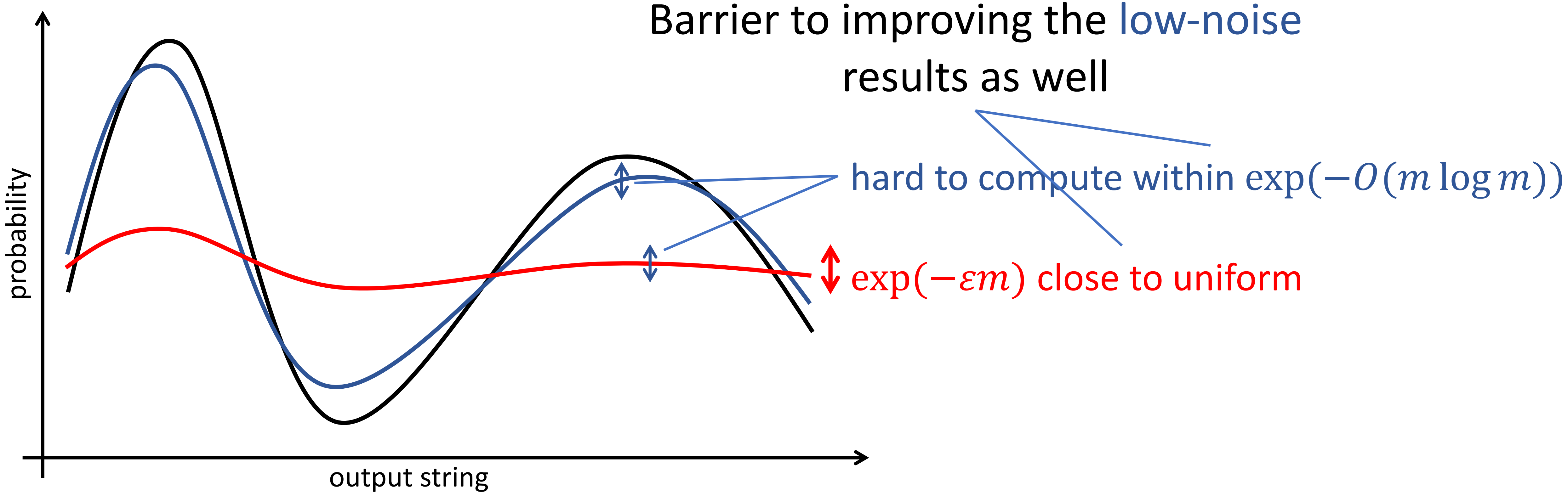
- Our result: we substantially improve the robustness of prior hardness results in the low noise setting

↕ optimality

- Our result: we give initial evidence of hardness with exponentially decreasing fidelity

High-noise result implies barrier to improving low-noise result

Barrier to improving the low-noise results as well



Barriers to proving hardness of sampling (in the low-noise regime)

Random circuit sampling

- Noise barrier [This work]
- Depth barrier [Napp et al'20] (see talk on Friday)
- Polynomial interpolation barrier [AA'11]

BosonSampling

- Polynomial interpolation barrier [AA'11]
- *Q: do noise and depth barriers apply?*

- Our result: $\exp\big(-O(n \log n)\big)$
- Goal: $O(2^{-n})$

- Our result: $\exp(-6n \log n)$
- Goal: $\exp(-n \log n)$