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The cost of universality

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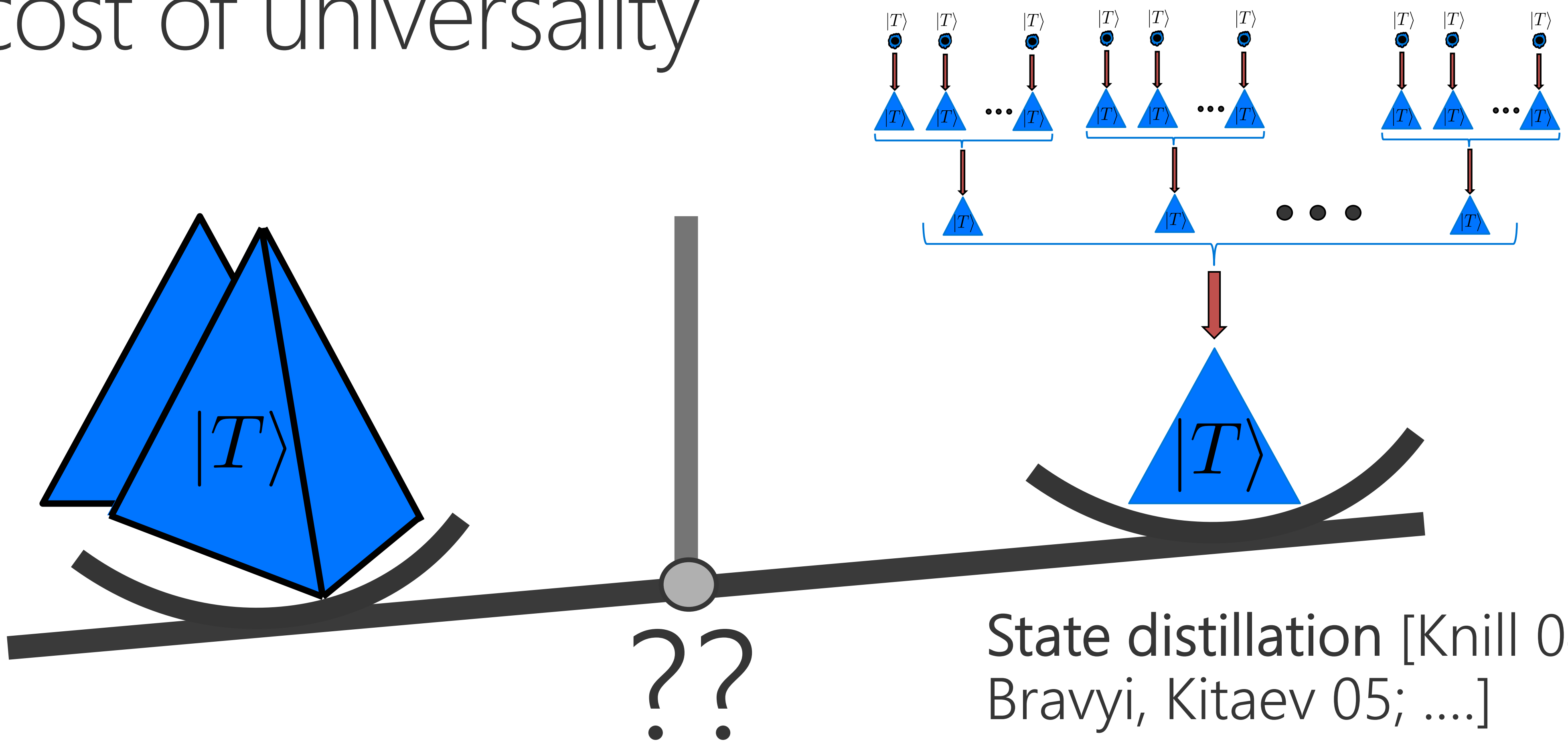
Microsoft Quantum

Work with: Aleksander Kubica (AWS) and Krysta Svore (Microsoft)
arXiv:2101.02211

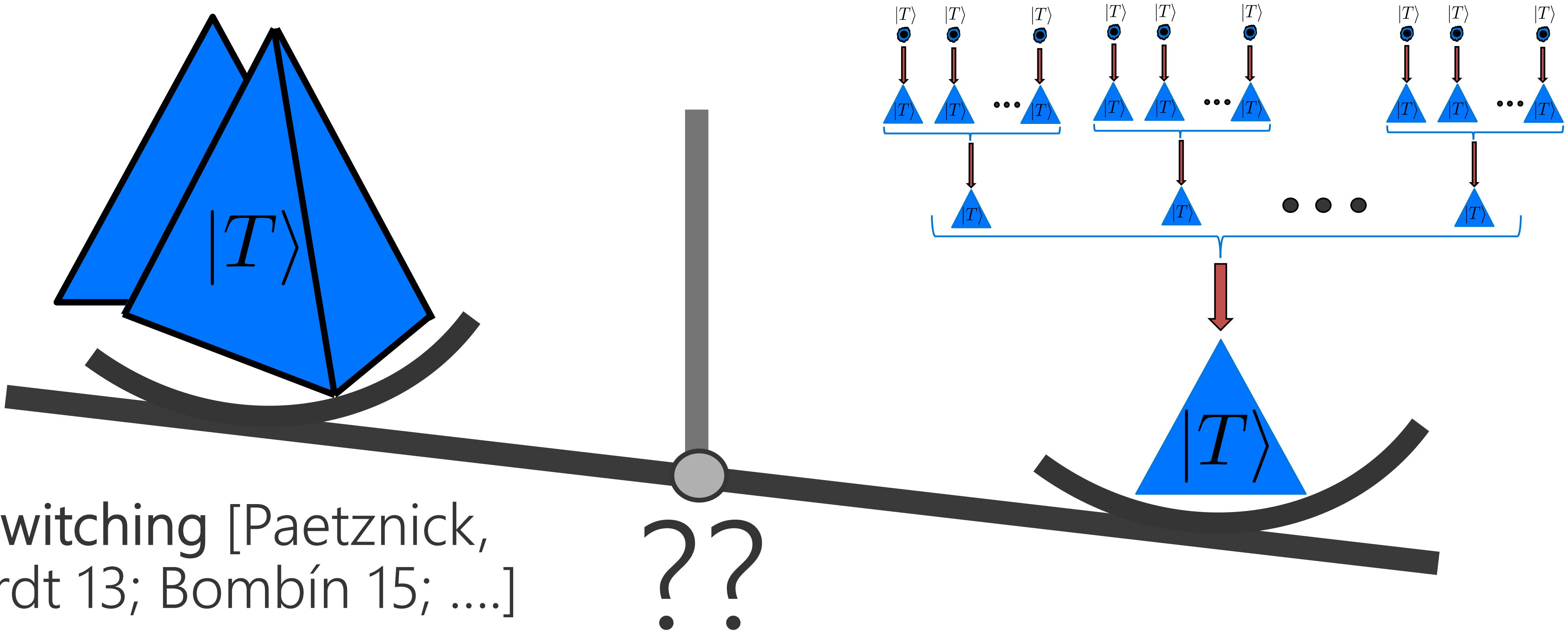
The cost of universality



State distillation [Knill 04; Bravyi, Kitaev 05; ....]

A comparative study of the overhead of state distillation and code switching with color codes

The cost of universality



Code switching [Paetznick, Reichardt 13; Bombín 15; ....]

A comparative study of the overhead of state distillation and code switching with color codes

The cost of universality

1. Assumptions
2. 2D hardware
3. Non-Clifford gates in 2D: Distillation
4. Non-Clifford gates in 3D: Code switching
5. Results and conclusions

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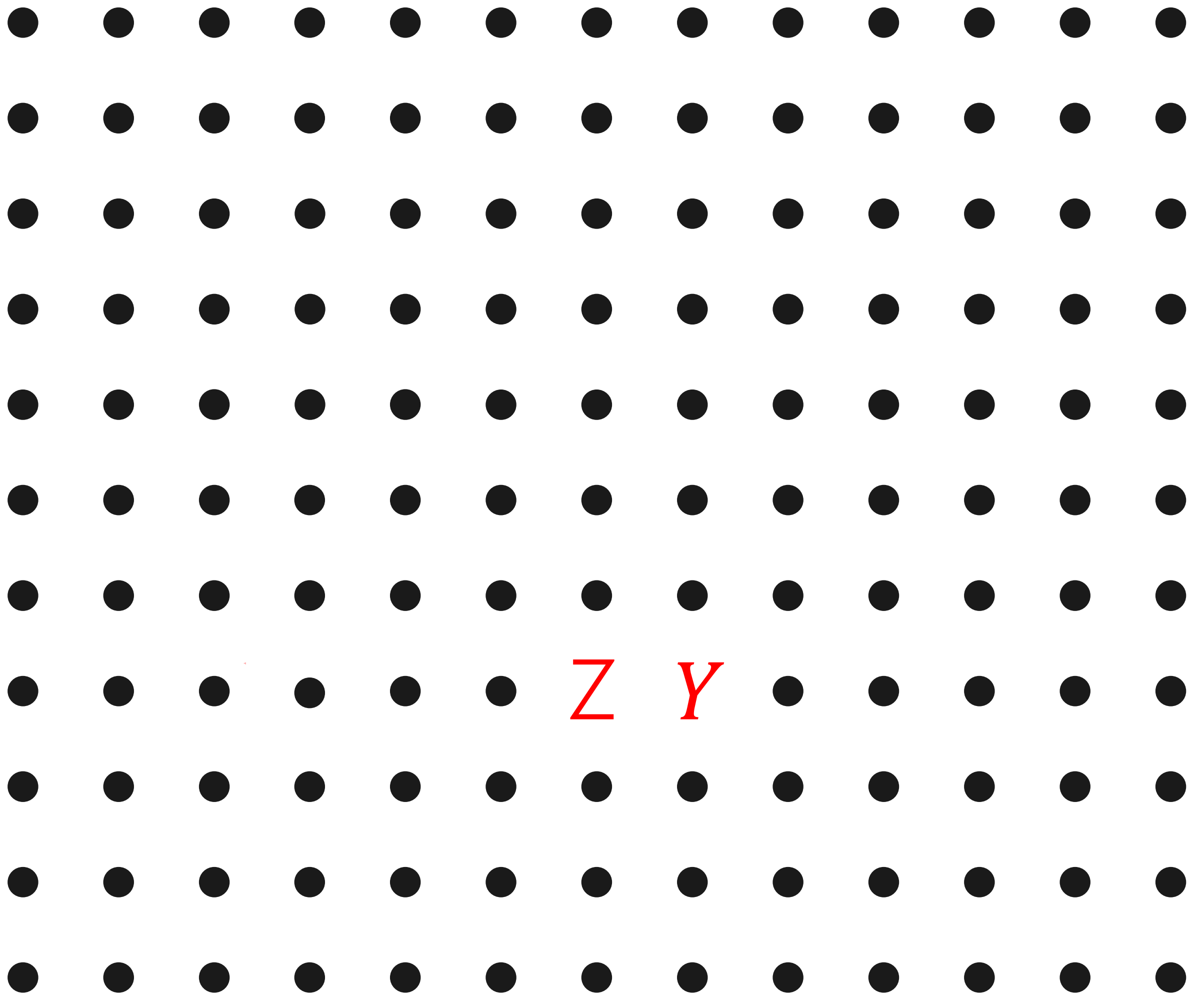
LOCAL OPERATIONS AND NOISE

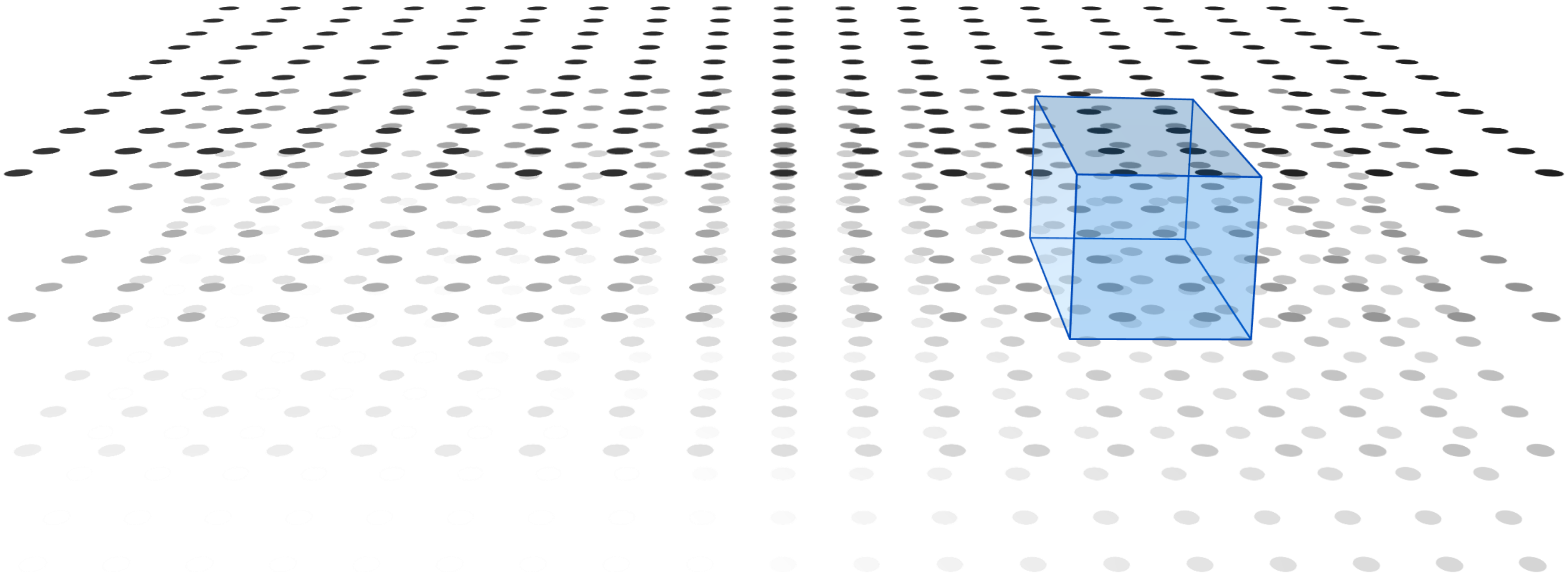
2D local operations

Quantum ops on nearby pairs of qubits.

Quantum operations fail with prob p.



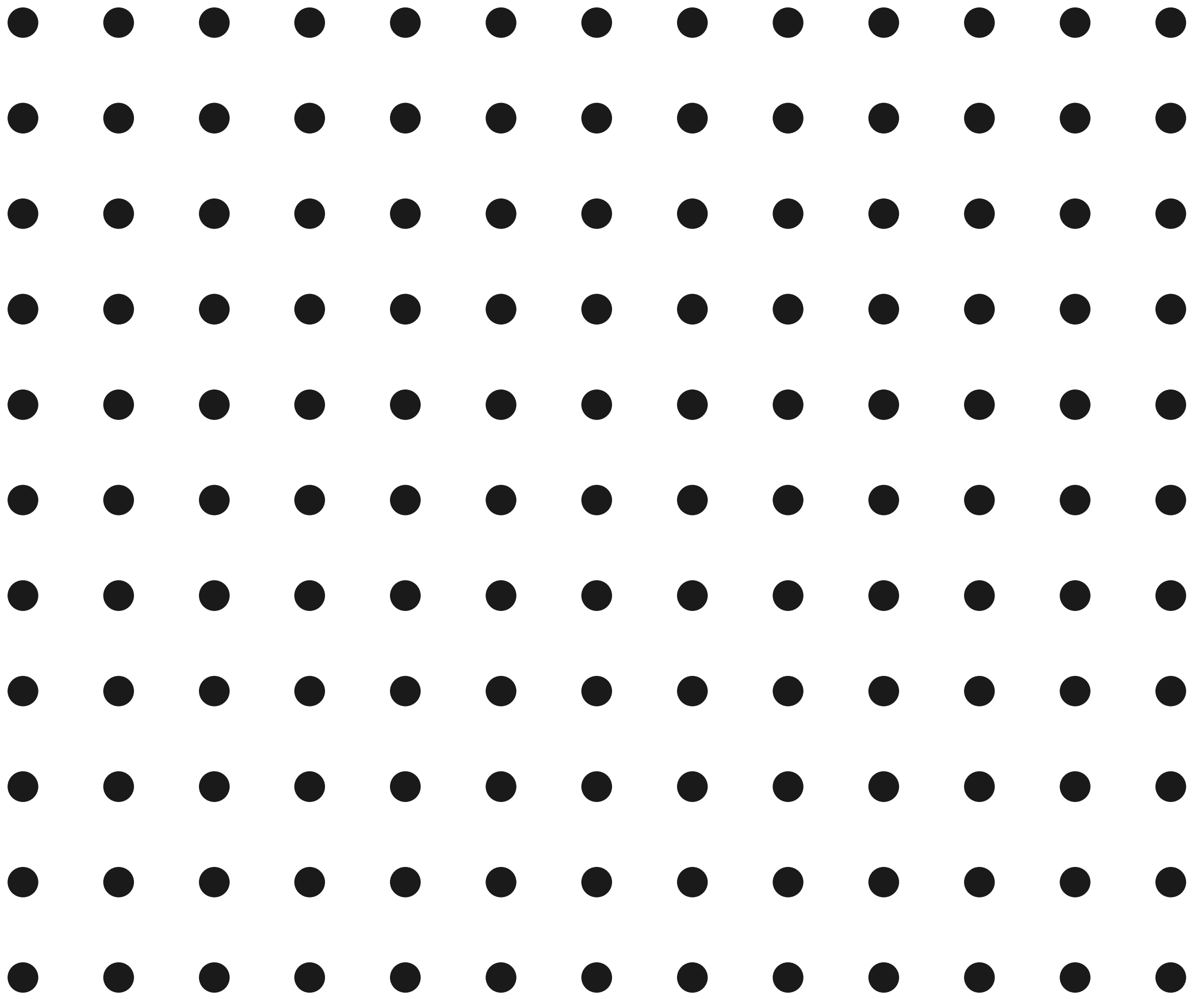
~~2D~~3D local operations



WHAT CAN BE DONE IN 2D?

2D quantum-local operations

Can build surface codes.



2D quantum-local operations

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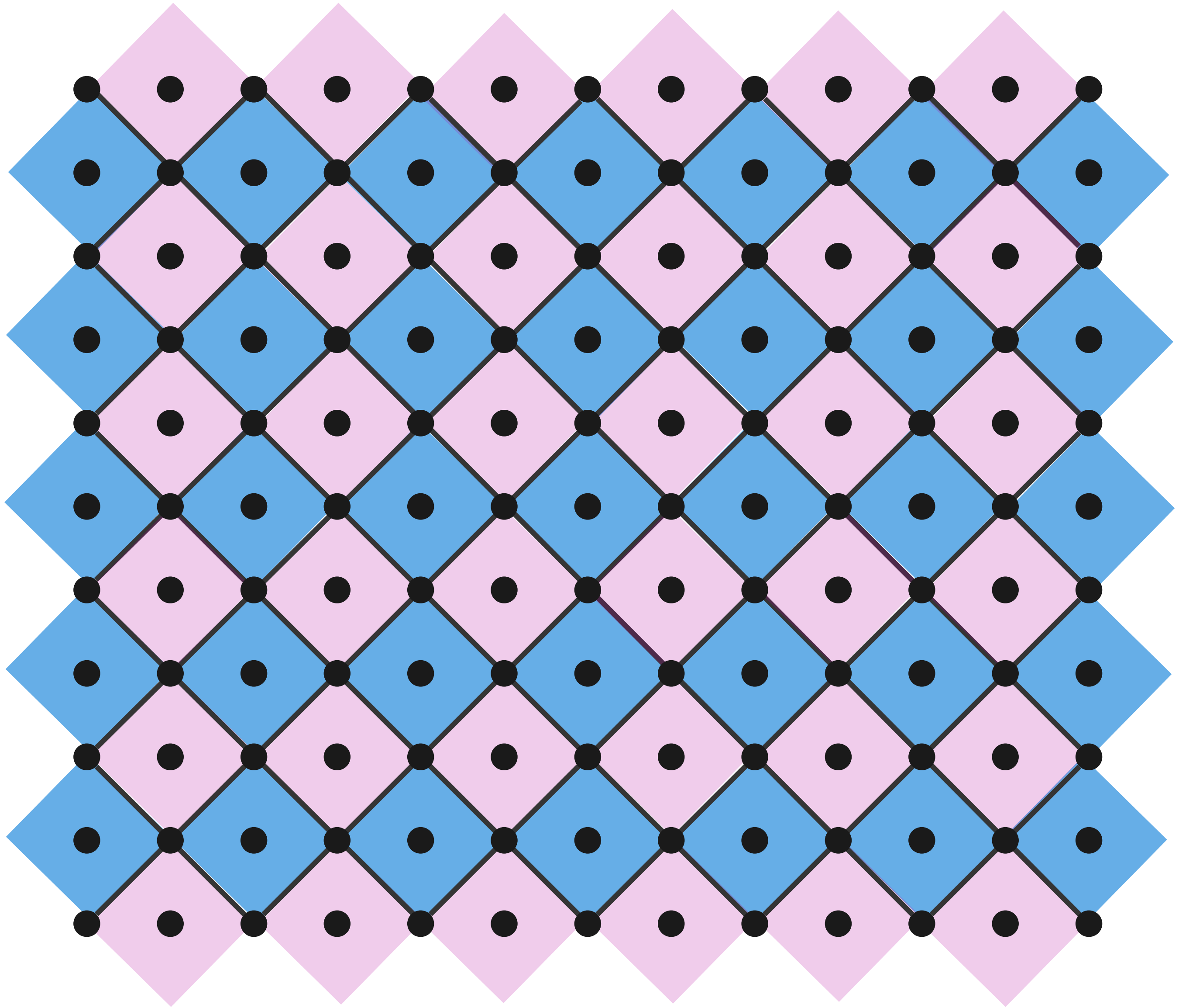

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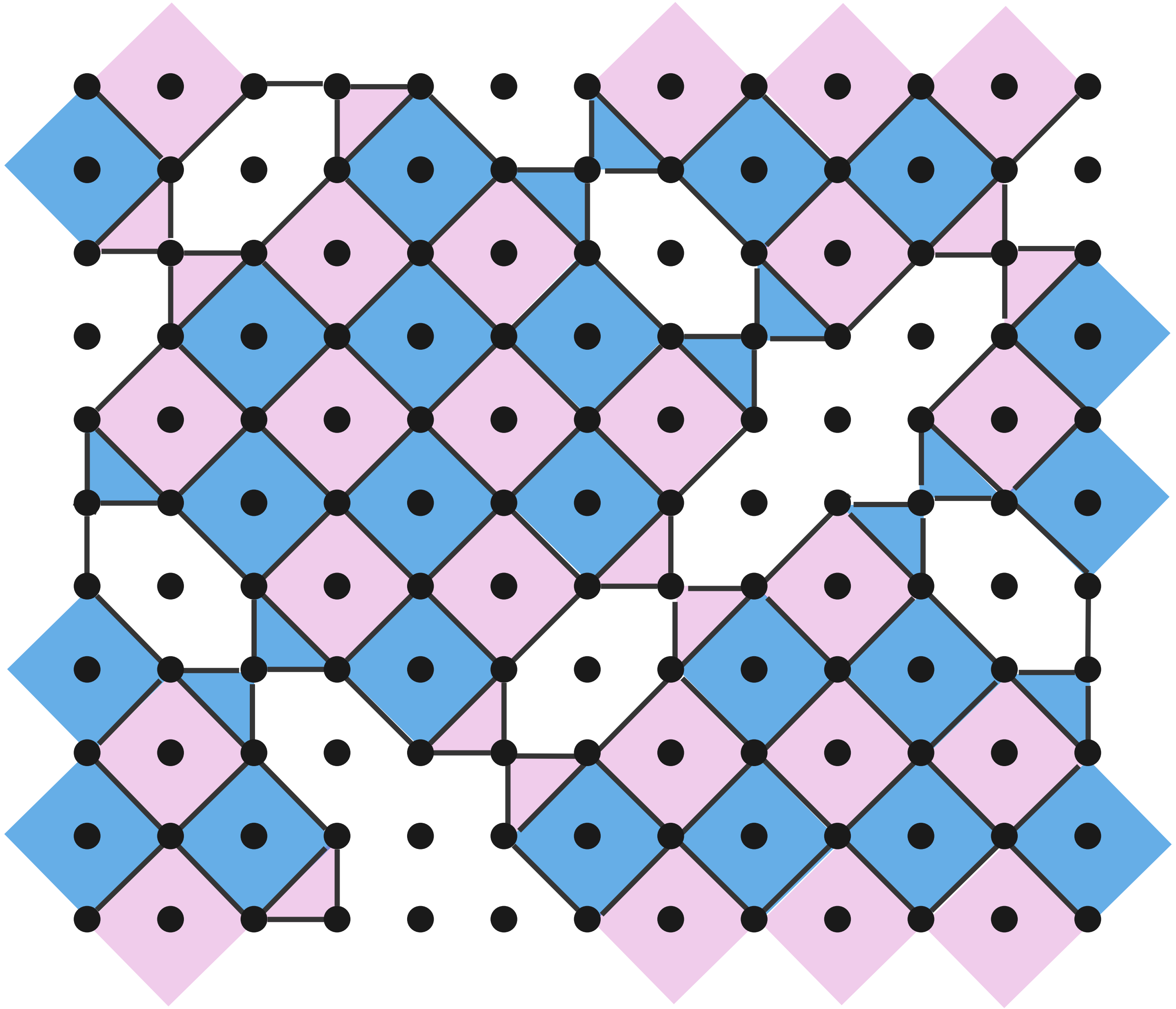

2D quantum-local operations

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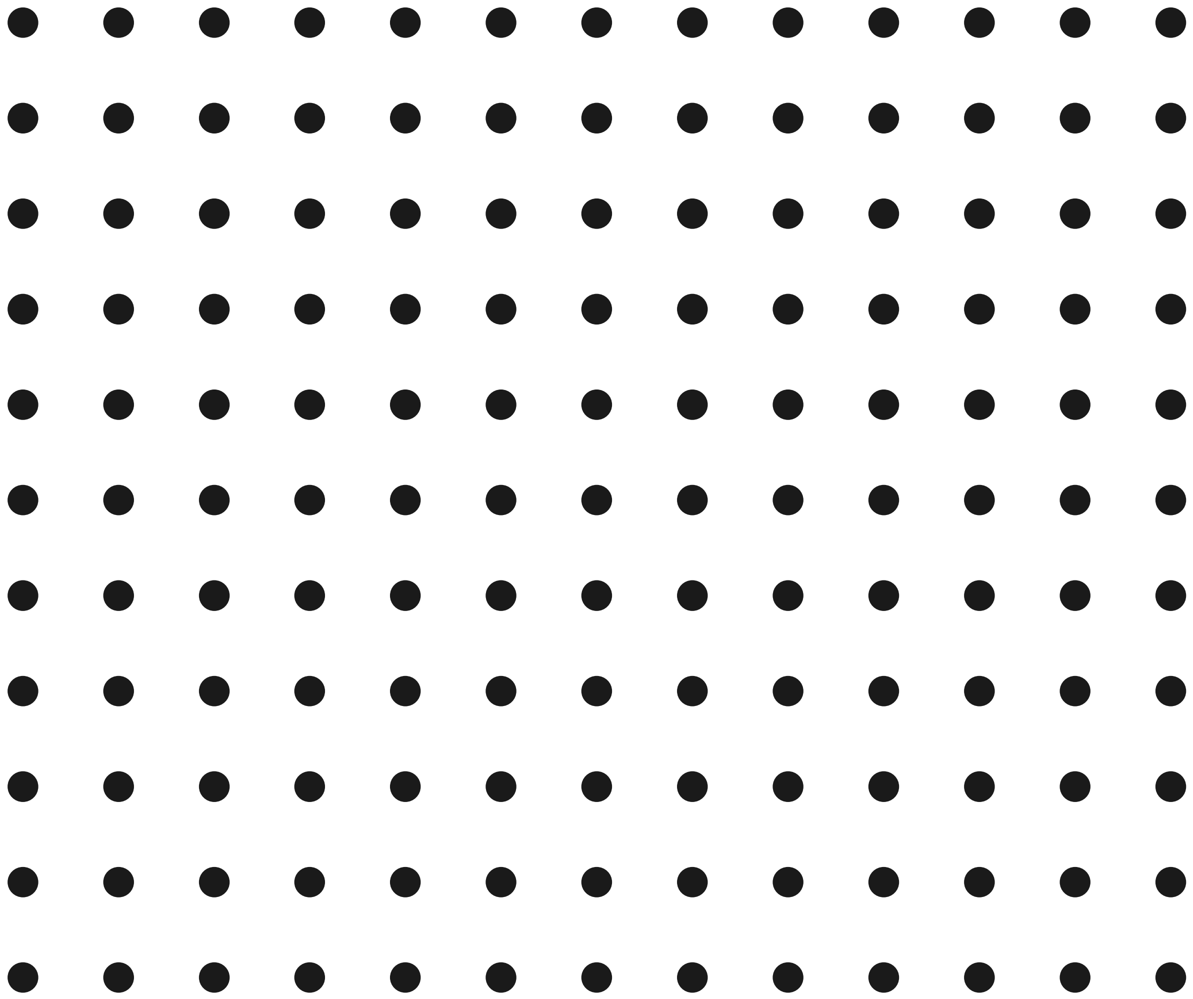

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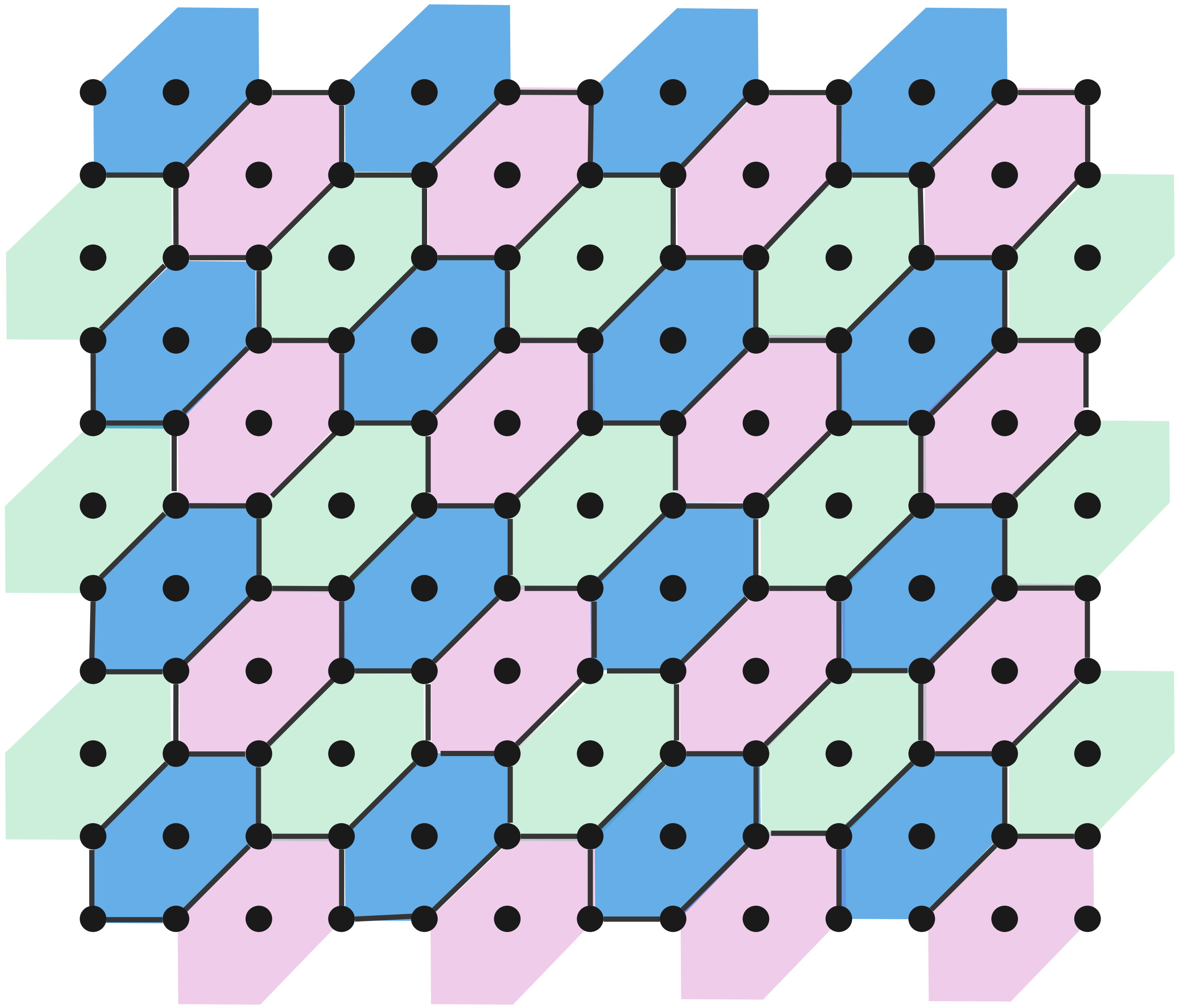
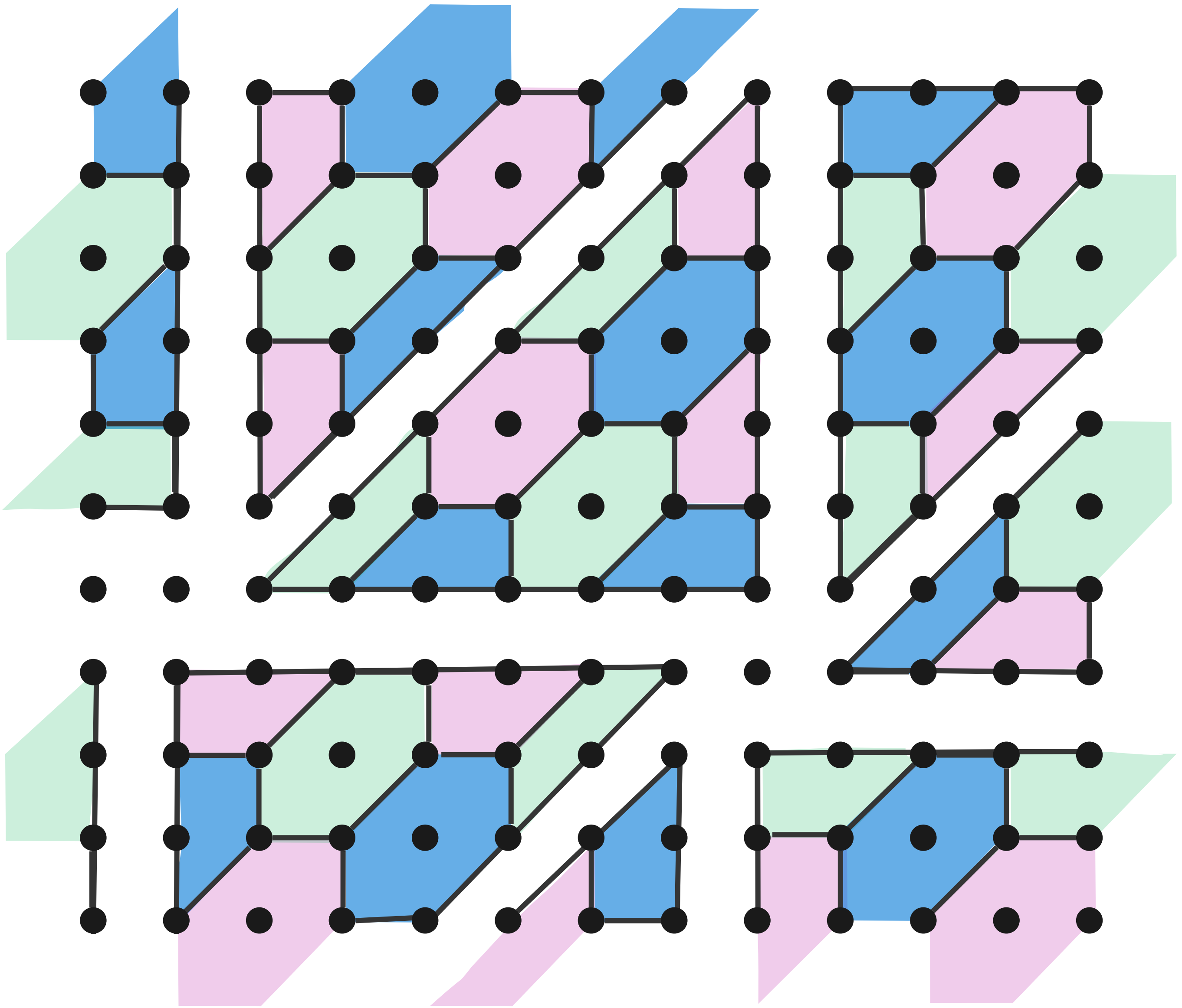


2D quantum-local operations

Can build color codes.

$\bar{p} \propto (p/p^*)^{Cd}$ protection.

with high threshold:
$\mathrm{p}^* \sim 10^{-3} - 10^{-2}$.



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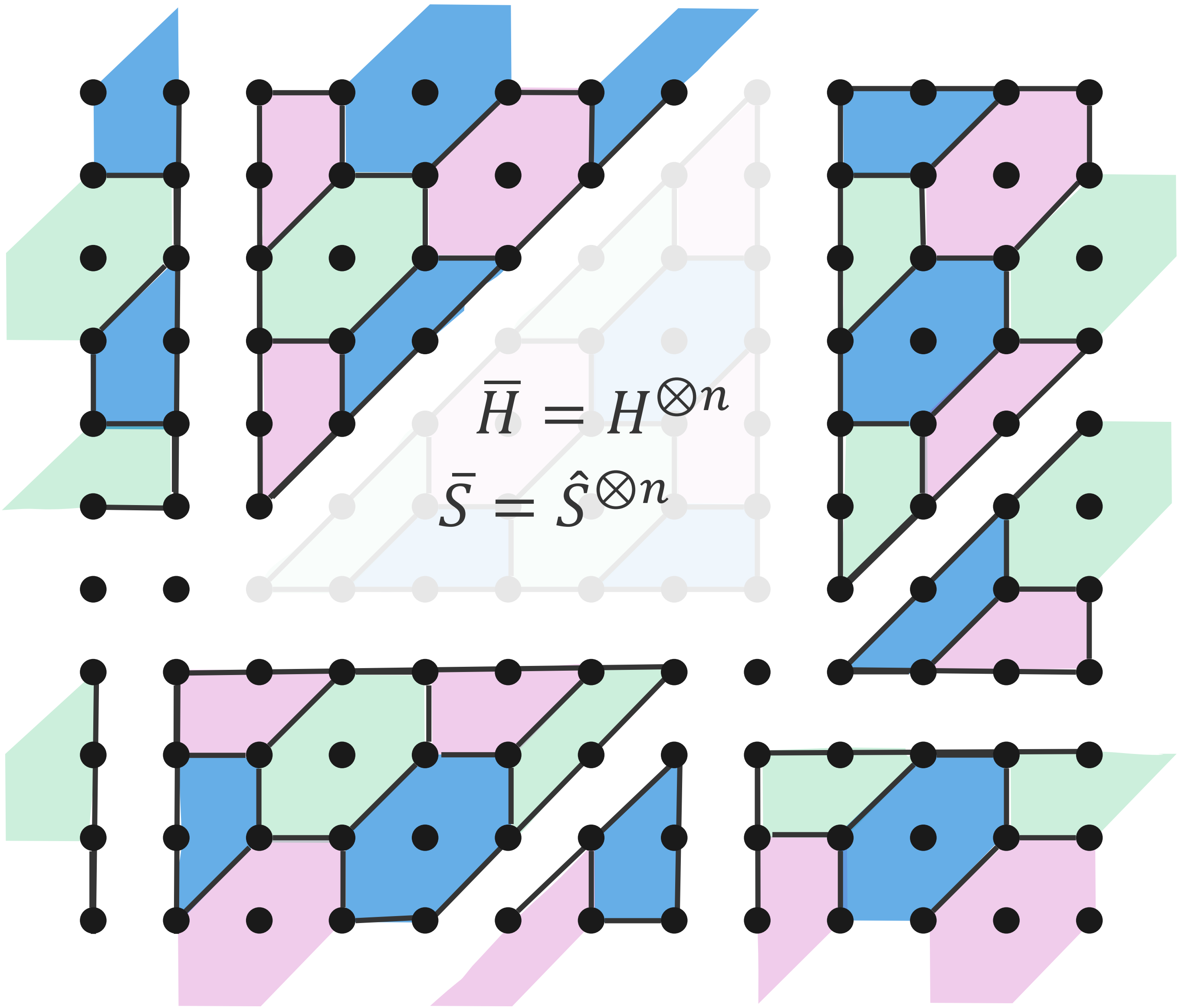
Transverse 1-qubit
Clifford group.



2D quantum-local operations

Can build color codes.

$\bar{p} \propto (p/p^*)^{Cd}$ protection.

with high threshold:
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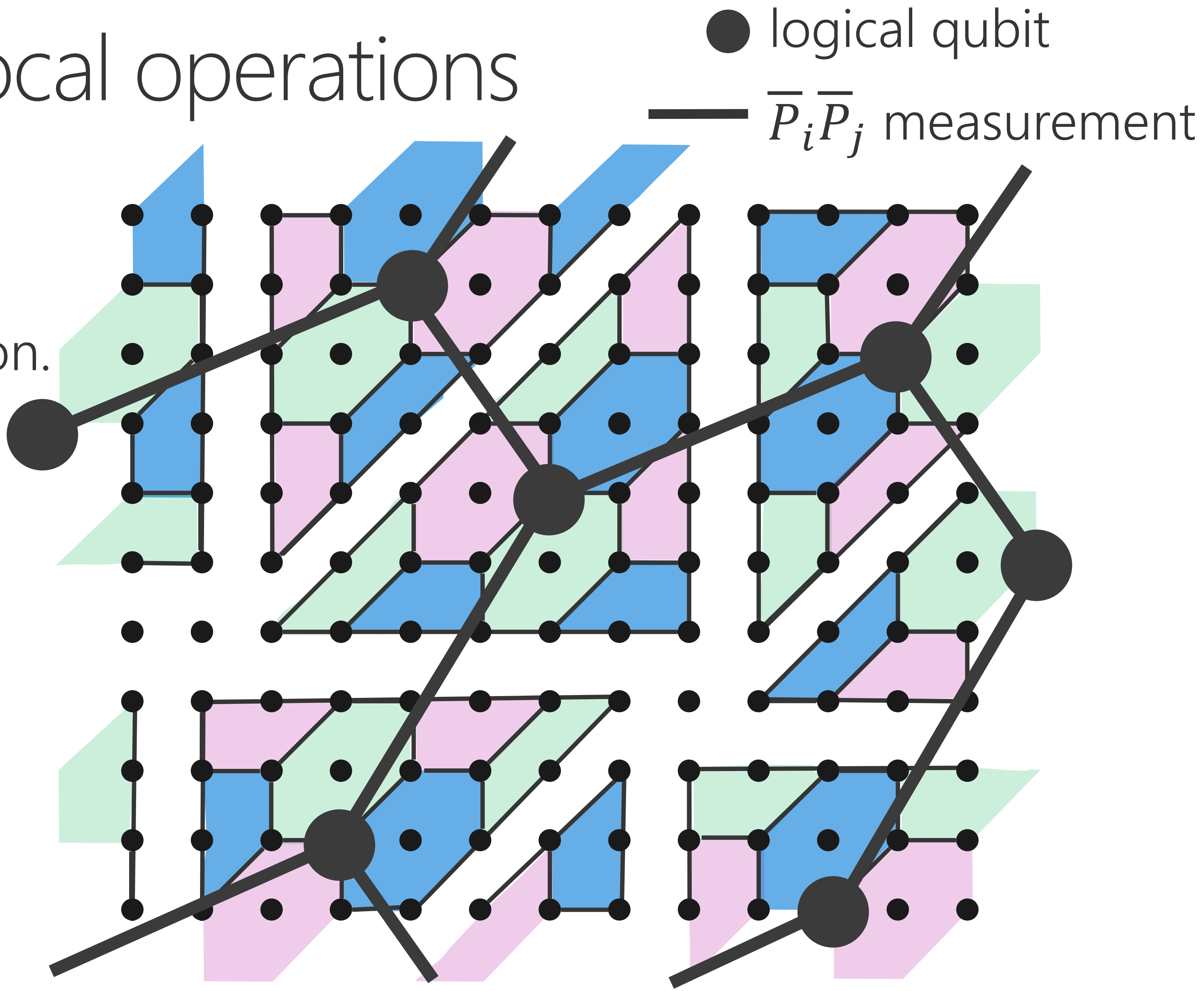
Transverse 1-qubit
Clifford group.

All Clifford operations
protected.



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NON-CLIFFORD GATES IN 2D

Strategies for non-Clifford gates

2D transverse gates *only* Clifford (non-universal). [Bravyi, König 13].

Cat-state method [Aharonov, BenOr 96]

State distillation [Knill 04; Bravyi, Kitaev 05; ….]

Applicable to 2D topological codes

Code switching to a code not restricted to Clifford operations [Paetznick, Reichardt 13; Bombín 15; ….]

Multiple partitions [Jochym-O'Connor, Laflamme 14]

Pieceable fault-tolerance [Yoder, Takagi, Chuang 16]

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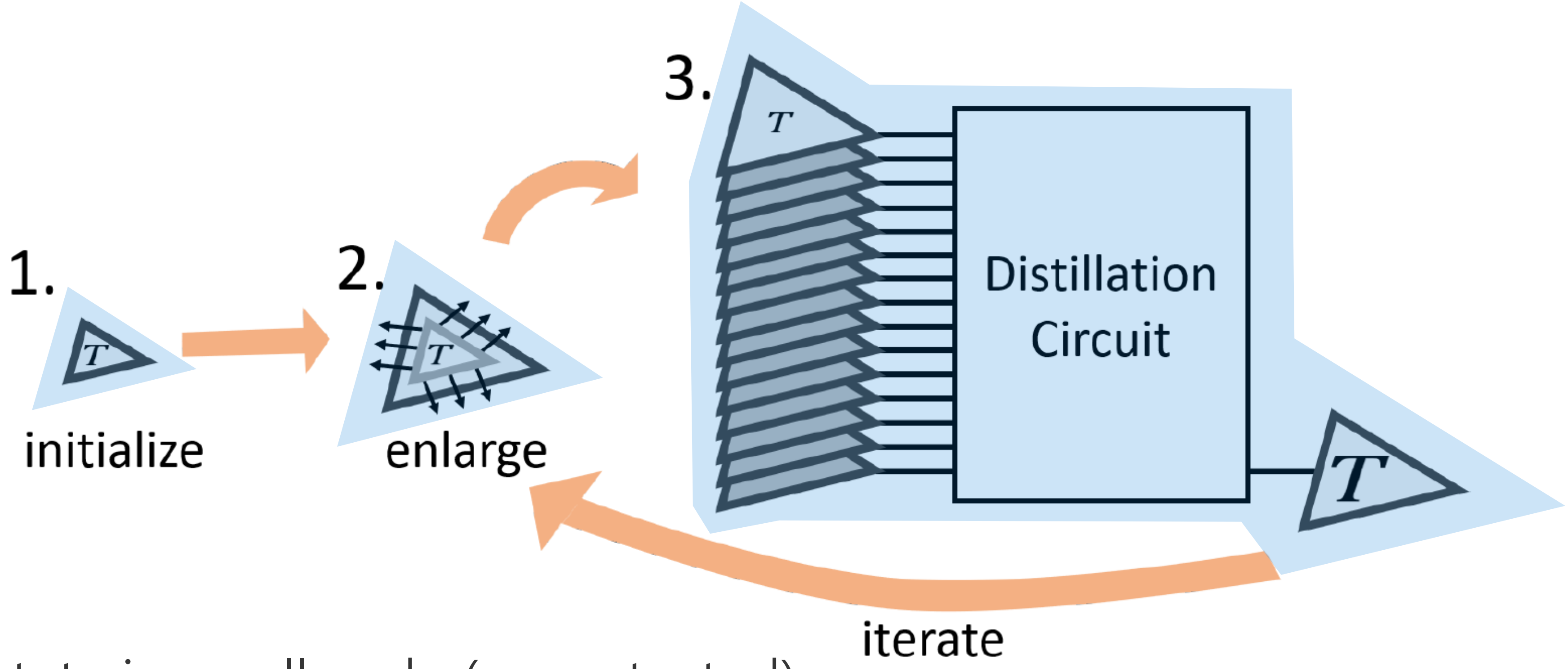
STATE DISTILLATION

State distillation

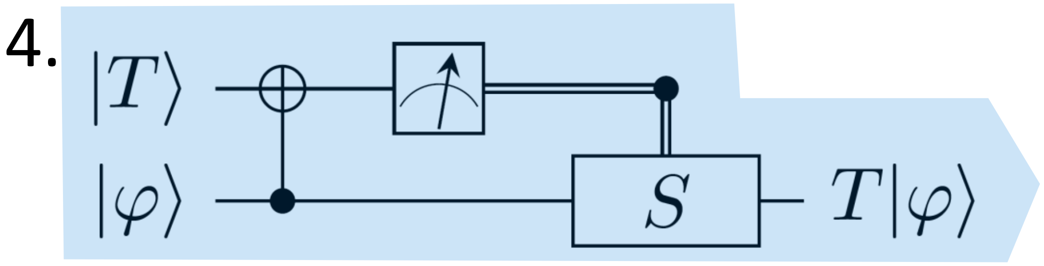


Steps:

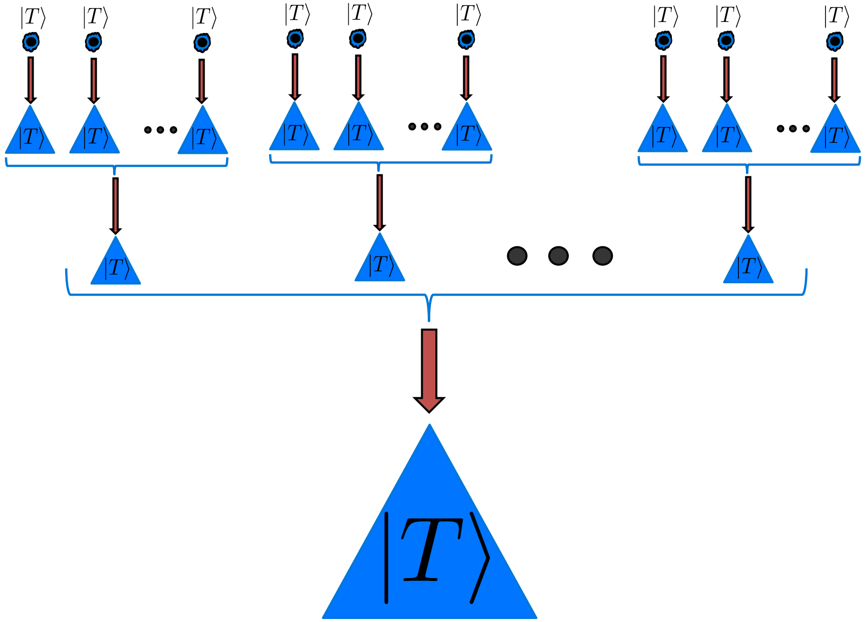
1) Initialize state in small code (unprotected).
2) Enlarge code distance (protected).
3) Run distillation circuit (protected).

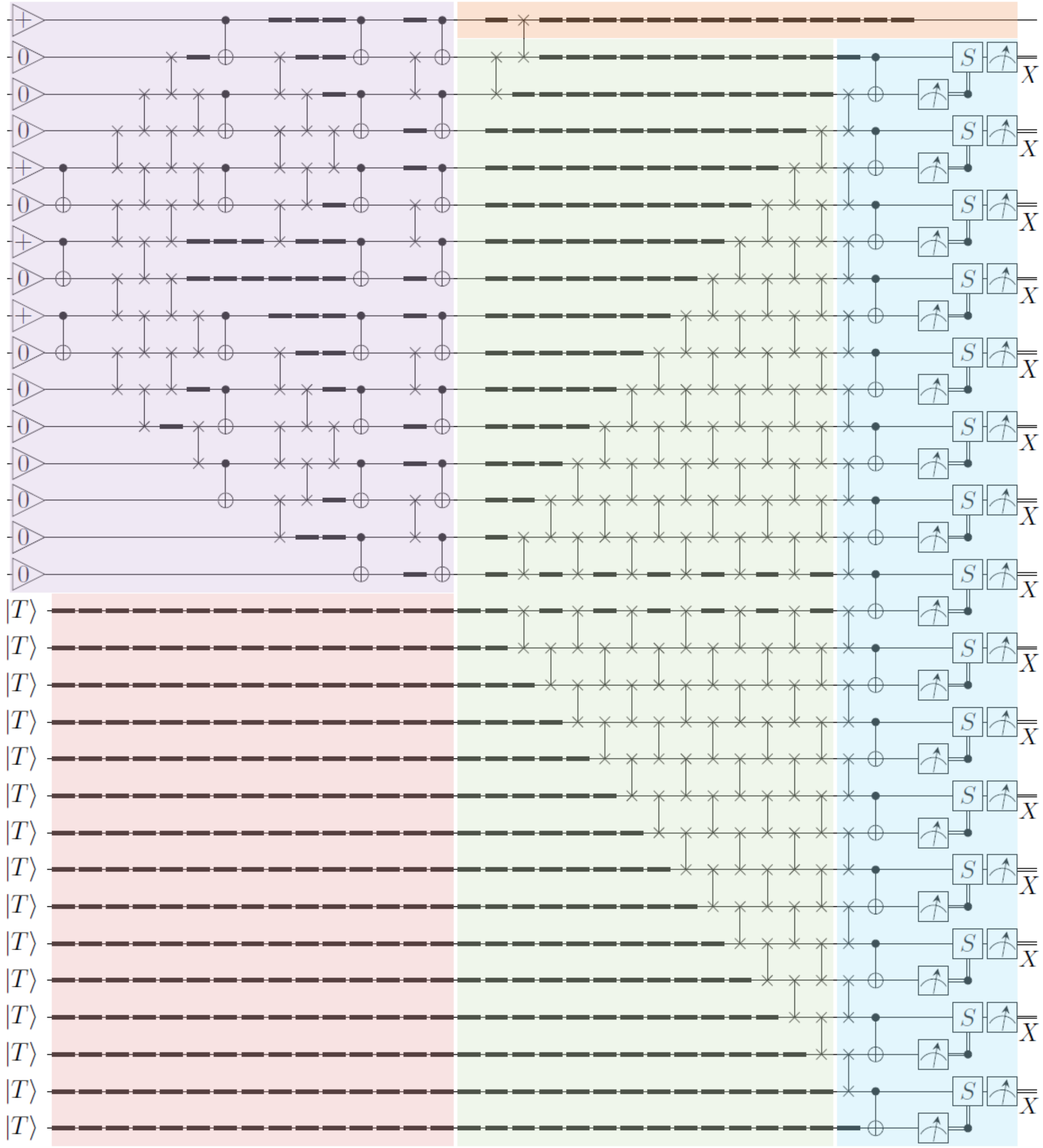
   Repeat 2) and 3) as needed to produce good $|T\rangle$.

4) Inject $|T\rangle$ to apply T gate (protected).



Distillation can be costly

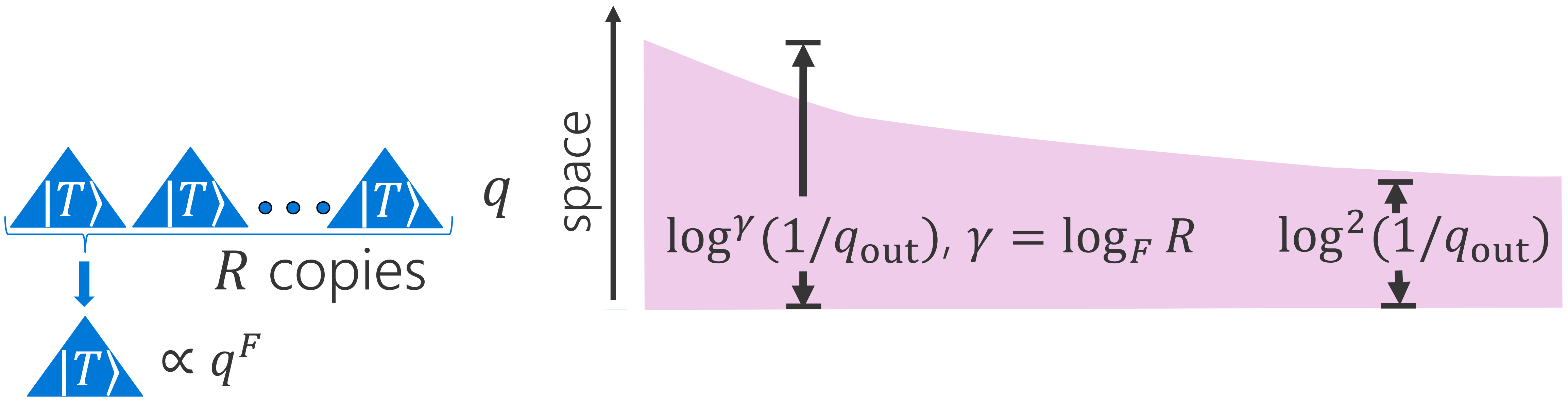




Many inputs needed.

Clifford distillation circuit large.

Distillation overhead (asymptotic)



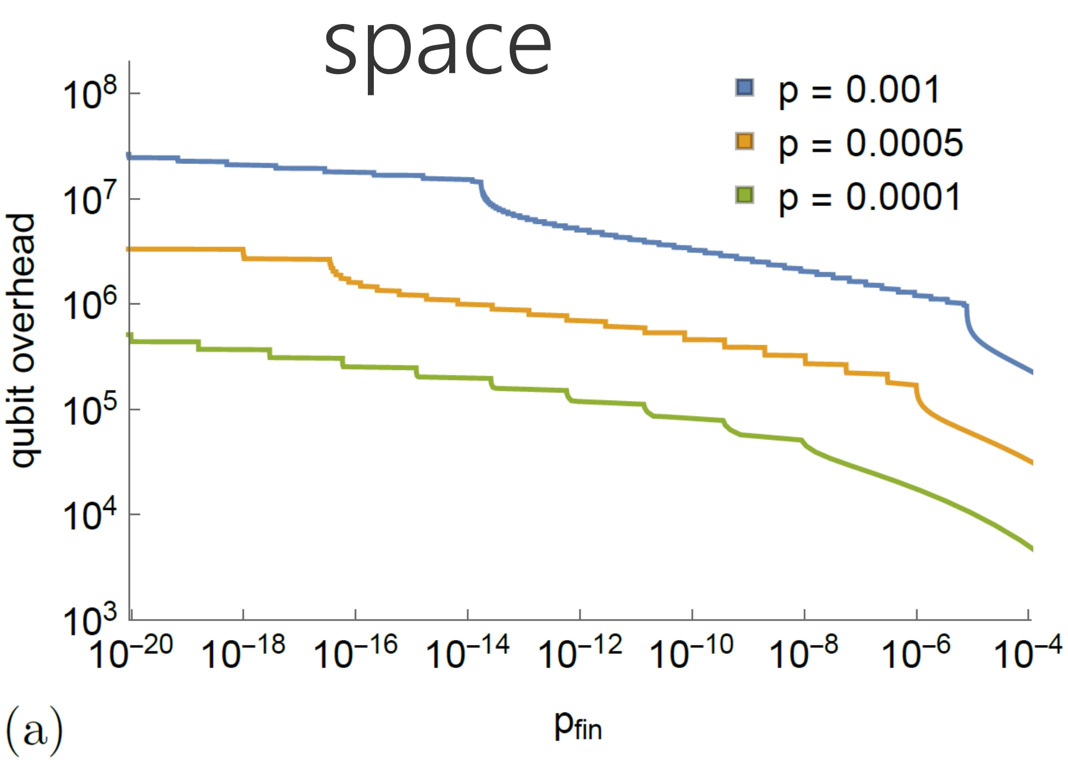
	round 1	round 2	⋯	round N
copies in	R^N	R^{N-1}		R
distance	d_1	d_2		d_N
space ∝	$R^N d_1^2$	$R^{N-1} d_2^2$		$R d_N^2$

Distillation overhead (numerics)

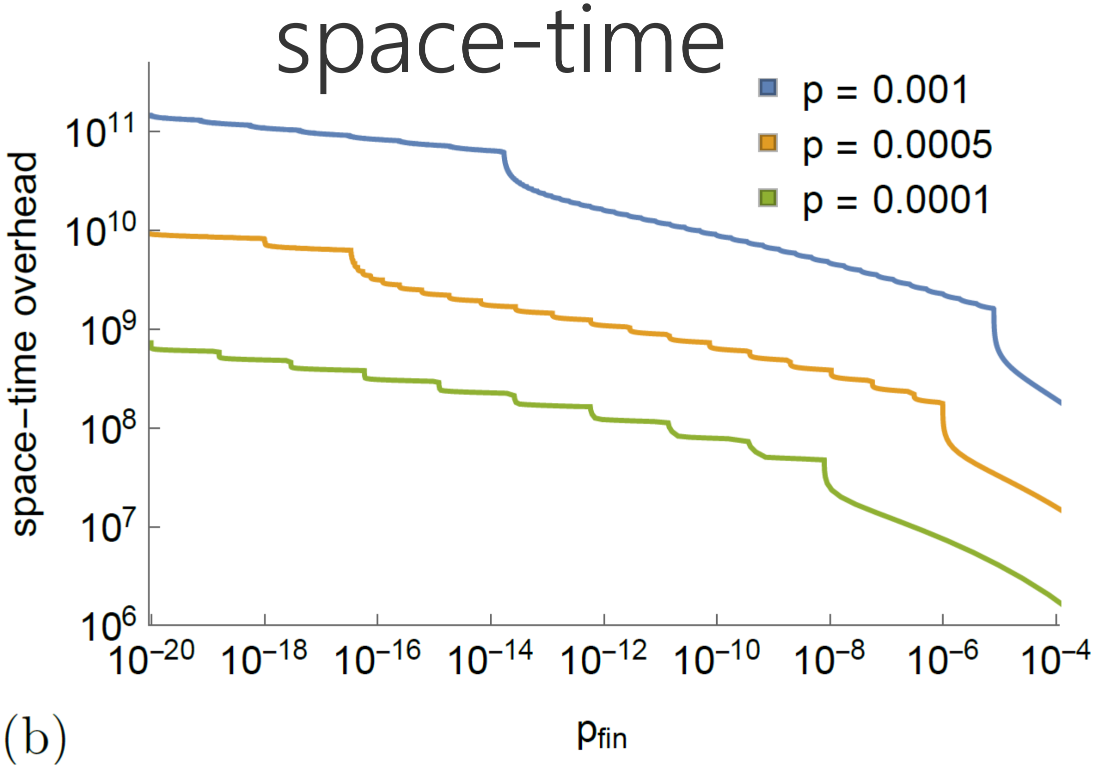
Optimized 2D color code (stabilizer circuits and decoder).

Simulated logical errors in a laid out 15-to-1 distillation circuit.

Searched over distances $\{d_1, d_2, \ldots, d_\mathrm{N}\} \rightarrow q_\mathrm{out}$ with min cost.



(a)



(b)

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NON-CLIFFORD GATES IN 3D

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Applicable to 2D topological codes

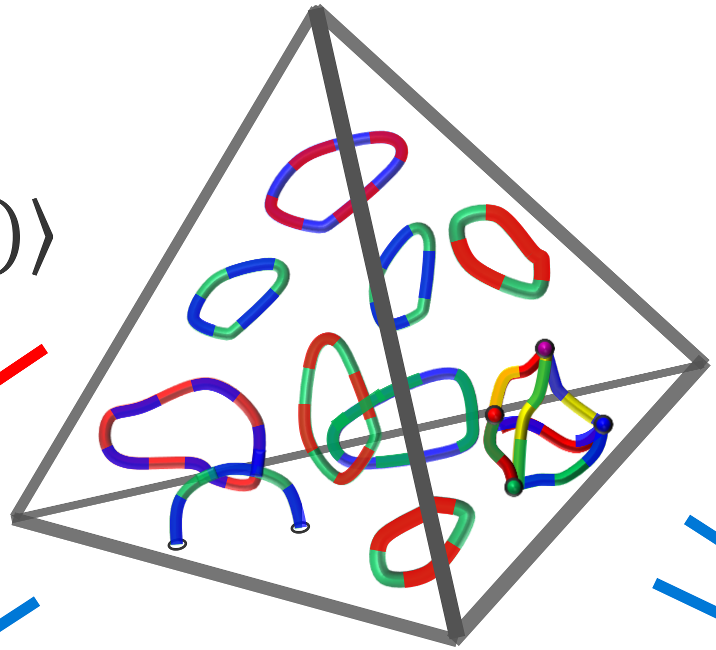
Applicable to 3D topological codes

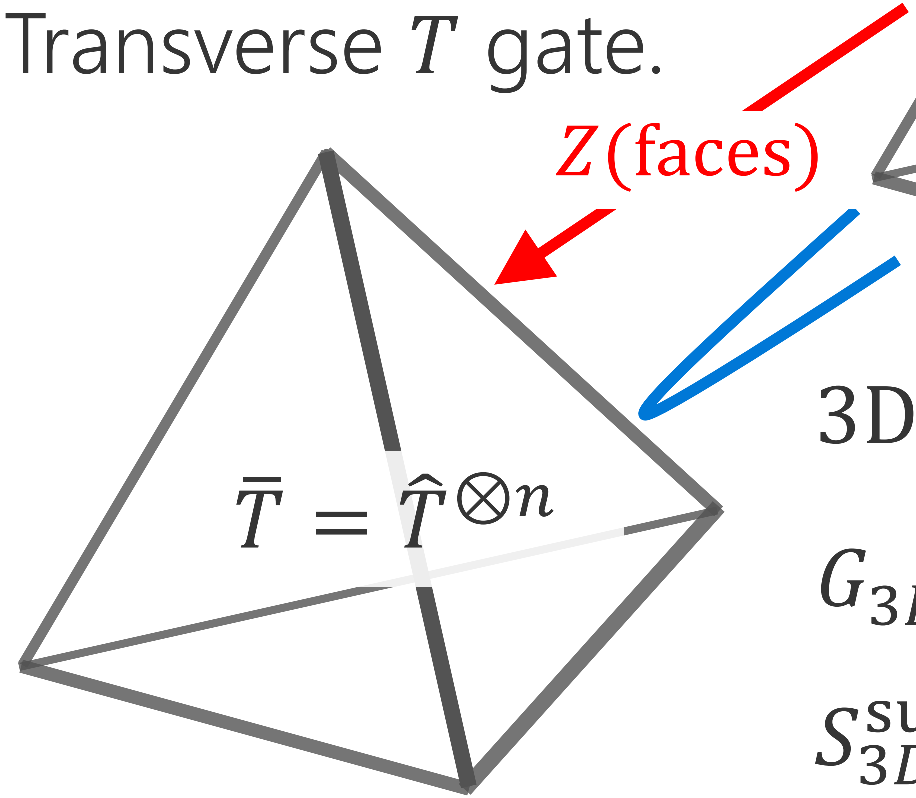
3D color codes

3D Stabilizer code:

$S_{3D} = \langle X(\text{vols}), Z(\text{faces})\rangle$

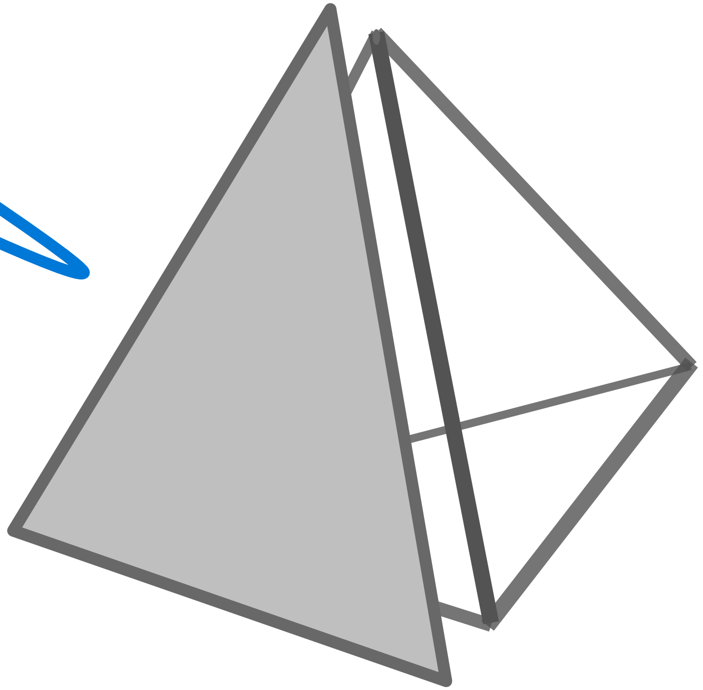
Transverse T gate.





2D Stabilizer code:

$S_{2D} \times S^{\text{int}}$



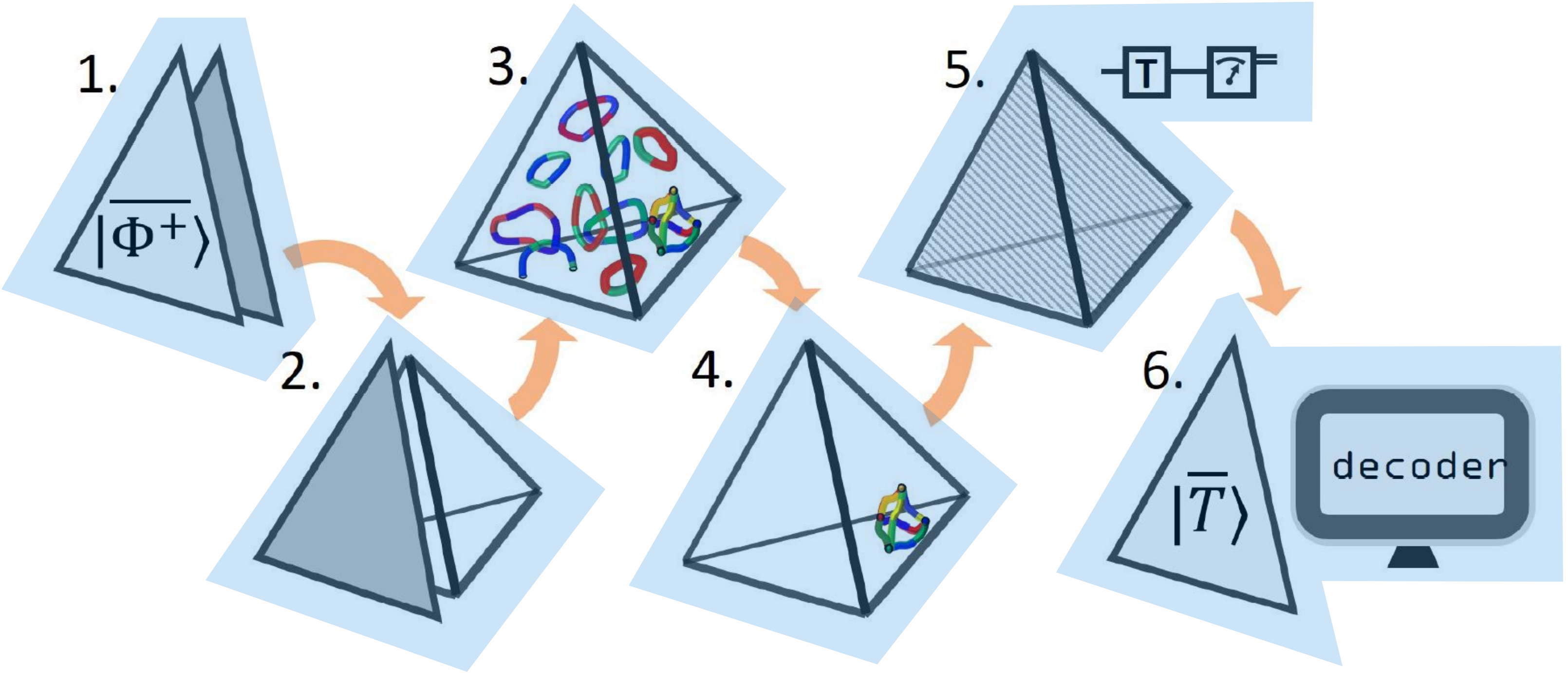
3D Subsystem code:

$G_{3D} = \langle X(\text{faces}), Z(\text{faces})\rangle$

$S_{3D}^{\text{sub}} = \langle X(\text{vols}), Z(\text{vols})\rangle$

CODE SWITCHING

Code switching



Code switching steps:

1) Prepare Bell state in 2D codes
2) Prepare the 3D interior
3) Measure gauge operators
4) Gauge fix
5) Apply $\bar{T}$ and measure
6) Decode Z errors in 3D code

$$|T\rangle = |0\rangle_{2D} + e^{i\,\pi/4}\,|1\rangle_{2D}$$

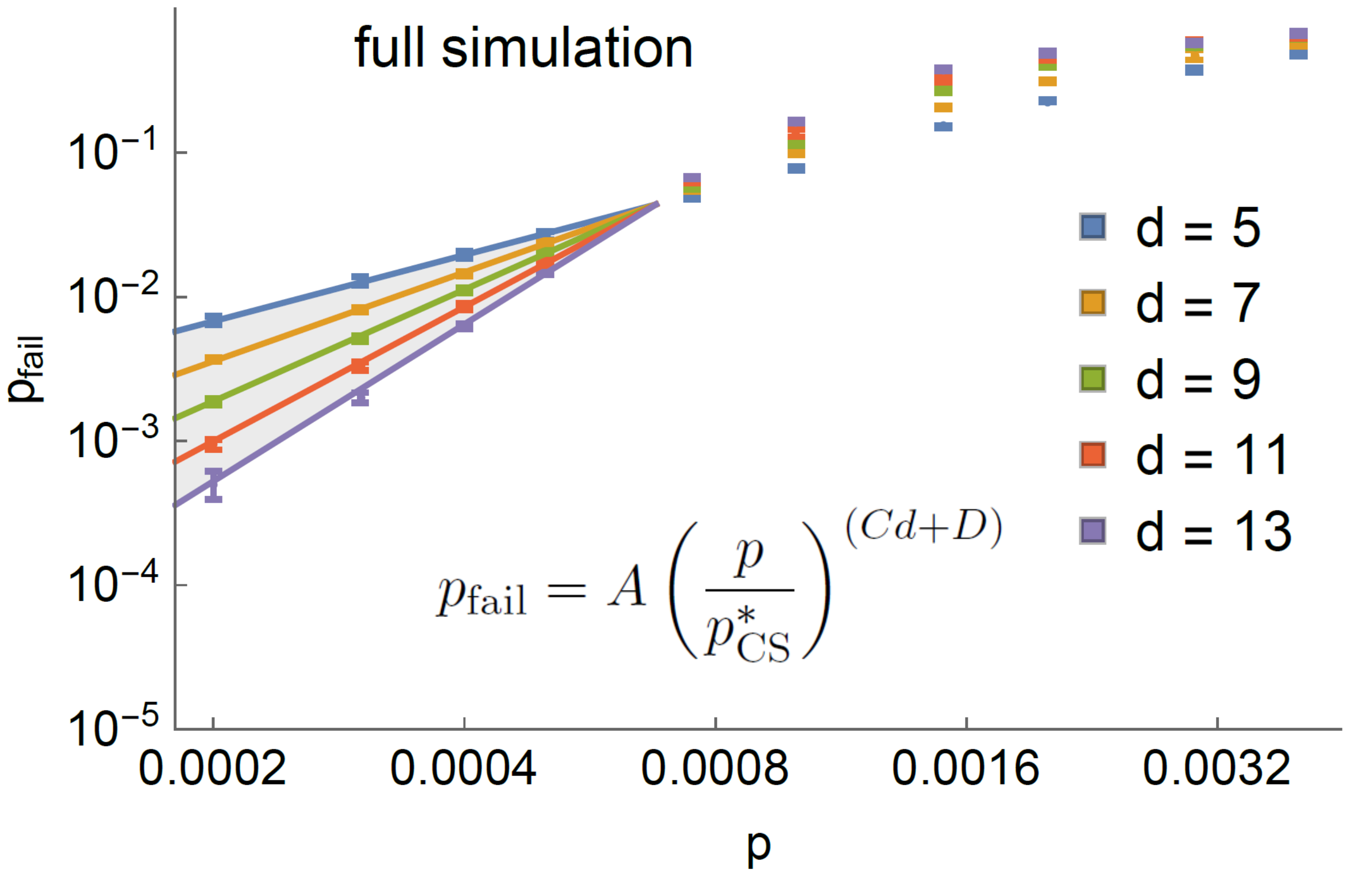
Code switching (numerics)

Optimized each step, finding:

- low-depth preparation and gauge-measurement circuits.
- efficient classical decoders for gauge fixing and error correction in 3D.

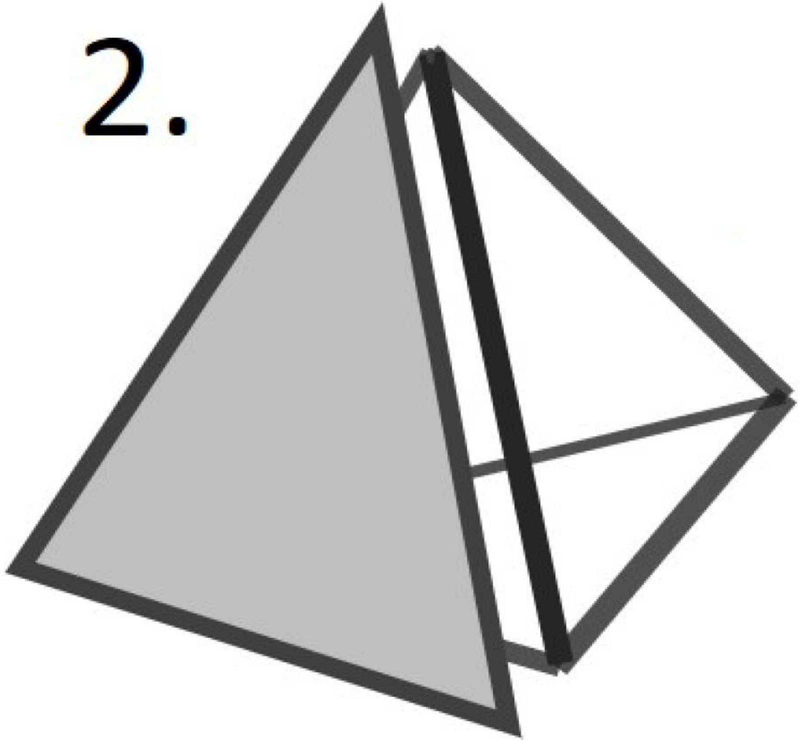
Overhead from inverting fit.

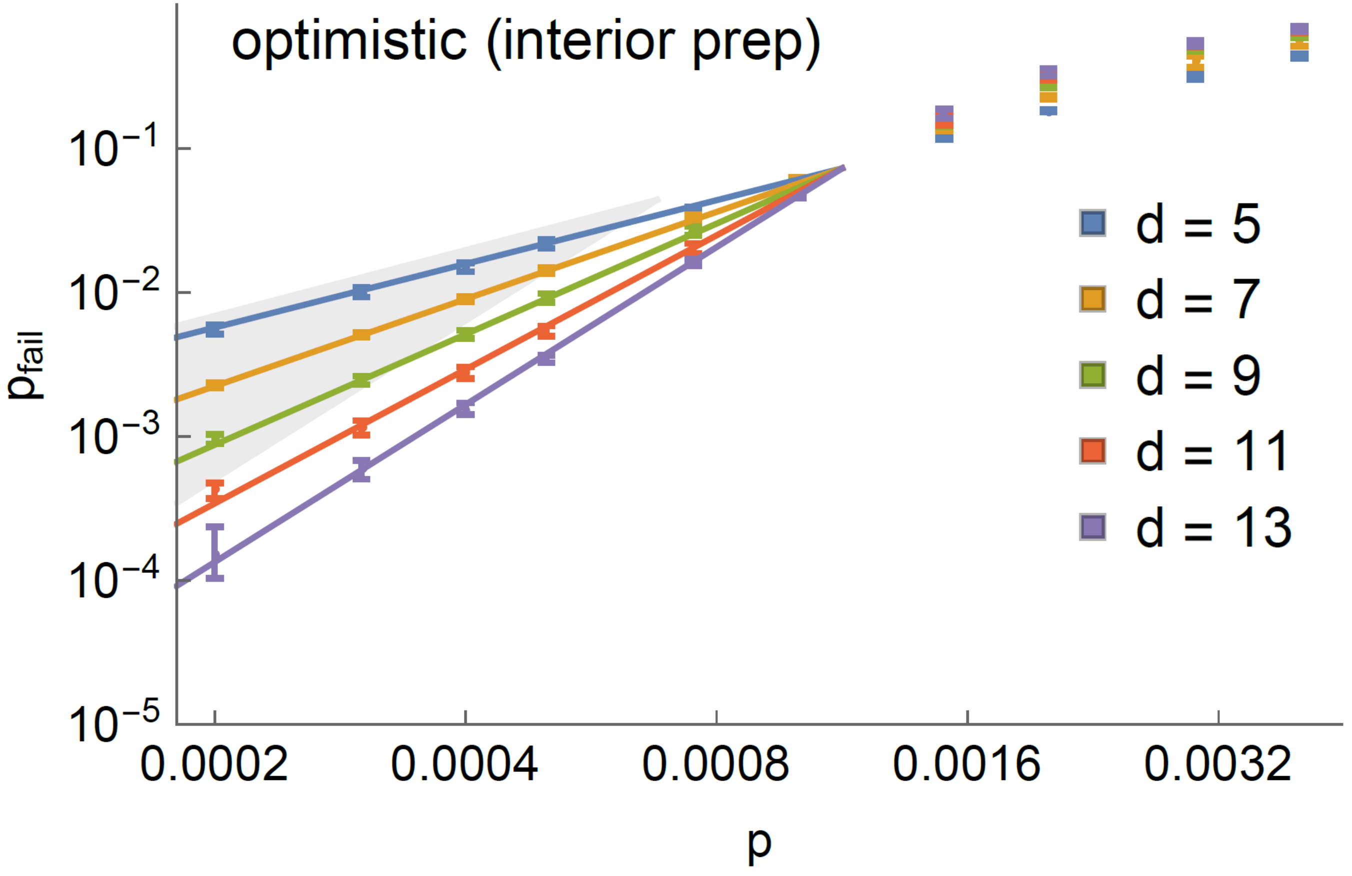
We bound impact of potential improvements by replacing each step by an optimal version.



Code switching (numerics)

e.g. What if there's a better way to prepare the interior?





(Replacing with 3 time units of single-qubit error).

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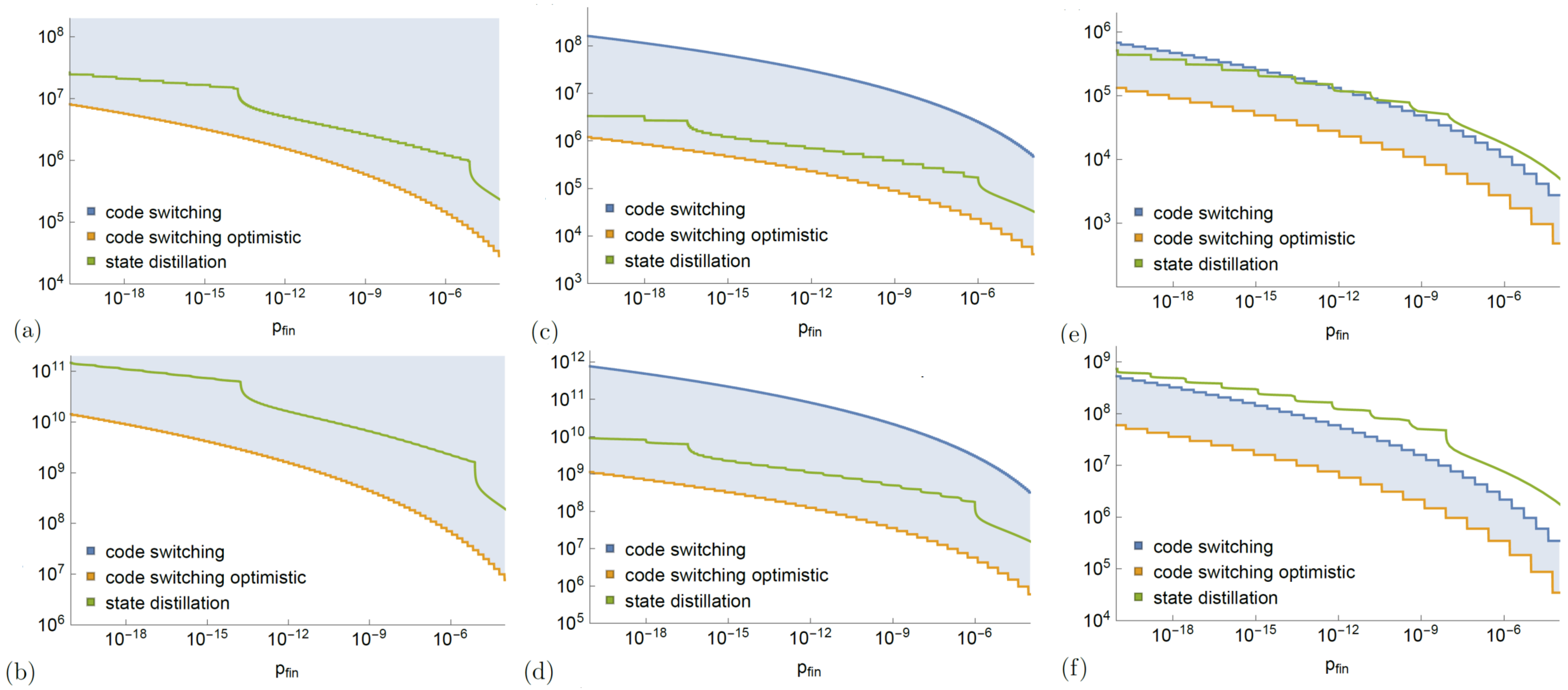
MAIN RESULTS

Numerical Simulation

p=0.001 | p=0.0005 | p=0.0001

space

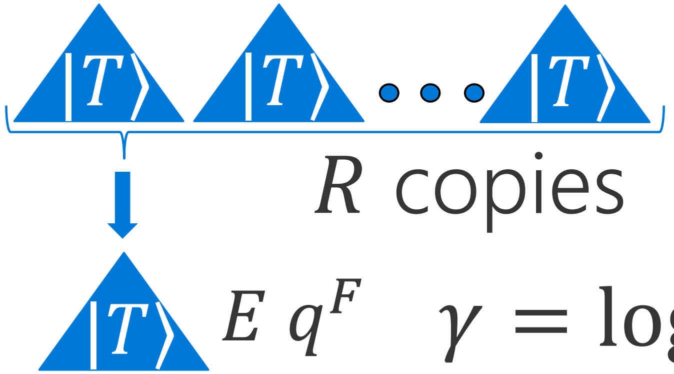
space-time



Asymptotic behavior for $q_{\text{out}} \to 0$

$$\text{space} \sim \log^{\Gamma_*}(1/q_{\text{out}})$$

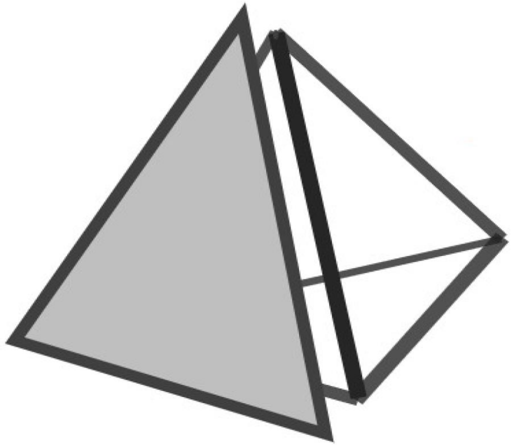
$$\text{space-time} \sim \log^{\Gamma_*+1}(1/q_{\text{out}})$$



q

$\gamma = \log_F R$

$\Gamma_{\text{SD}} = \max(\gamma, 2) \approx 2.46$



$\Gamma_{\text{CS}} = 3$

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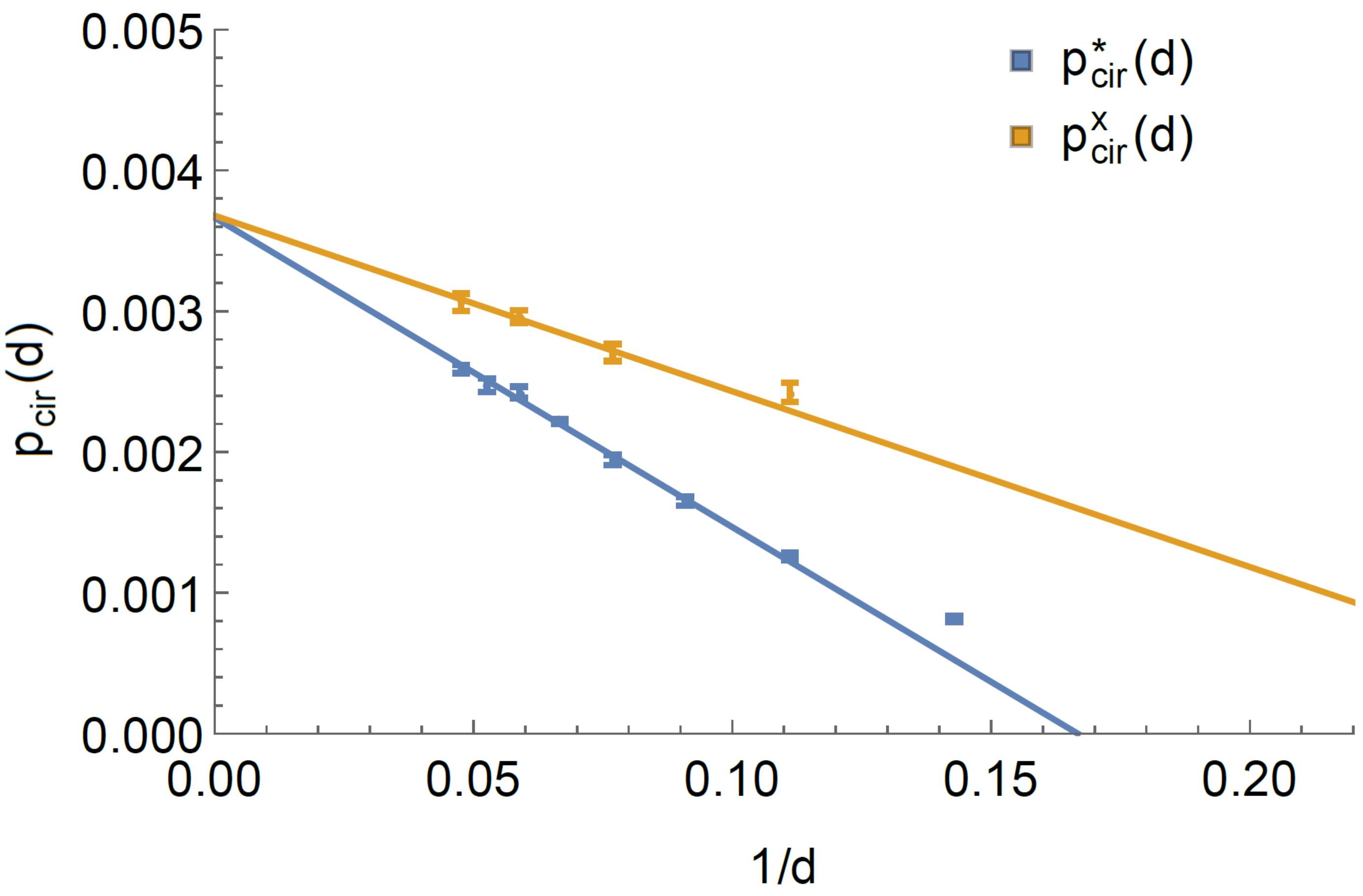
INTERMEDIATE RESULTS

2D color code circuit-noise threshold



Syndrome extraction requires carefully choosing order of CNOTs.

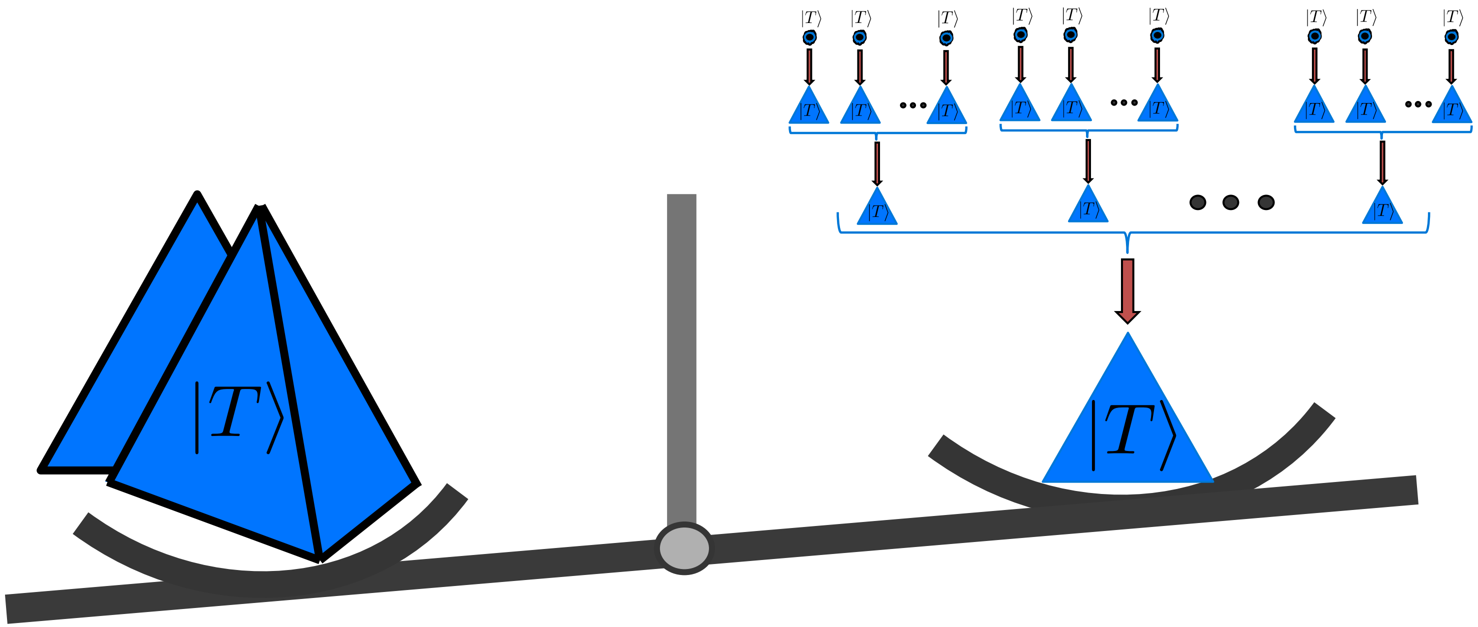
Threshold (>0.3%) is very close to surface code (1%). We have not used X-Z correlations, or hook errors in decoder.

Very slow convergence here toward threshold with distance.

Full distance is NOT achieved. For example d=3 color code not FT for all single faults.

Main conclusions

- CS not substantially cheaper than SD for $p \geq 10^{-4}$.
- Holds even with highly-optimistic improvements to CS.
- Unlikely that other (more constrained) CS approaches are better.



Further conclusions

- Fruitful general approach to bound impact of potential improvements.
- 2D color code threshold above 0.3% (approaching surface code's).
- Distillation schemes with $\gamma < 2$ appear to have overhead limited by the last not first distillation round when implemented using 2D codes.