Fault-tolerant qubit from a constant number of components

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In recent years, significant experimental progress has been made towards building a large-scale quantum computer. In platforms such as superconducting qubits and trapped ions, the error rates for small systems have been successfully suppressed below the threshold error rate of the surface code [1–3]. Using newly developed techniques for neutral atoms trapped in optical tweezer arrays, the coherence time, gate fidelity, and read-out fidelity for large assemblies of qubits are being rapidly improved [4–7]. These advances give us hope that we will one day be able to perform fault-tolerant quantum computation by scaling up these systems while maintaining low error rates.

However, the scalability of leading approaches remains an important challenge. Current estimates suggest that the engineering effort needed to build even a *single* logical qubit with logical error rate low enough for useful quantum computation could be enormous [8]. Quantum algorithms with practical ramifications can involve applying at least $\sim 10^8$ logical gates to ~ 100 logical qubits [9, 10]. To ensure that the outcome of the computation is correct with high probability, the logical error rate would then need to be below $\sim 10^{-8}$. Based on the sub-threshold error scaling in Ref. [11], this would require at least ~ 400 physical qubits per logical qubit if the physical error rate is half the threshold.

Manufacturing, calibrating and controlling physical qubits in such large numbers will be tremendously difficult. The fabrication process for components of solid-state quantum devices, such as quantum dots or superconducting circuits [1], is inevitably imperfect, leading to variations in the properties of individual qubits and their interactions. Even in systems where qubits are encoded in identical particles, *e.g.*, trapped ions [2, 3, 12] or neutral atoms [4–7], experimental control parameters such as the strengths of laser excitation pulses or trapping potentials may exhibit inhomogeneity. Thus, in order to control these qubits with high fidelity, an experimental system needs to be accurately calibrated across the entire quantum computer. In superconducting circuits, for instance, inhomogeneity is unavoidable, and stray couplings between ideally independent qubits are an experimental fact of life that must be mitigated through control logic (see *e.g.*, [1].) The difficulty of doing so increases significantly with the number of qubits [13].

To circumvent these challenges, we propose a novel approach to fault-tolerant quantum computation, in which a well-protected logical qubit can be built using only a handful of experimental components. Consequently, the engineering effort required to develop the computer's components can be significantly reduced, potentially opening a simpler and more easily scalable route to fault-tolerant quantum computation. At a high level, our approach succeeds by shedding the limitations implicit in two assumptions that usually guide fault-tolerant circuit design: first, that the computer's qubits are all of the same type and are thus fairly homogeneous, and second, that good fault-tolerant gates should not propagate errors.

Specifically, we construct a fault-tolerant protocol for generating the three-dimensional cluster state of Ref. [14], with which universal fault-tolerant computation can be performed via adaptive single-qubit measurements. While there is already a well-known procedure for preparing this state, our method has the advantage of being compatible with a much simpler experimental setup than what was originally envisaged in Refs. [14–16]. We take an approach similar to existing proposals for building large one- and two-dimensional cluster states using a small number of physical components [17–20]. However, while two-dimensional cluster states are universal for quantum computation, they are not known to support *fault-*

tolerant quantum computation.[1] The step from universality to fault-tolerance is not obvious and, in fact, quite surprising considering the structure of our proposed scheme.

Our protocol is built around a special ancilla qubit, $\mathcal{Q}$, which interacts sequentially with a stream of data qubits propagating through a delay line. These data qubits are encoded in degrees of freedom sharing a common physical implementation, *e.g.*, different temporal modes of photons or phonons in a waveguide. The only interactions are between $\mathcal{Q}$ and data qubits (and not between data qubits themselves), and these interactions are fixed and periodic, requiring a modest amount of calibration. We show, moreover, that all of the operations required in our protocol can be implemented using existing technologies in quantum photonic and phononic systems.

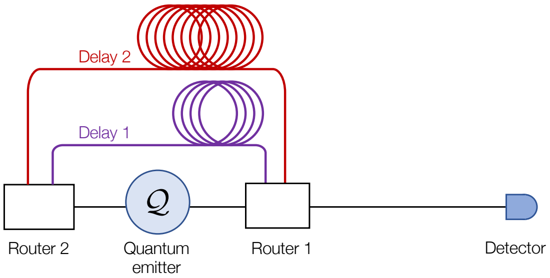

FIG. 1. A schematic illustration of the apparatus for implementing our protocols. Propagating modes, *e.g.*, photons or phonons, are stored in delay lines between interactions with the control qubit $\mathcal{Q}$ (which, in the context of the proposed experimental platforms, is a quantum emitter). Each qubit is measured after a constant number of interactions with $\mathcal{Q}$.

From a theoretical perspective, our protocol has a novel feature that is counterintuitive given the traditional to fault-tolerance. The design of fault-tolerant protocols usually aims to prevent the propagation of single-qubit errors to many qubits. This is achieved, naturally enough, by applying gates that do not spread errors, *e.g.*, transversal gates, or “long” gates that are interspersed with error correction steps, such as in lattice surgery [21]. In all of these methods, one actively avoids interacting one qubit with many others in a code block, since errors occurring on that qubit could propagate to the others, exceeding the error-correcting capabilities of the code.

However, in our protocol, we are actually deliberately taking this seemingly ill-advised approach: a single qubit ($\mathcal{Q}$) is coupled to *every* data qubit. The depth of the circuit scales linearly with the number of data qubits, and no error detection or correction is performed during the process. Nevertheless, the procedure is fault-tolerant in that any single-qubit error occurring in the circuit results in a constant-weight error on the final state. An interesting subtlety is that even though a single-qubit circuit-level error can in general be propagated by the subsequent gates to an highly nonlocal error, this nonlocal error is always equivalent under stabilisers of the prepared cluster state to some geometrically local error. More generally, we show that any m-qubit circuit-level error results in at most m geometrically local errors on the final state.

In fact, this counterintuitive property applies not only to our protocol for generating the specific three-dimensional cluster state of Ref. [14], but to our general algorithm, Algorithm 1 in the paper, for preparing cluster states corresponding to *arbitrary* graphs. In Algorithm 1, there is likewise a single qubit $\mathcal{Q}$ that interacts with every data qubit, yet the effect of any m-qubit circuit-level error on the prepared state is the same as that of at most m geometrically local errors.[2] By leveraging this fact, we were able to construct fault-tolerant quantum circuits whose depth necessarily scales with the system size. Fault-tolerant quantum computing protocols usually avoid circuits structured like ours because of the danger that they will spread errors too widely. This often restricts the design of these protocols, leading them to rely on a small number of trusted and manifestly fault-tolerant building blocks, such as transversal gates or “catch-and-correct” [22]. Our work shows that there can be a subtle form of fault-tolerance in which errors spread but without adverse effects. This observation may prove useful for generalising our methods to other fault-tolerance schemes. Indeed, our Algorithm 1 can immediately be used to fault-tolerantly generate cluster states obtained by

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[1] More precisely, two-dimensional cluster states of *unprotected physical qubits* are not known to be a useful resource for fault-tolerant quantum computation.

[2] For cluster states defined by arbitrary graphs, an error is said to be geometrically local if it is supported on some subset of $i \cup N(i)$, where $N(i)$ denotes the nearest neighbours of i in the graph.

foliating other stabiliser codes [23, 24].

Our paper includes detailed proposals showing how our theoretical protocol for preparing the three-dimensional cluster state of Ref. [14] could be realised in a system consisting of a single quantum emitter (*e.g.*, an atom, ion, transmon, or quantum dot) coupled to a photonic or phononic waveguide [25, 26]. In such a system, any stable internal states of the emitter can be used to encode qubit degrees of freedom for $\mathcal{Q}$, while any radiative states of the emitter that are coupled to the waveguide can be leveraged to realise certain gates between $\mathcal{Q}$ and a photon or phonon propagating in the waveguide. We show that the set of available gates in these technologies is sufficient; moreover, the required routing of the photons or phonons is simple enough to be implemented with just two routers [cf. Fig. 1].

To demonstrate fault-tolerance, we analyse the robustness of our protocol against both circuit errors and memory errors. We use a standard depolarising model to describe circuit errors, which are associated with imperfect gates, measurements, and state initialisation. Memory errors refer to errors that occur while qubits are idle, for which we study the effect of dephasing and qubit loss. In the absence of memory errors, there is a threshold of 0.39% for the circuit error rate, below which the logical error can be arbitrarily suppressed by increasing the number of physical qubits. In the presence of memory errors, the logical error rate cannot be arbitrarily suppressed. However, provided that the circuit error rate is below threshold, the logical error rate decays rapidly with the inverse of the memory error rate. More precisely, suppose that the coherence time of the data qubits is lower-bounded by T. Then, for a sufficiently large but finite T, the logical error rate can be made exponentially small in $\sqrt{T/\tau}$. Here, τ is the inverse of the frequency with which gates are applied. The number of logical gates that can be reliably executed will therefore scale exponentially with $\sqrt{T/\tau}$. A large separation between T and τ is often observed in certain experimental platforms, such as trapped ions or neutral atoms utilising atomic clock transitions [4–6, 12]. Indeed, because of the strict separation in the roles of $\mathcal{Q}$ and the data qubits, maximising the ratio T/τ while maintaining high gate fidelity is an invitation to design a hybrid system consisting of two types of qubits with different physical substrates. That is the context in which we expect our scheme to be the most promising. Photonic [27] and phononic [28] delay lines are known to be good quantum memories, and can be coupled to controllable qubits capable of playing the role of $\mathcal{Q}$.

To illustrate the potential of our scheme, suppose that memory errors are dominated by loss. Then, if the circuit error rate is 10^{-3}—an aspirational but realistic target—our protocol can in principle attain a logical error rate of 10^{-8} for $\tau/T \approx 1.4 \times 10^{-5}$, and 10^{-15} for $\tau/T \approx 3.2 \times 10^{-6}$. Although these numbers are beyond the reach of current experiments, these estimates suggest that extremely low logical error rates can be achieved by improving a very small number of experimental components. In particular, if the operations involving $\mathcal{Q}$ can be calibrated such that circuit error rate is below the threshold value of 0.39%, incremental improvements of a *single* component—the delay line—can lead to drastic reductions in the logical error rate.

To summarise, our proposal and analysis indicate that fault-tolerant quantum computation could be achieved through the incremental improvement of a small number of key components, avoiding most of the systems engineering challenges inherent in leading approaches. This is possible because of three important features of our scheme. First, it only requires manufacturing and calibrating a constant number of experimental components, independent of the number of data qubits. Second, there are readily available experimental platforms that can realise our protocol. Third, any constant-weight error occurring during our protocol results in a constant-weight error on the prepared cluster state.

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