Fault-tolerant qubit from a constant number of components

Kianna Wan, Soonwon Choi, Isaac H. Kim, Noah Shutty,
& Patrick Hayden

qip/21

motivation

previous work:

[Lindner & Rudolph '09] – 1D cluster states

[Pichler, **Choi**, Zoller, Lukin '17] – 2D cluster states (universal MBQC)

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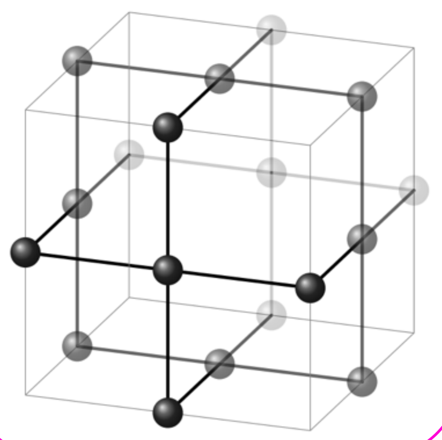

our goal: prepare 3D cluster states on bcc lattice (*fault-tolerant* universal MBQC [Raussendorf *et al.*])

1. using an experimentally feasible setup…
2. …while preserving fault-tolerance

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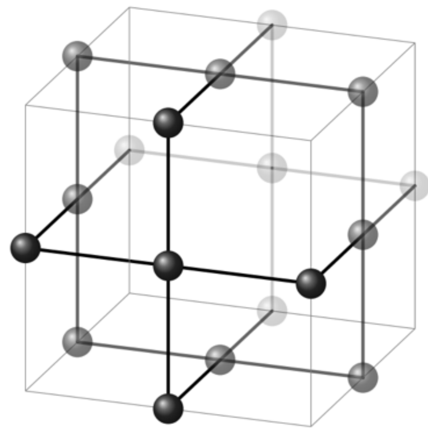

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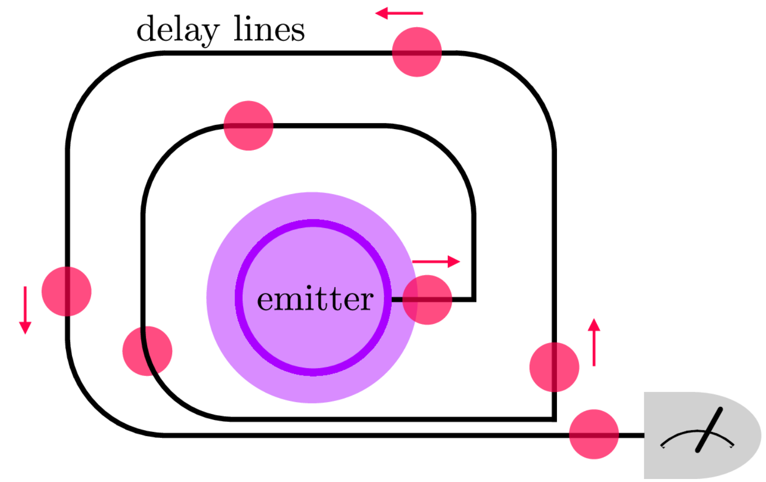

cluster states

any undirected graph $G = (V, E)$ defines a cluster state $|\psi_G\rangle$:

$$|\psi_G\rangle := \left[\prod_{(i,j)\in E} \mathrm{c}Z_{i,j}\right] \bigotimes_{k\in V} |+\rangle_k$$

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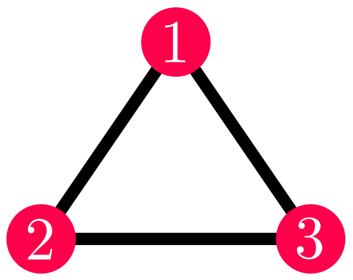

K_3

$$\leftrightarrow \qquad |\psi_{K_3}\rangle = Z_{1,2}Z_{2,3}Z_{3,1}|+\rangle_1|+\rangle_2|+\rangle_3$$

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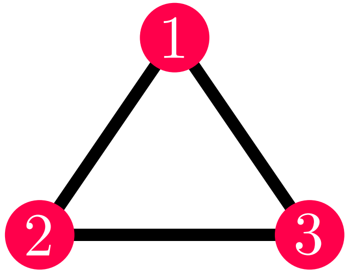

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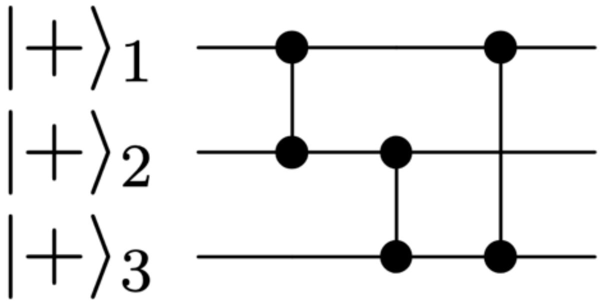

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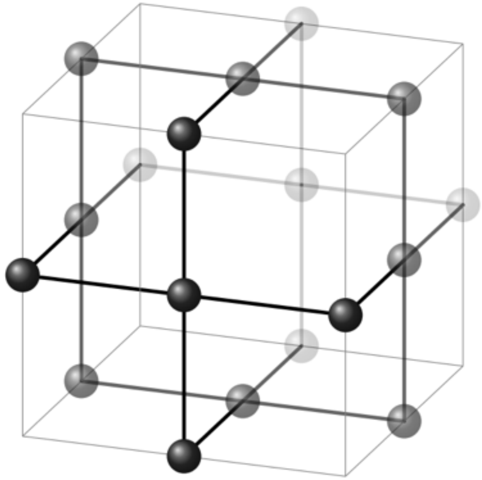

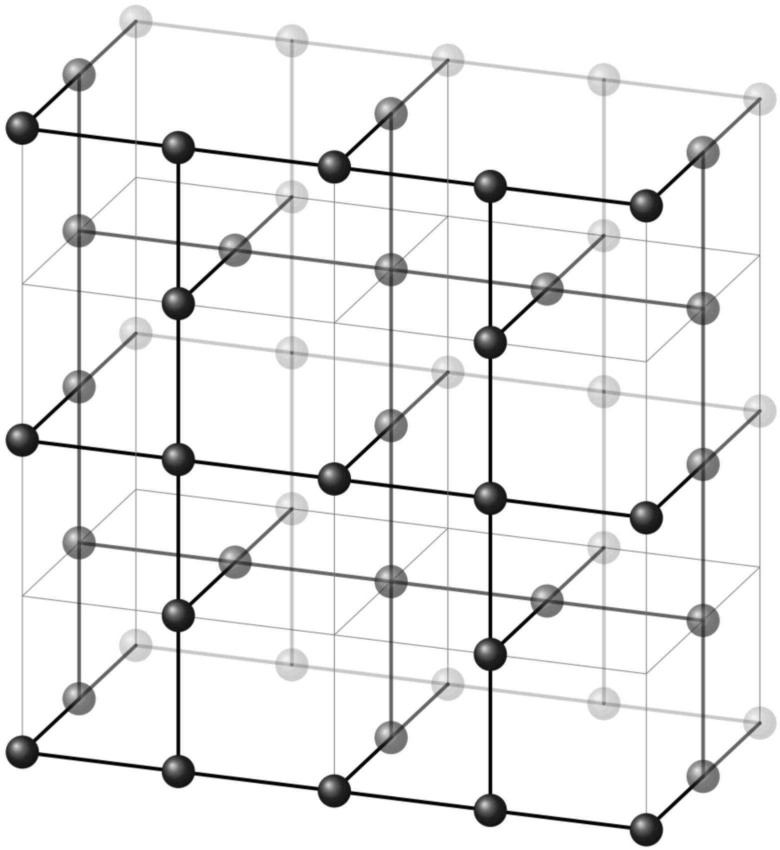

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however, requires interactions between $|E|$ distinct pairs of qubits

instead, introduce a single ancilla, $\mathcal{Q}$, that interacts with each of the "identical" data qubits ($i \in V$) one by one
$\rightarrow$ calibrate only a constant number of physically distinct interactions

preparing cluster states

abstract problem: prepare cluster states using interactions only between $\mathcal{Q}$ and data qubits (no two-qubit gates between data qubits)

simplest solution: use SWAP gates

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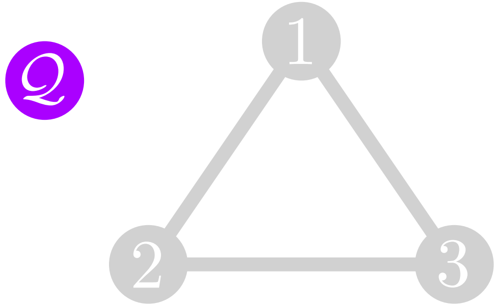

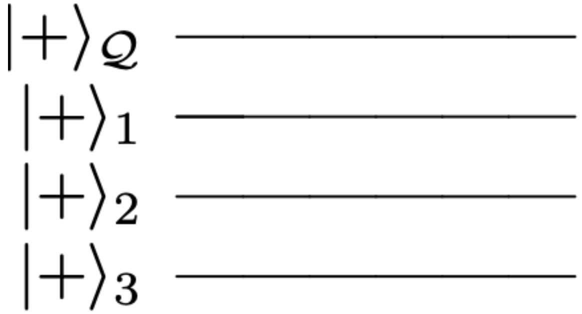

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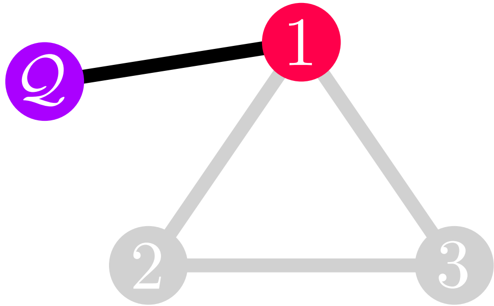

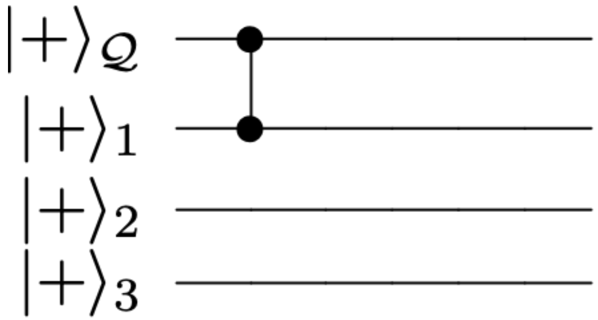

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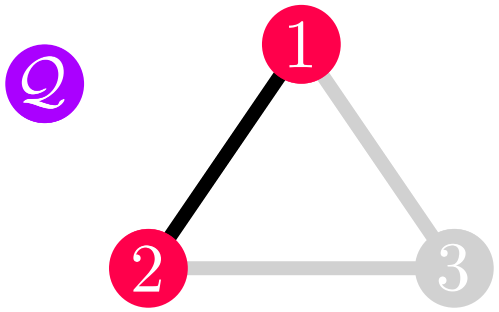

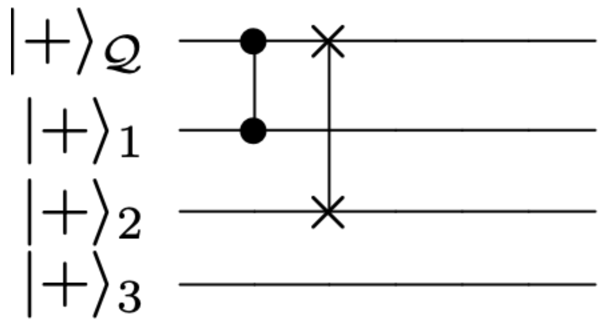

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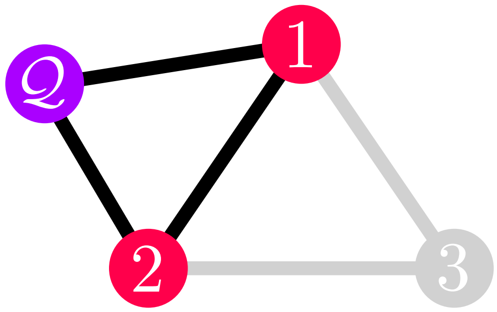

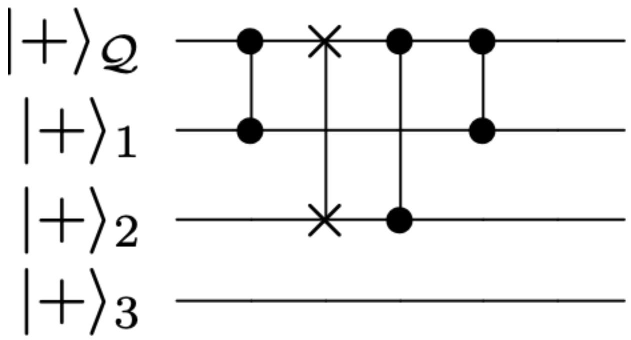

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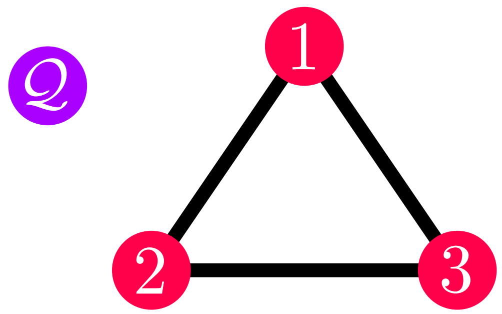

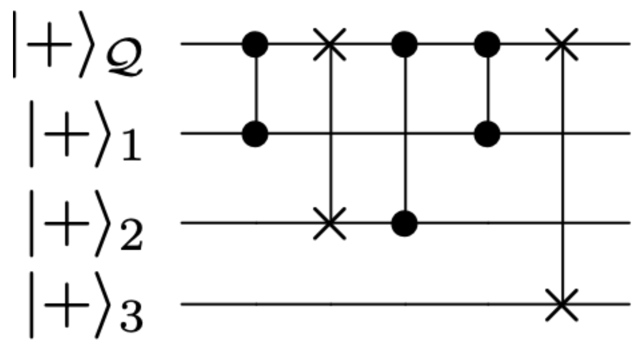

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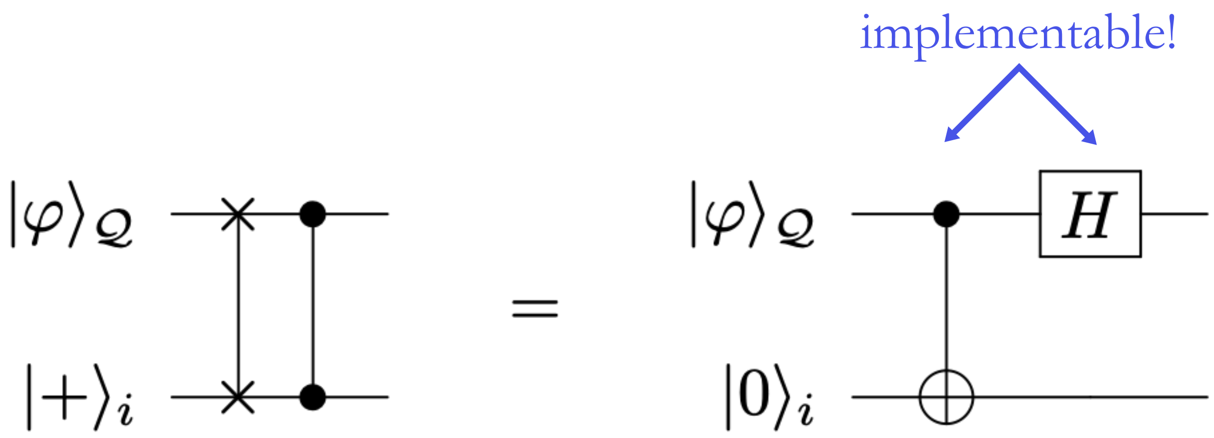
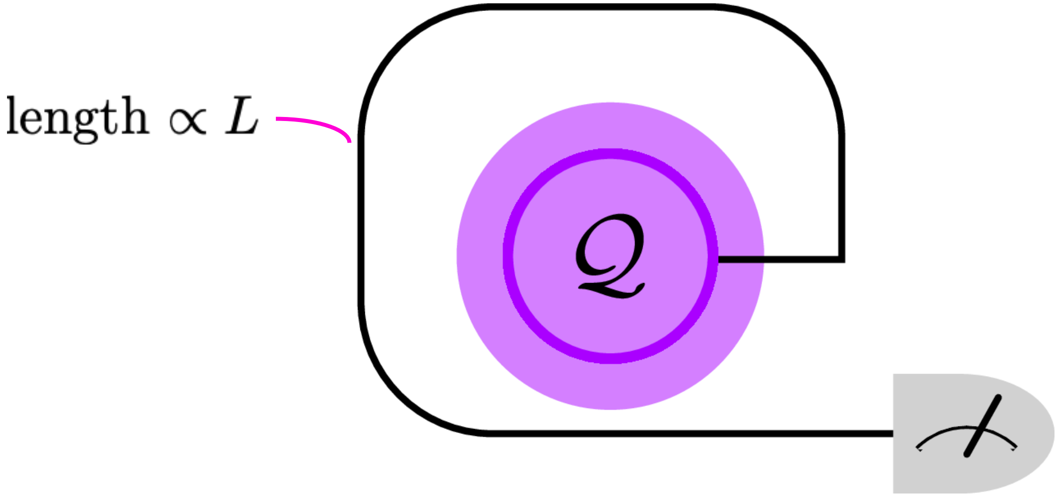
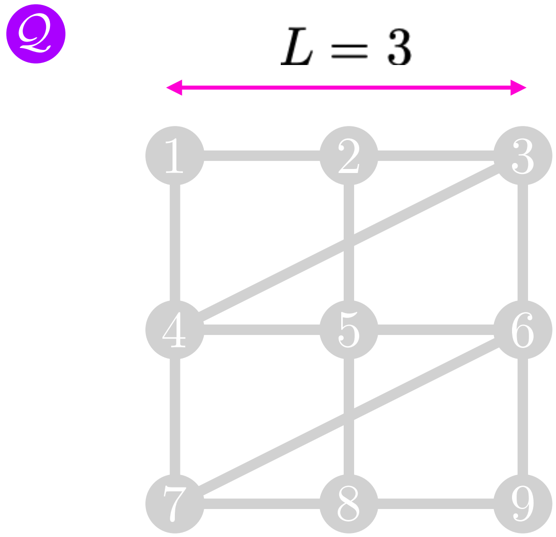
practical challenge: dual-rail SWAP gate ??

(encoding scheme in which qubit loss is detectable)

fix:




length $\propto L$
$\mathcal{Q}$


$\mathcal{Q}$
$L = 3$
1
2
3
4
5
6
7
8
9


$|+\rangle_{\mathcal{Q}}$
$|0\rangle_1$
$|0\rangle_2$
$|0\rangle_3$
$|0\rangle_4$
$|0\rangle_5$
$|0\rangle_6$
$|0\rangle_7$
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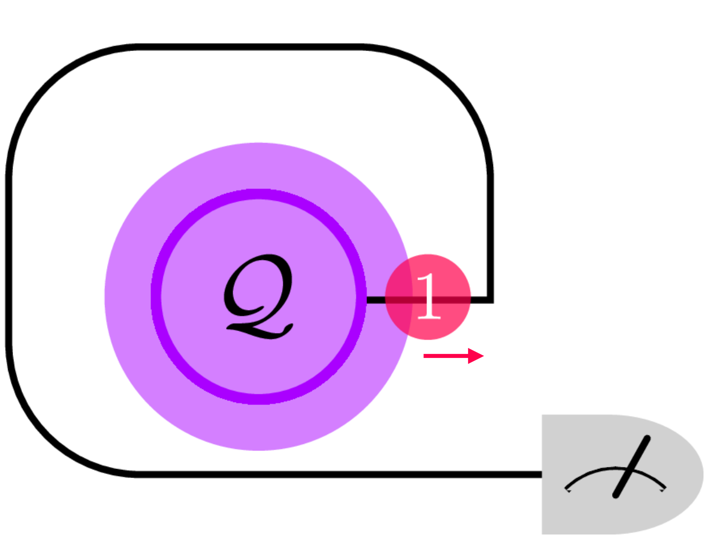
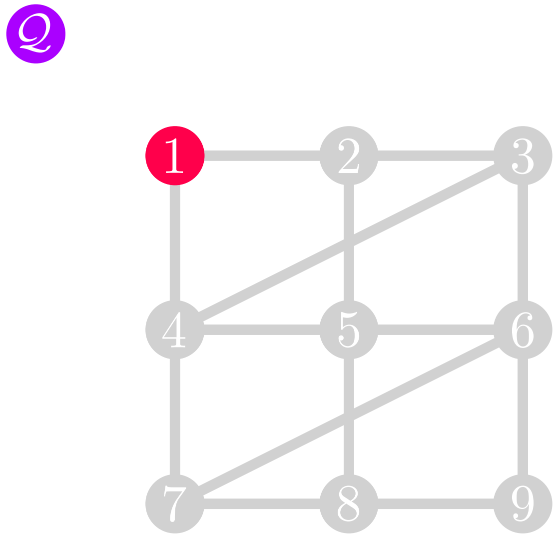
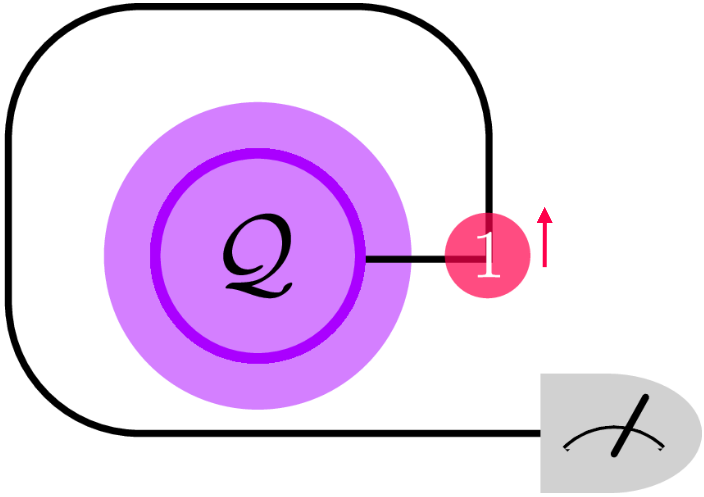
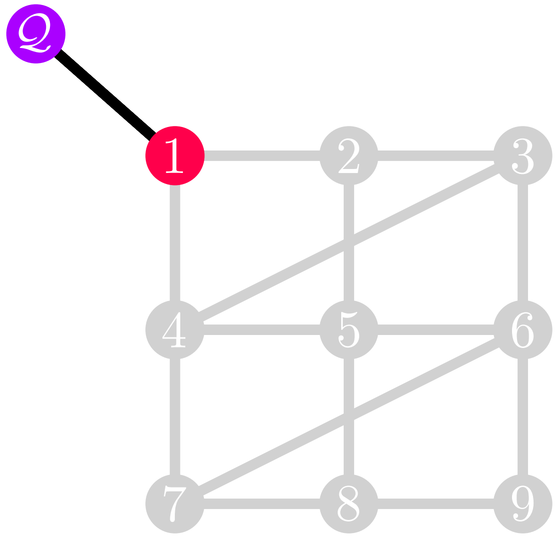
$|0\rangle_9$

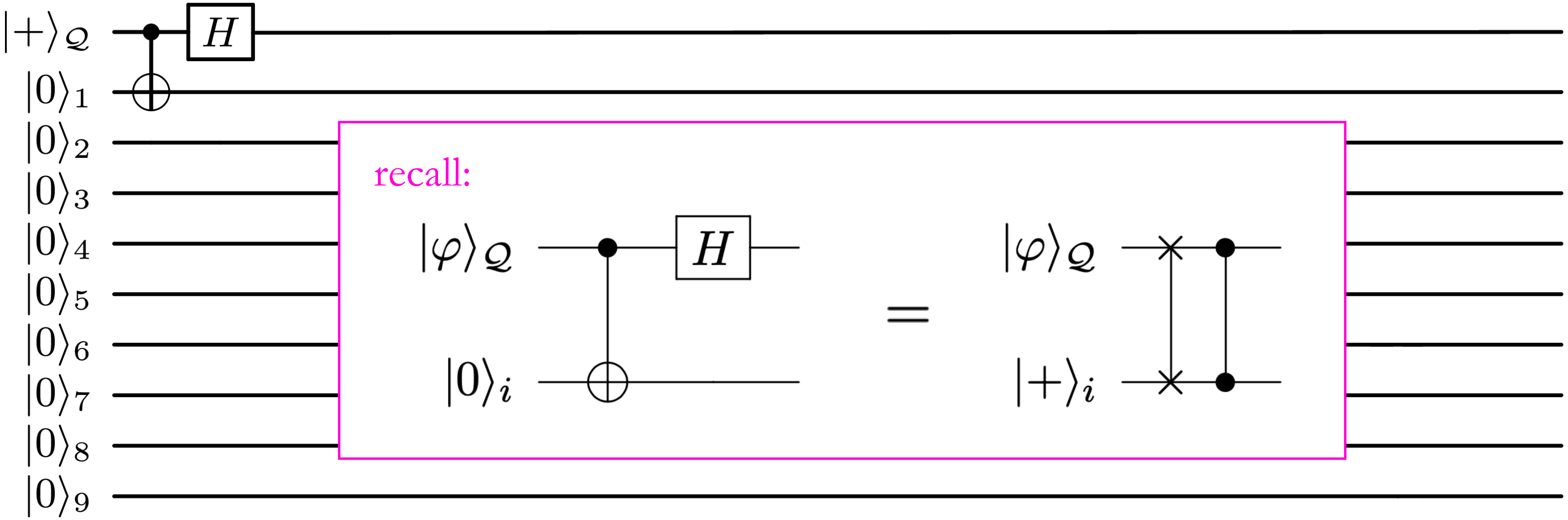




$$\text{“CNOT”}: \ (\alpha|0\rangle + \beta|1\rangle)_{\mathcal{Q}}|\varnothing\rangle_i \mapsto \alpha|0\rangle_{\mathcal{Q}}|0\rangle_i + \beta|1\rangle_{\mathcal{Q}}|1\rangle_i$$







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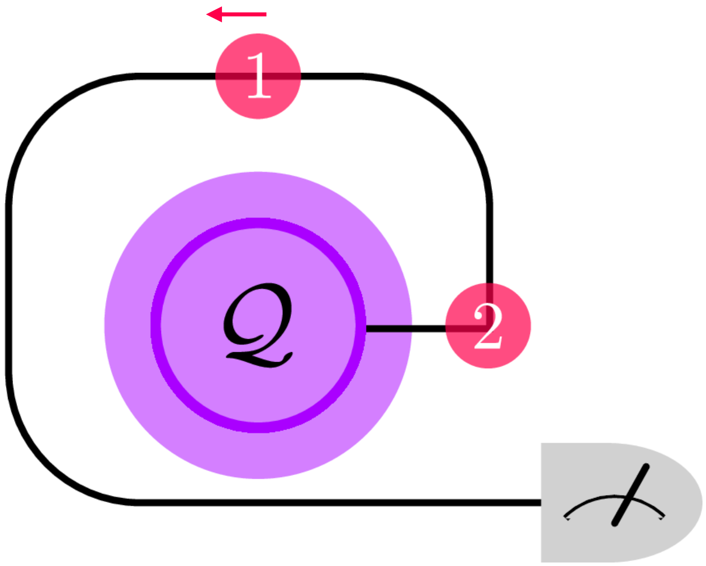

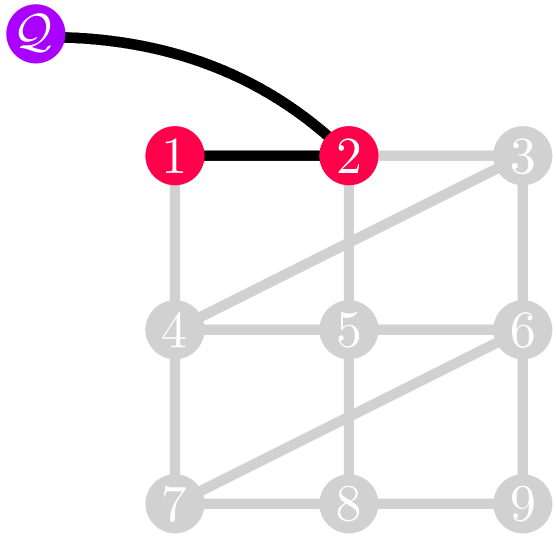

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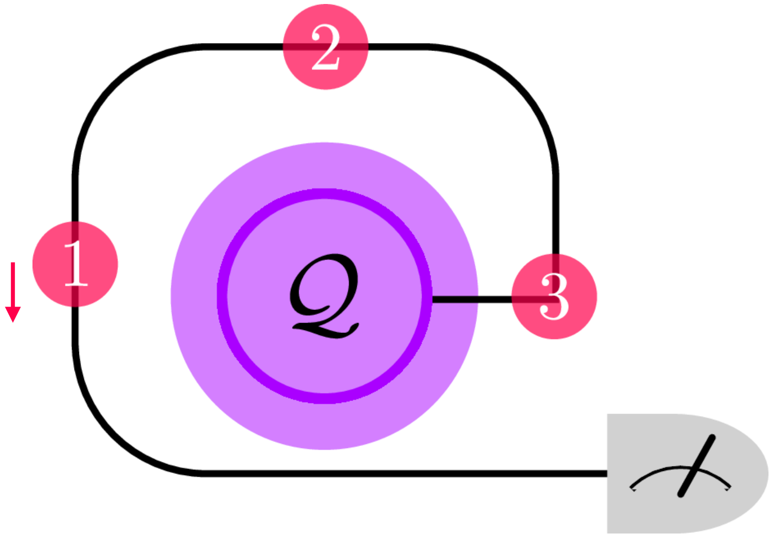

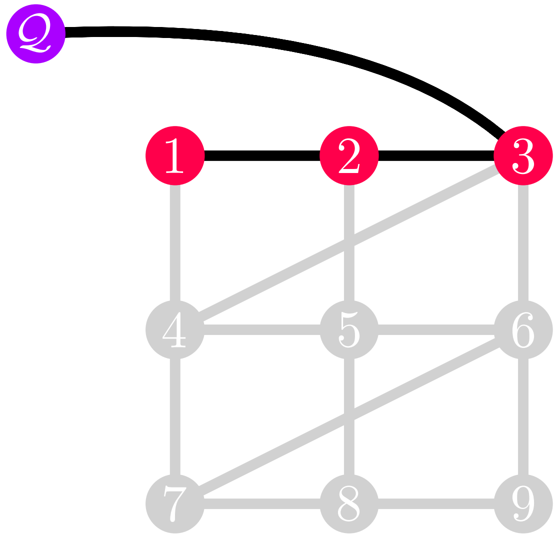

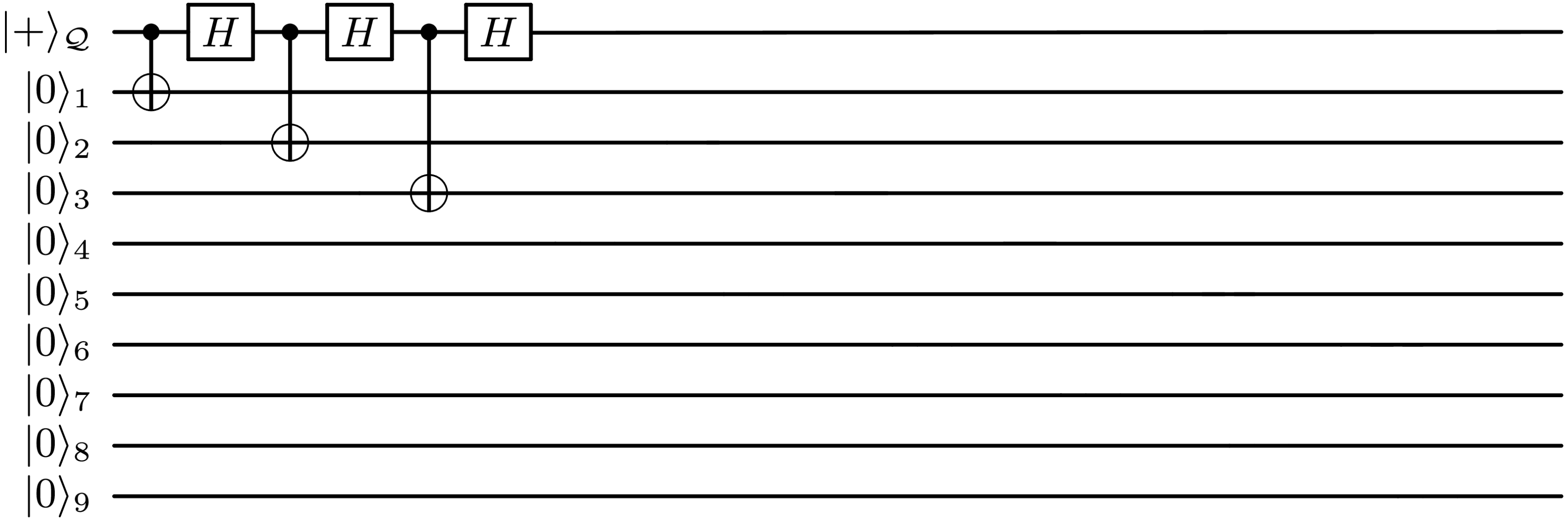

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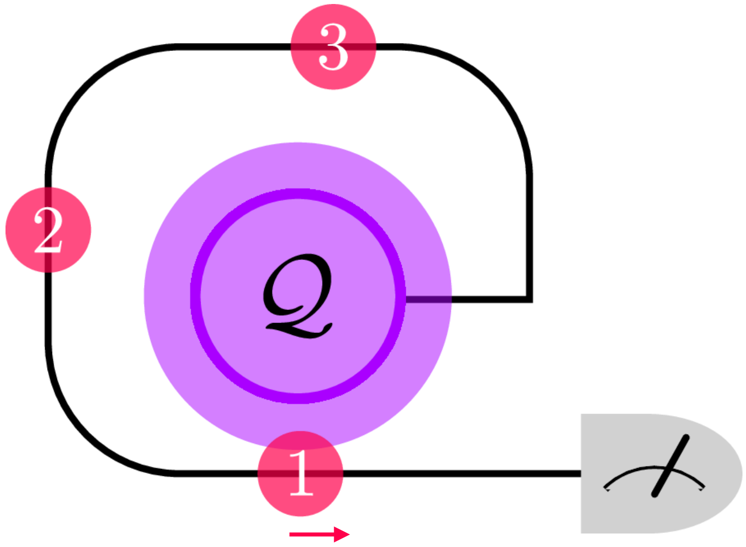

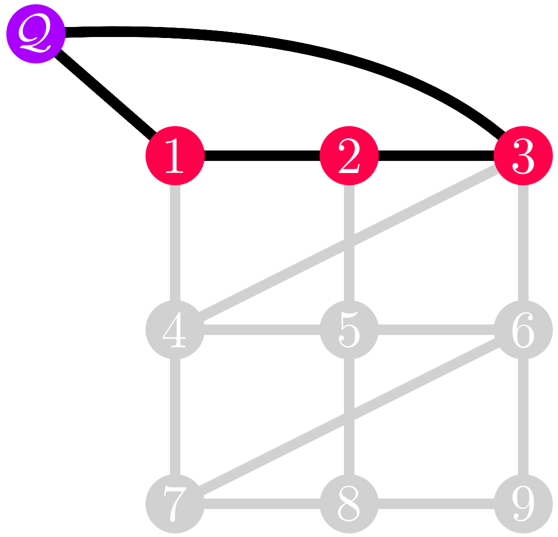

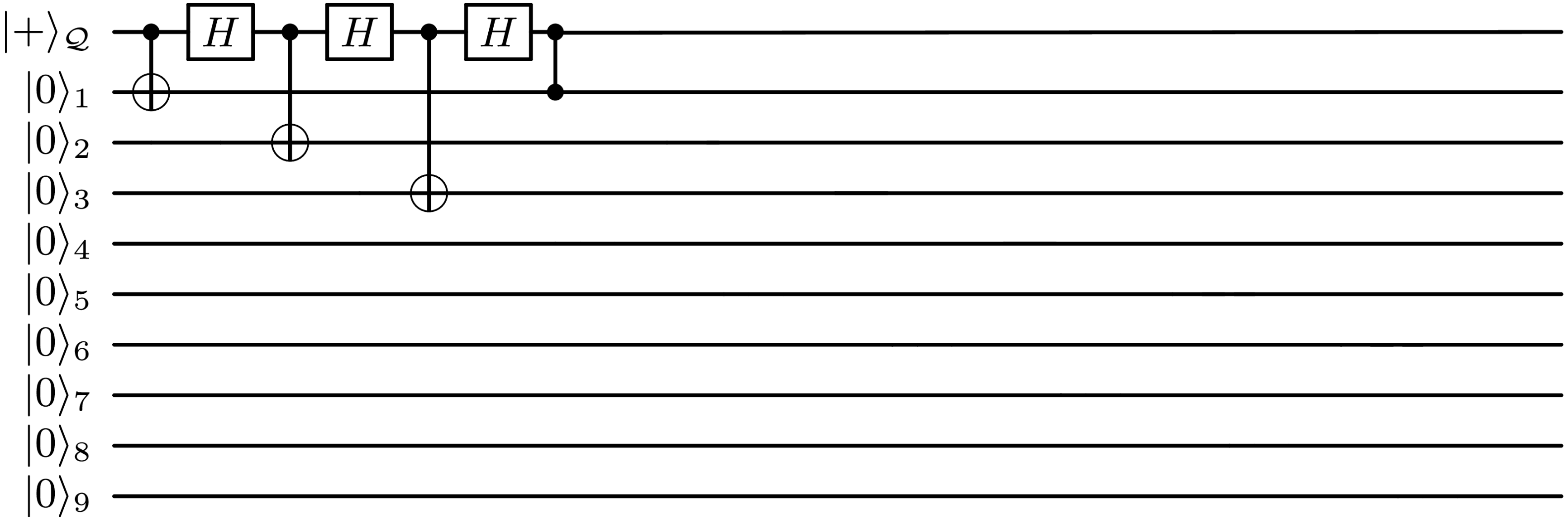

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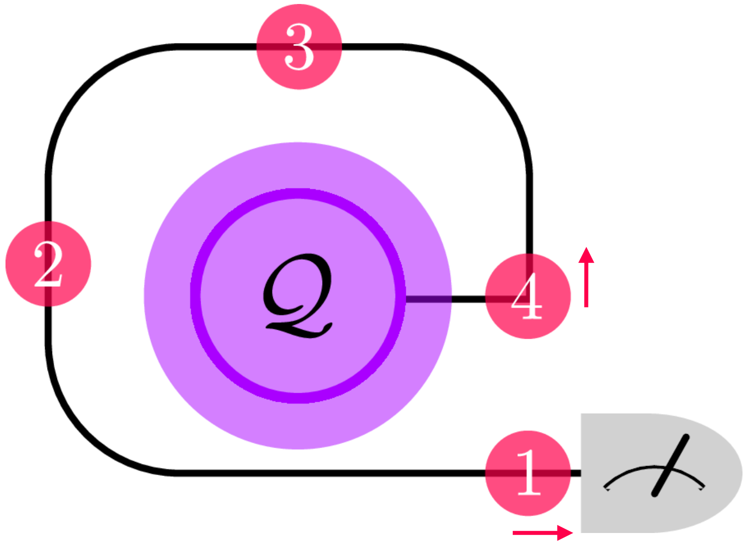

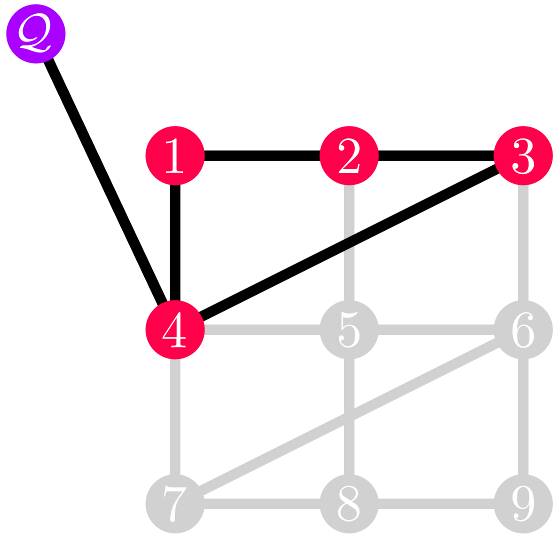

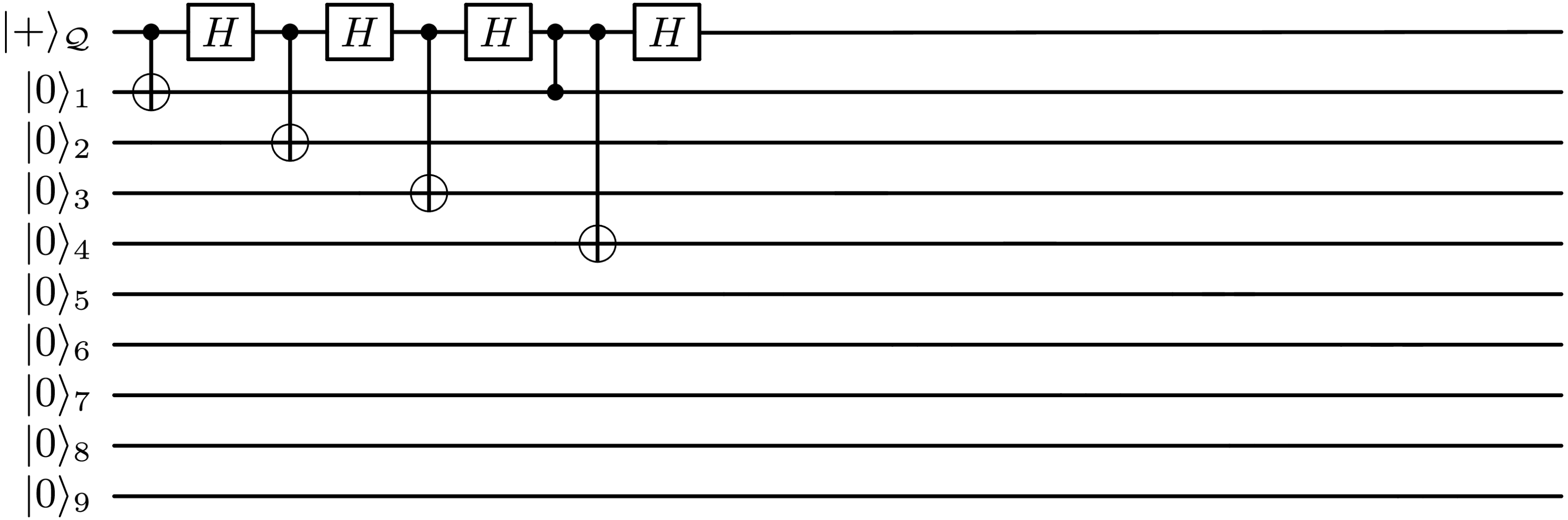

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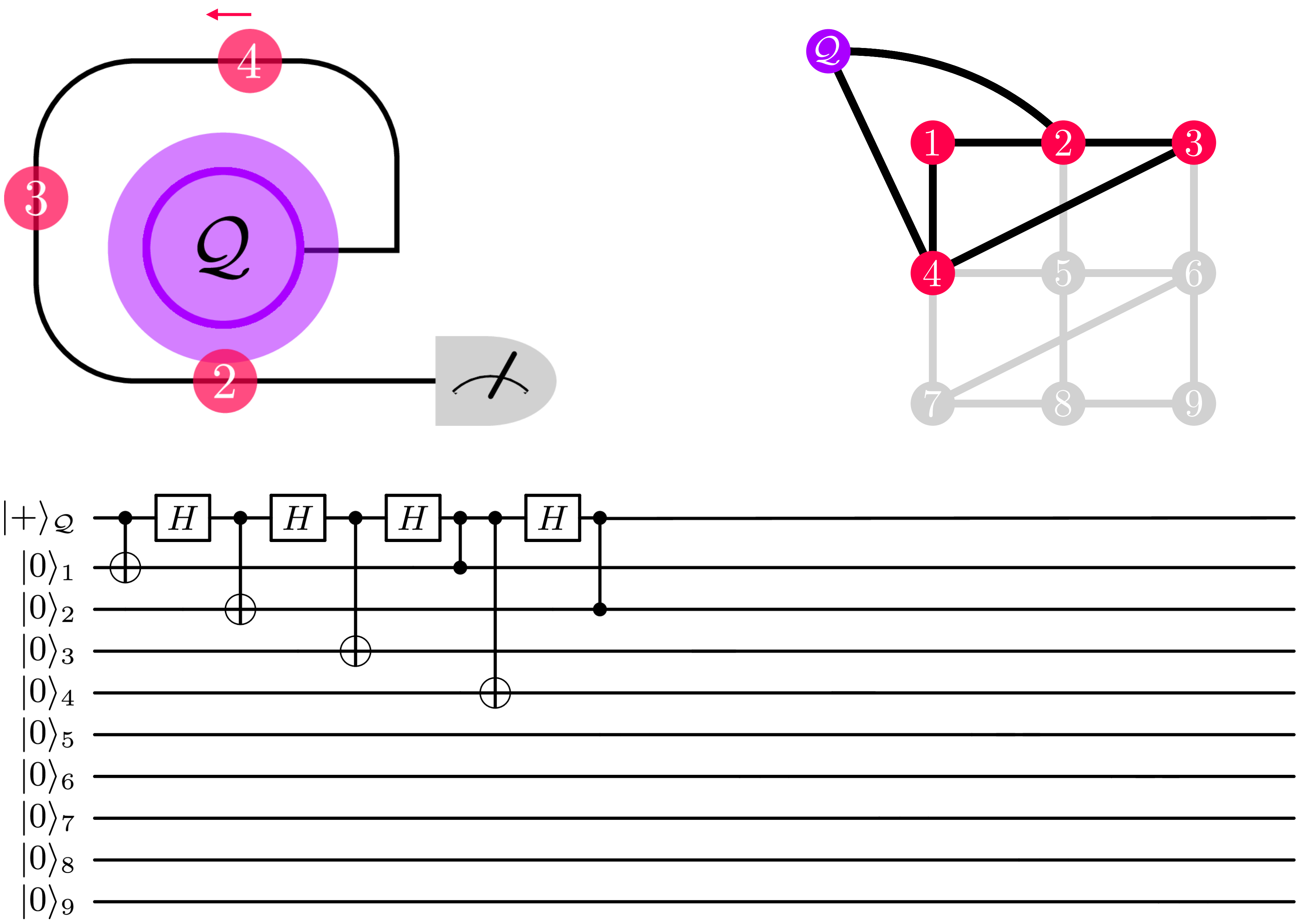

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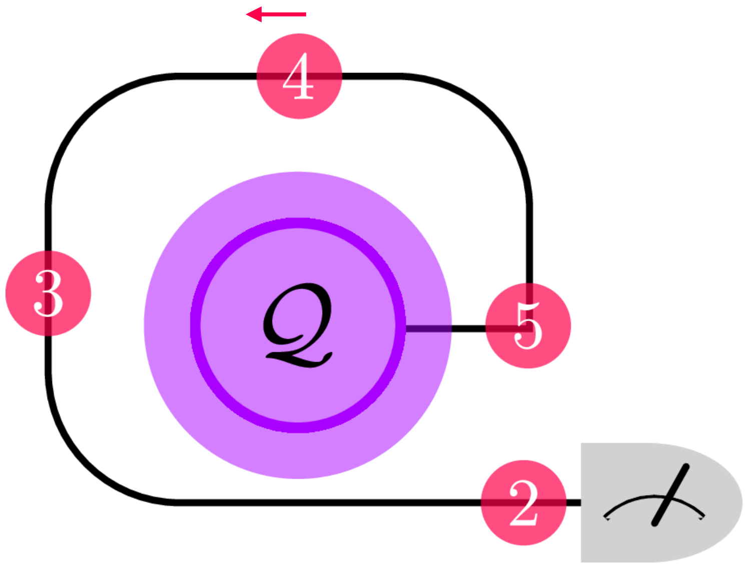

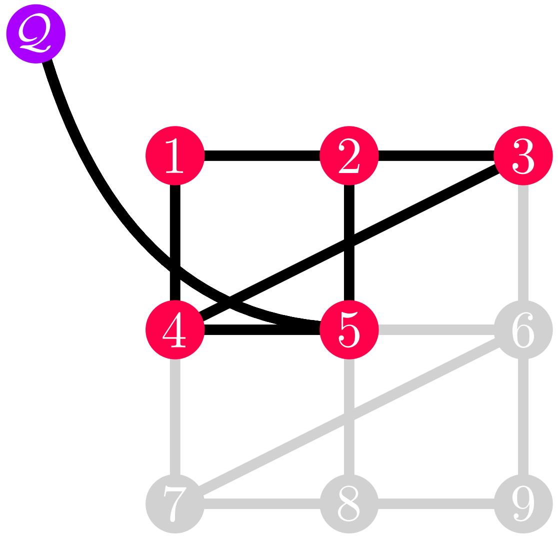

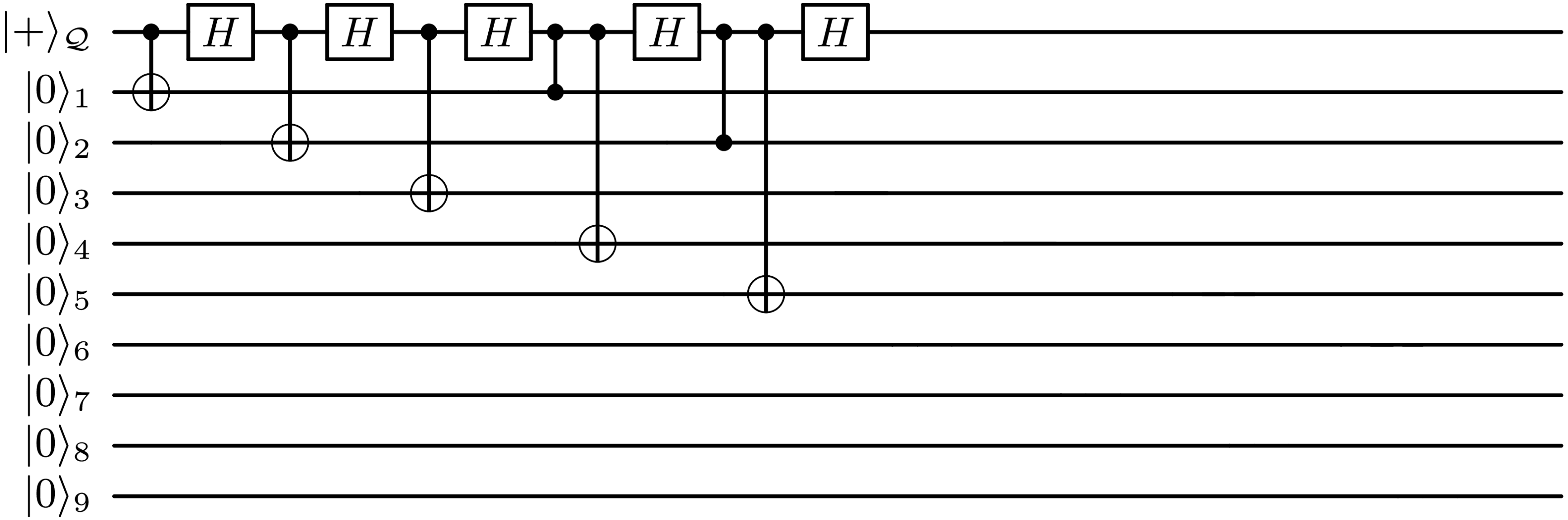

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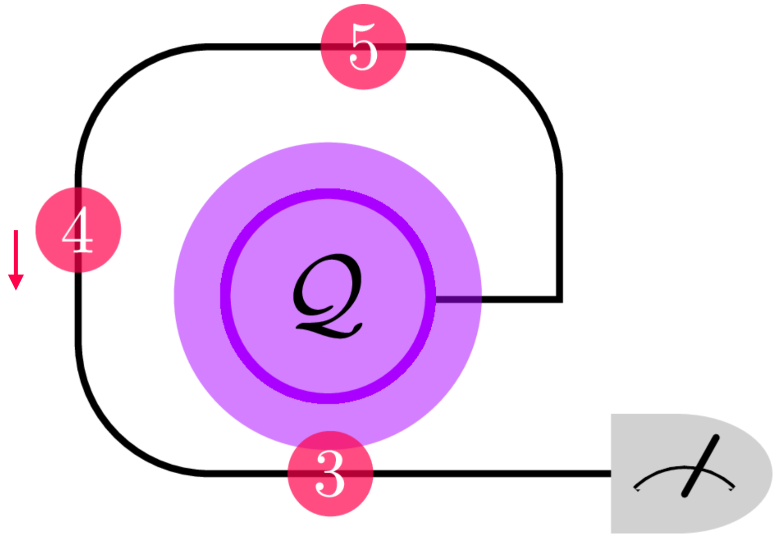

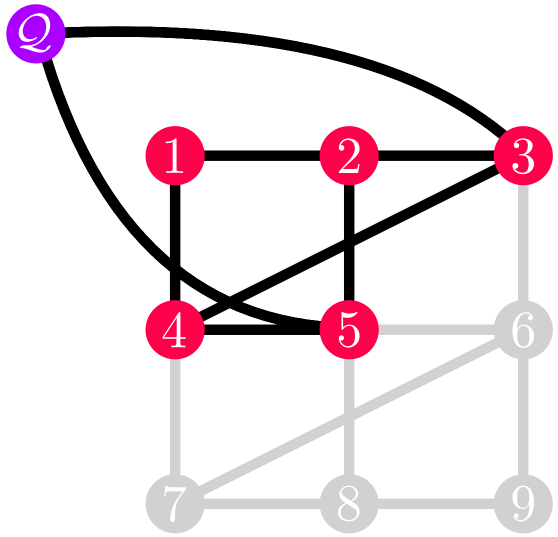

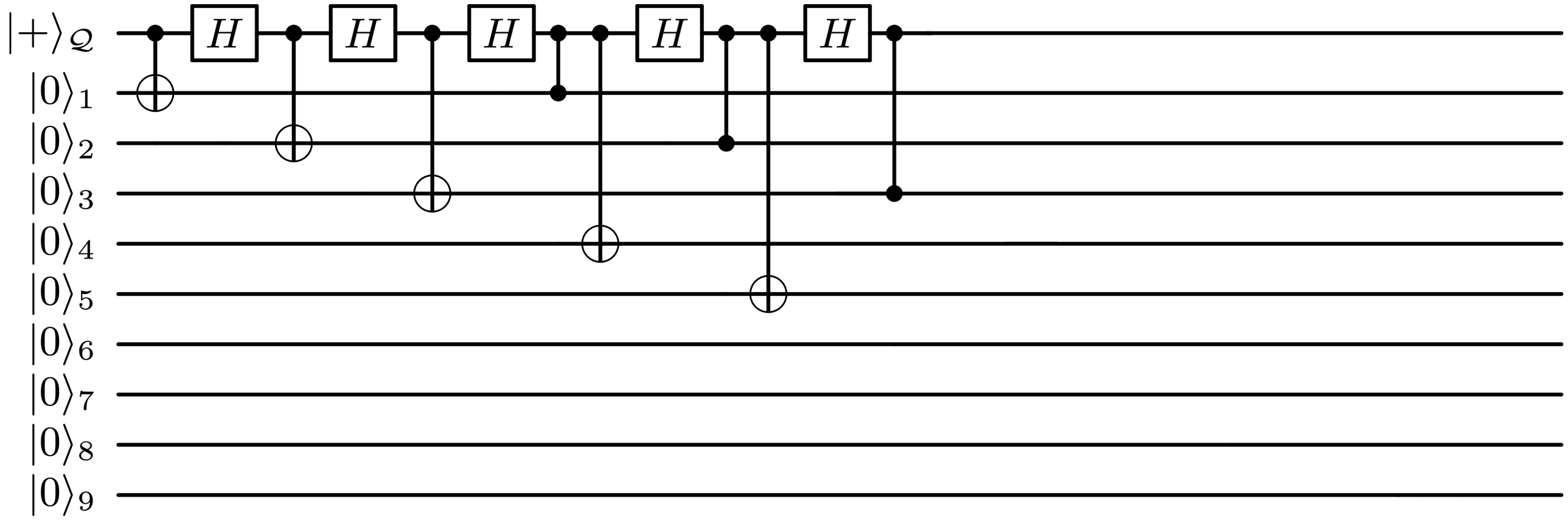

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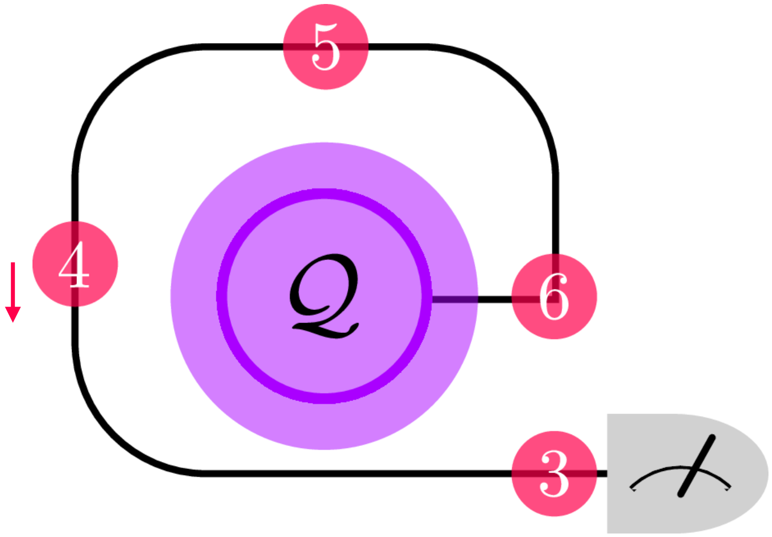

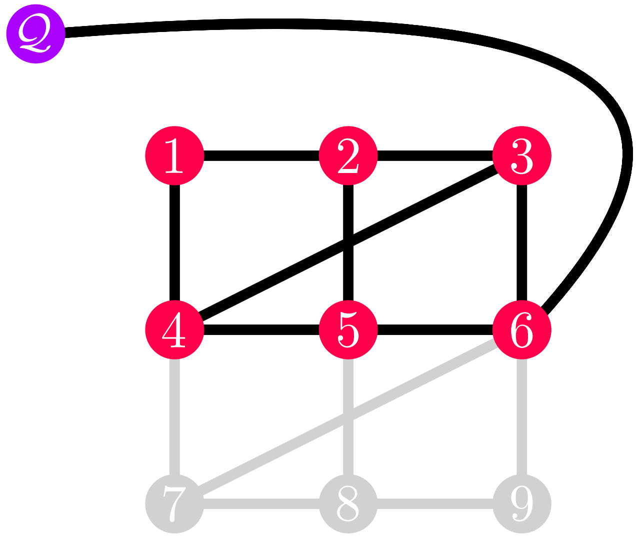

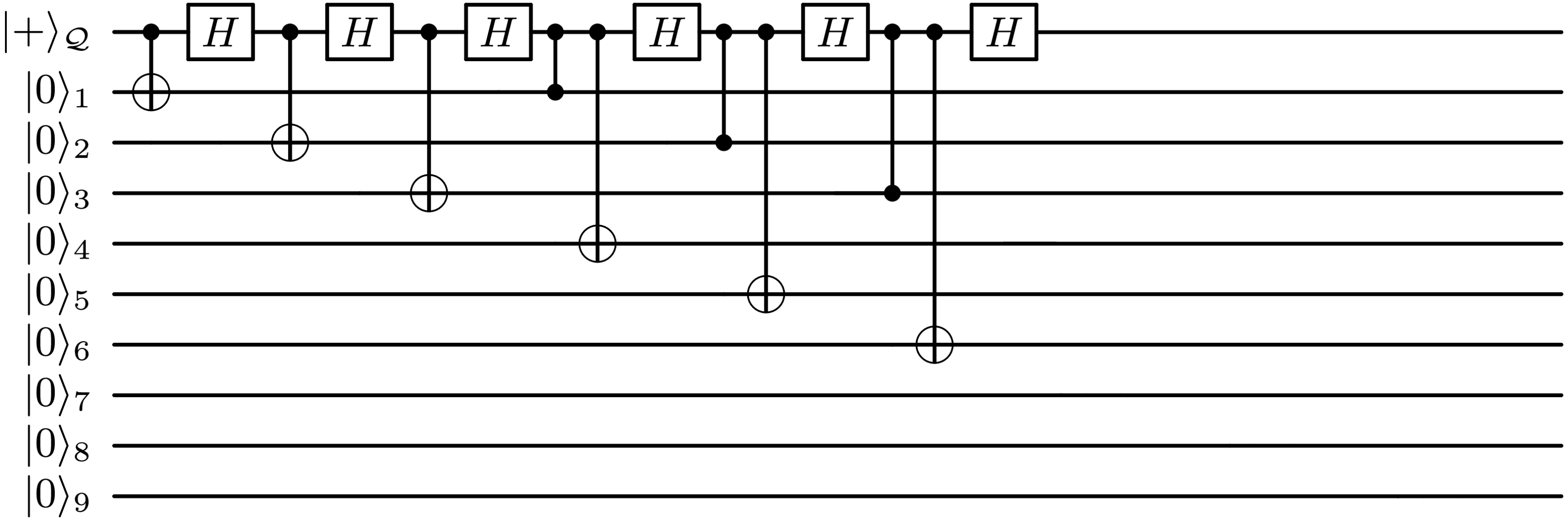

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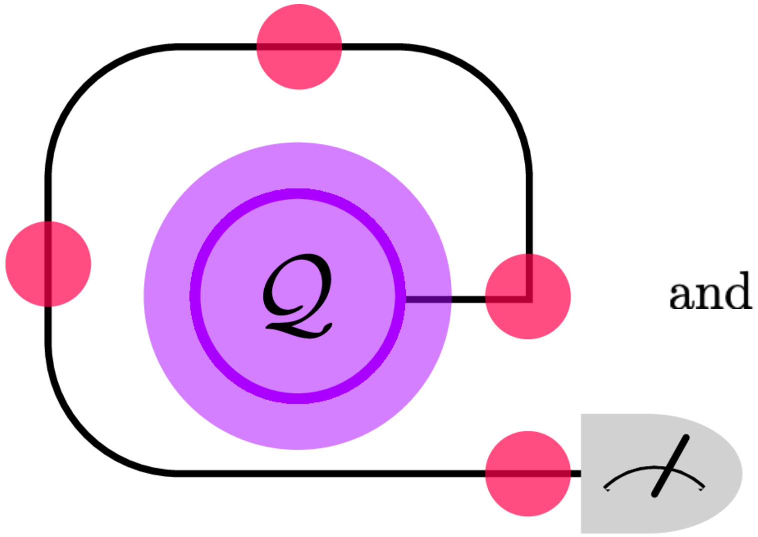

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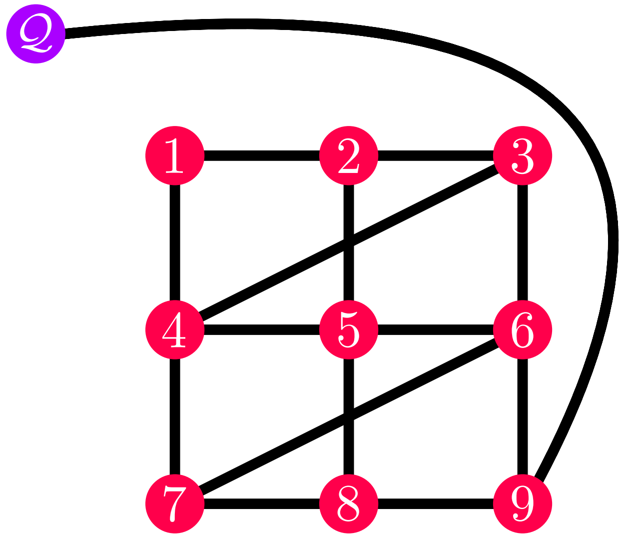

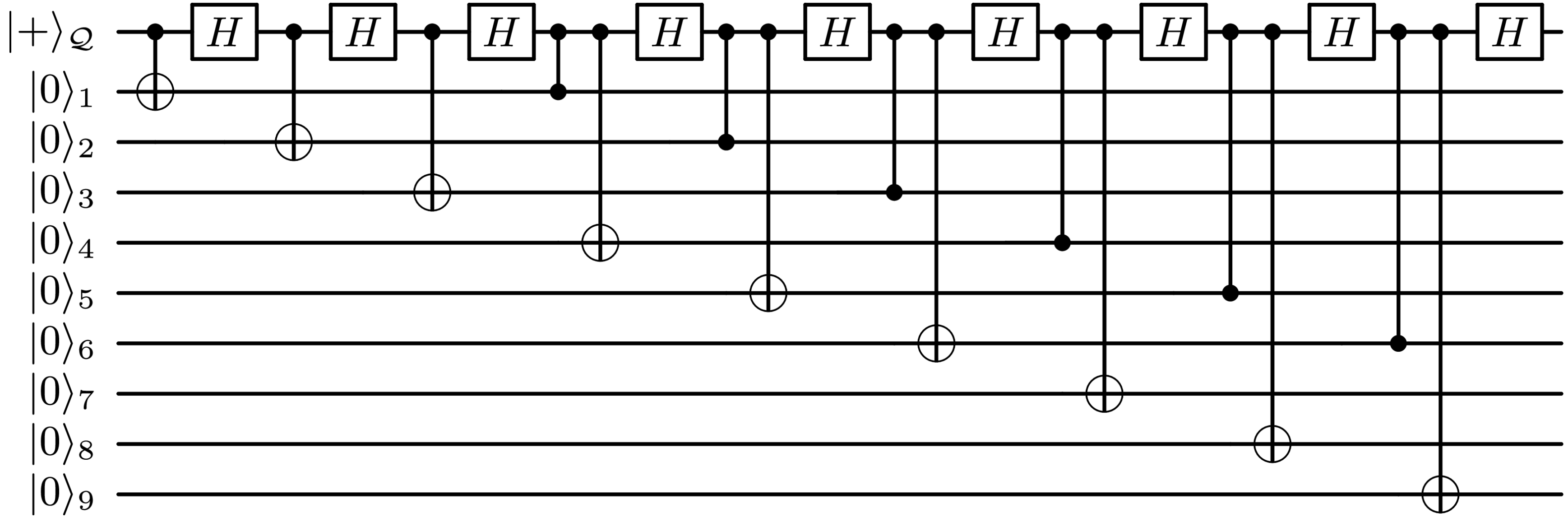

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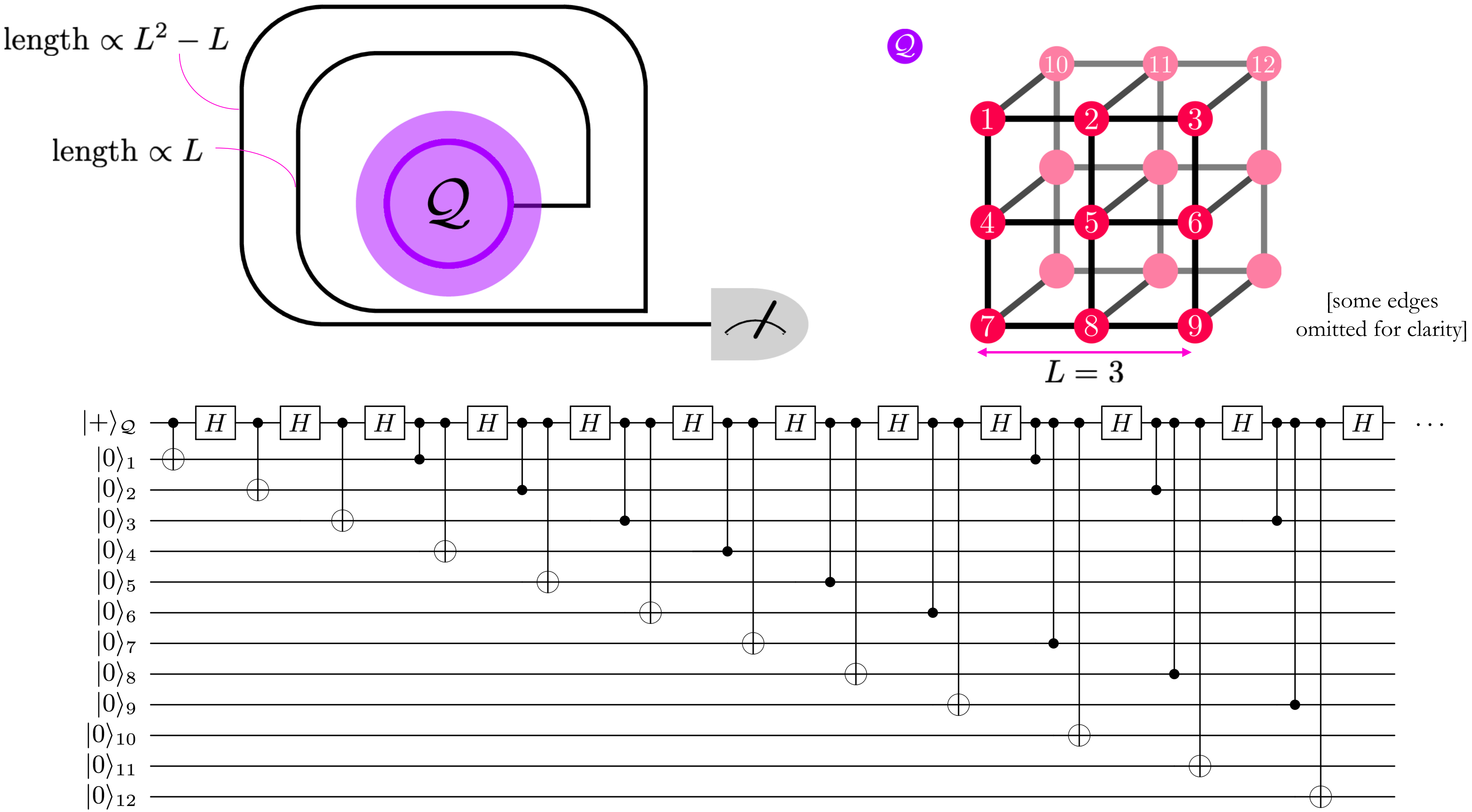
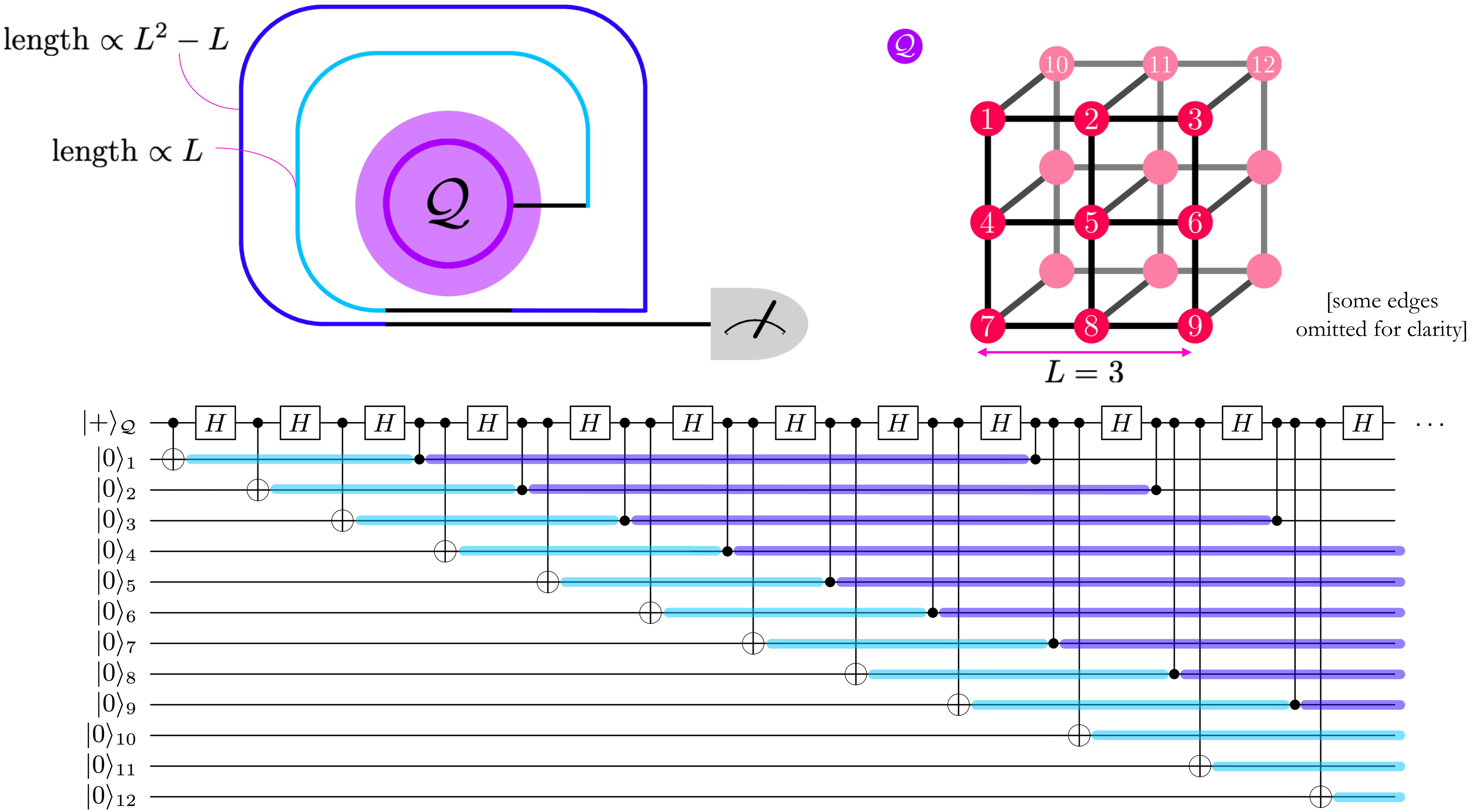
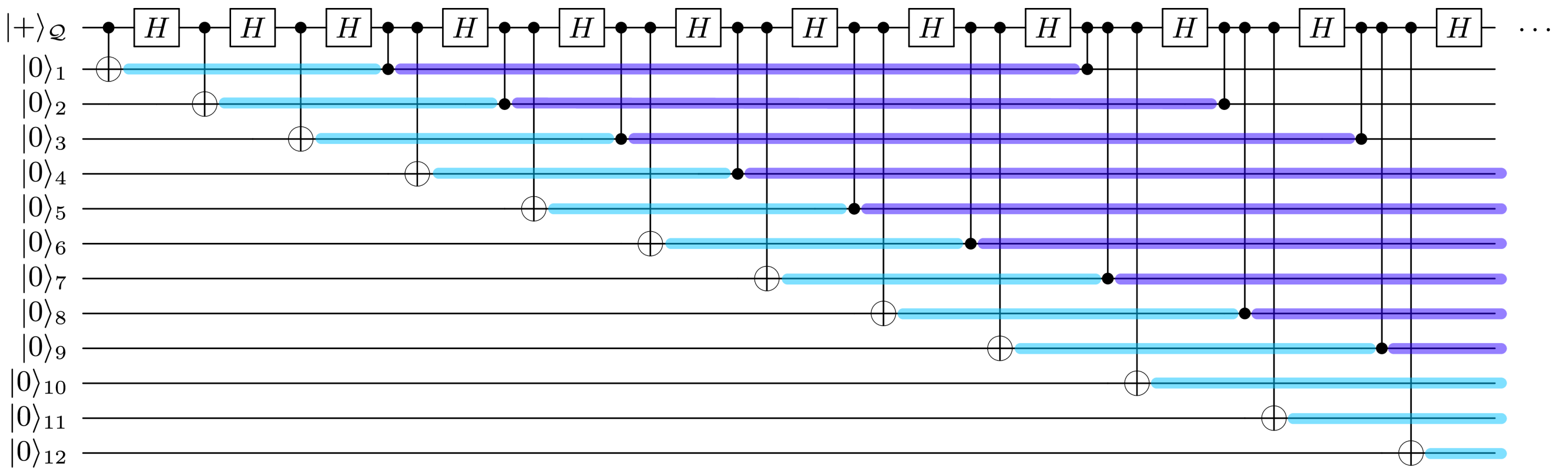


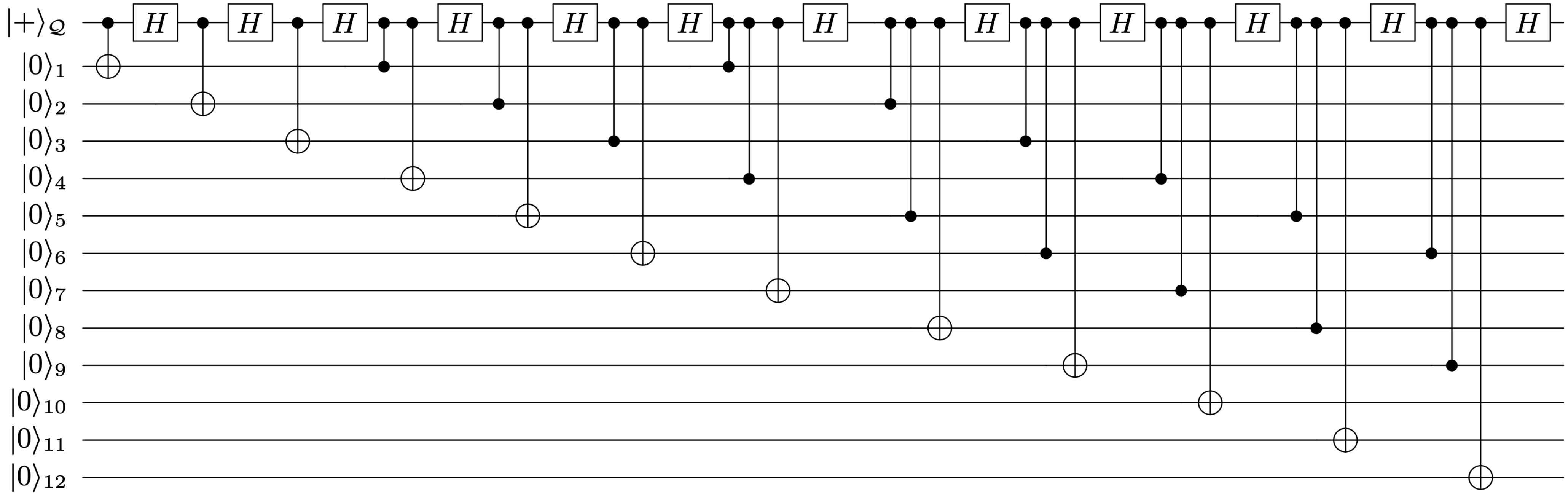
length ∝ L² − L
length ∝ L
[some edges omitted for clarity]
L = 3
|+⟩_Q
|0⟩_1
|0⟩_2
|0⟩_3
|0⟩_4
|0⟩_5
|0⟩_6
|0⟩_7
|0⟩_8
|0⟩_9
|0⟩_10
|0⟩_11
|0⟩_12

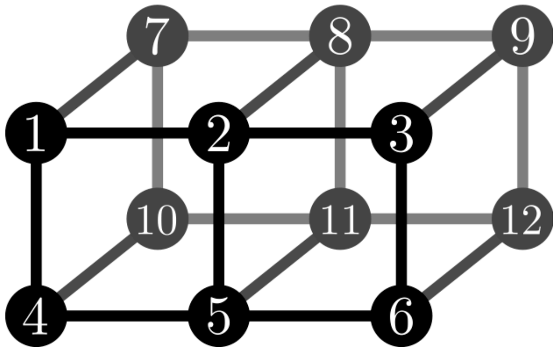

length ∝ $L^2 - L$
length ∝ L
$\mathcal{Q}$
$\mathcal{Q}$
10
11
12
1
2
3
4
5
6
7
8
9
[some edges
omitted for clarity]
$L = 3$
$|+\rangle_{\mathcal{Q}}$
$|0\rangle_1$
$|0\rangle_2$
$|0\rangle_3$
$|0\rangle_4$
$|0\rangle_5$
$|0\rangle_6$
$|0\rangle_7$
$|0\rangle_8$
$|0\rangle_9$
$|0\rangle_{10}$
$|0\rangle_{11}$
$|0\rangle_{12}$
H
$\cdots$

potential concerns

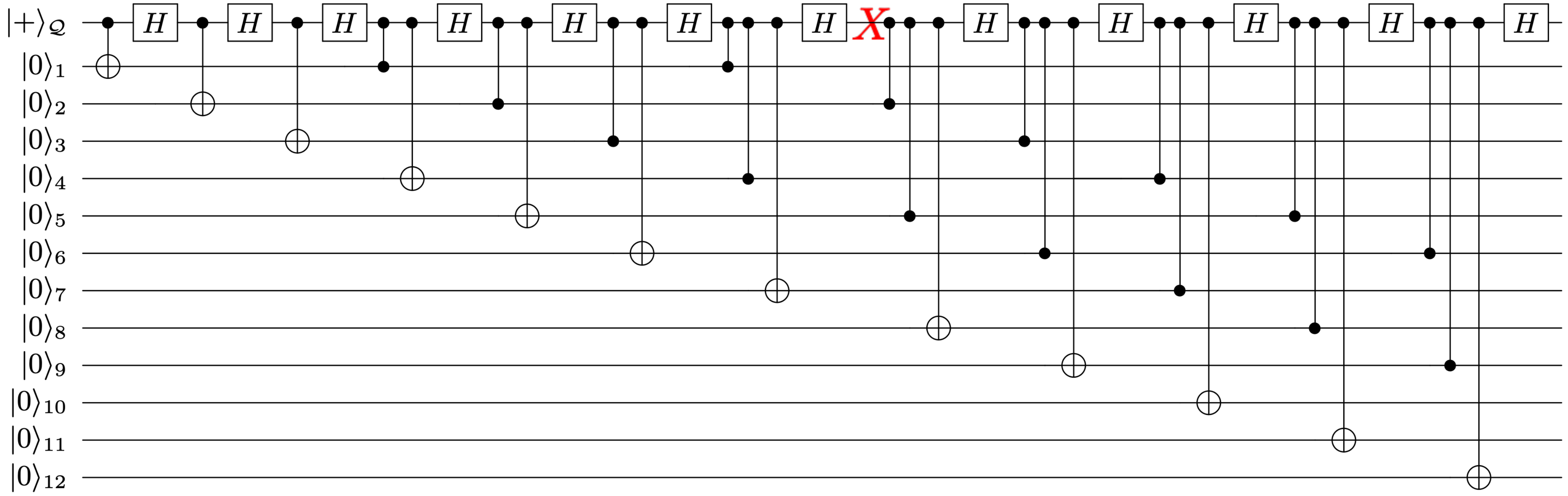
1. propagation of circuit-level errors

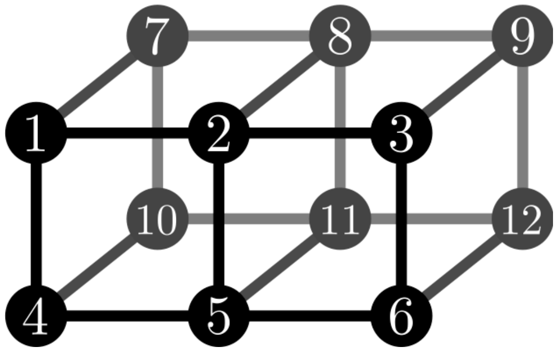






[some edges (periodic BCs) omitted for clarity]





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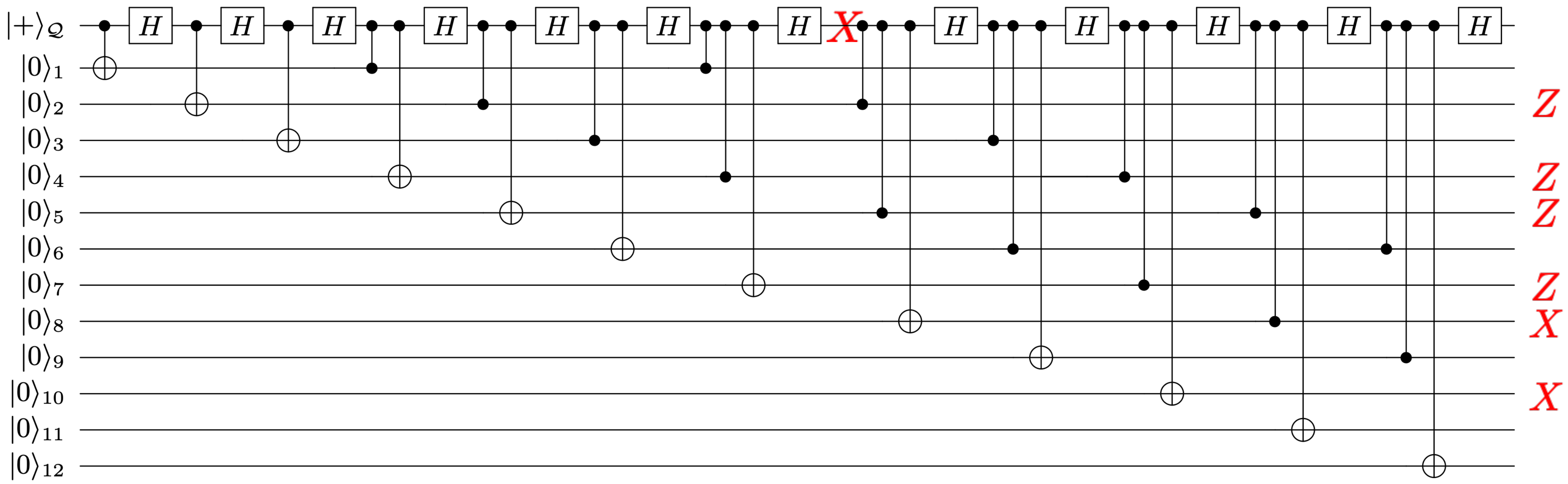

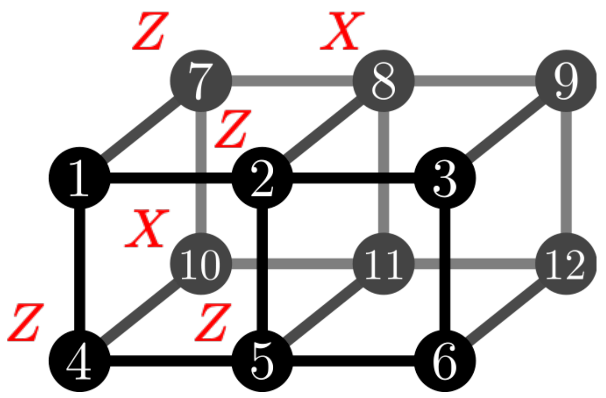

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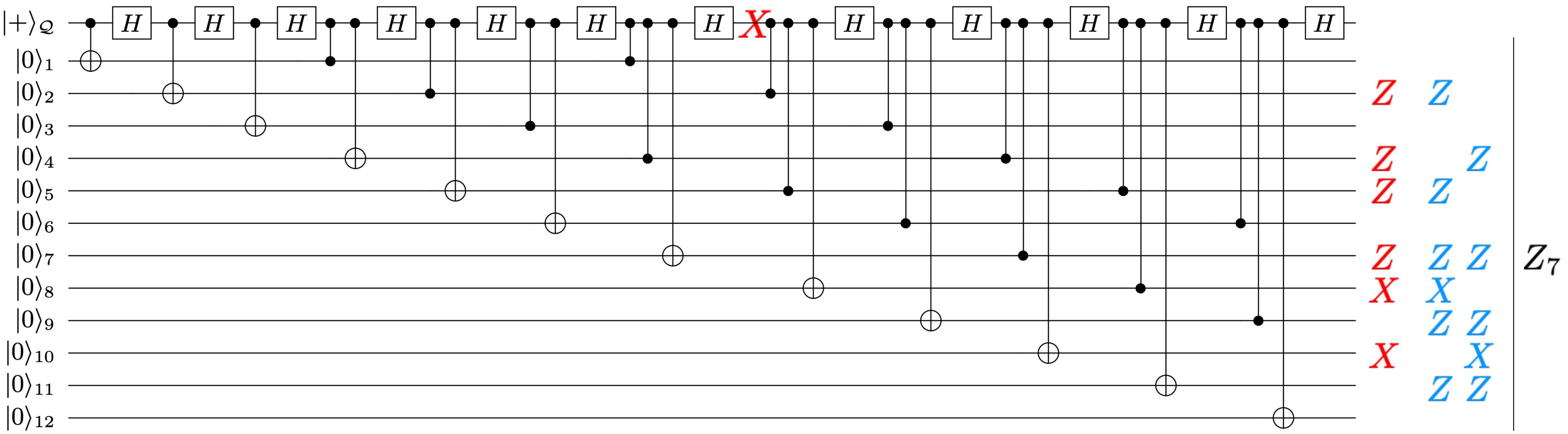
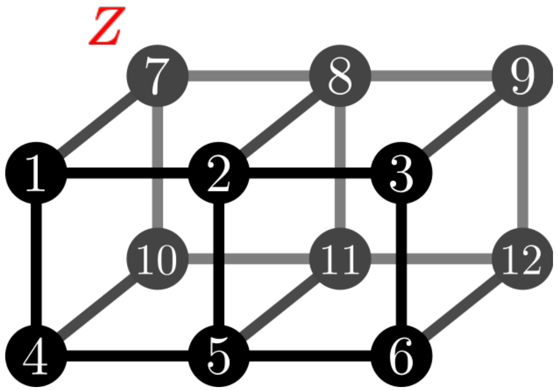




[some edges (periodic BCs) omitted for clarity]

cluster state stabiliser generators:

$$S_i = X_i \bigotimes_{j \in N(i)} Z_j, \qquad N(i) := \{j : (i,j) \in E\}$$

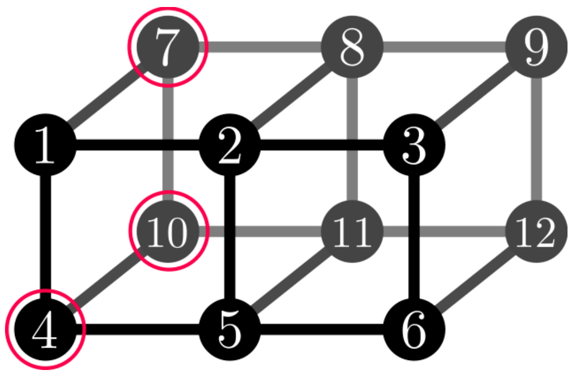
for this lattice,

e.g., $S_8 = X_8 Z_2 Z_5 Z_7 Z_9 Z_{11}$

e.g., $S_{10} = X_{10} Z_4 Z_7 Z_9 Z_{11}$

all effective errors are local!

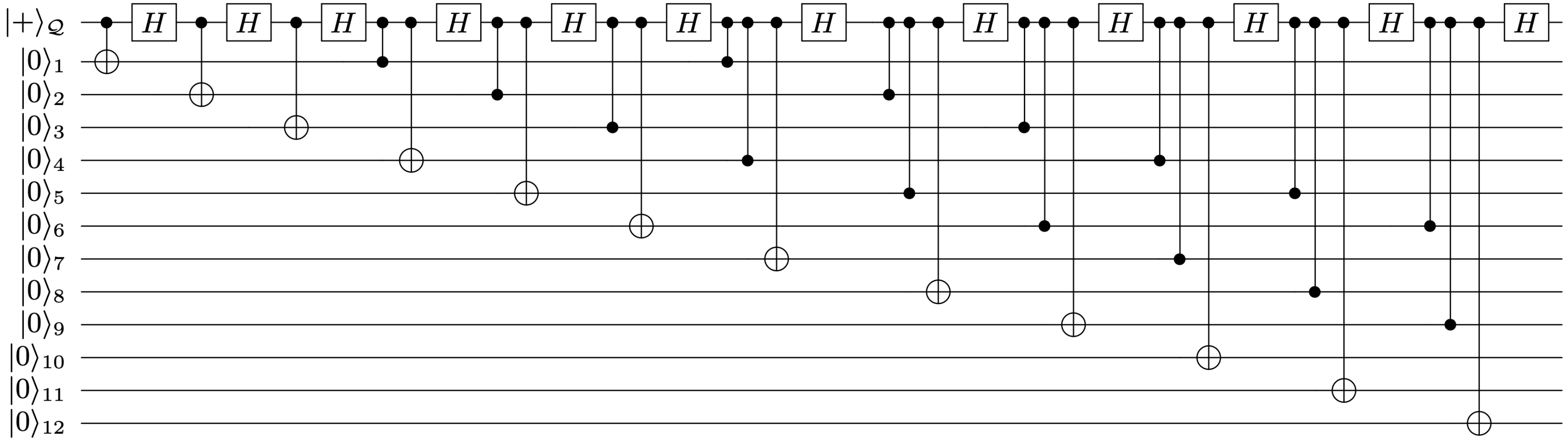
claim: *any* single-qubit circuit-level error $\Rightarrow$ error supported within $\{i\} \cup N(i)$ on the prepared cluster state, for some data qubit i



e.g., $S \subset \{i\} \cup N(i)$ for $i = 10$

claim: single-qubit circuit-level error $\Rightarrow$ error within $\{i\} \cup N(i)$

proof idea:



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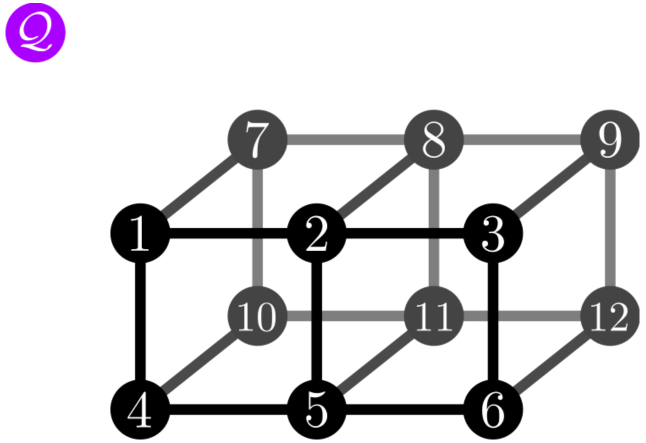

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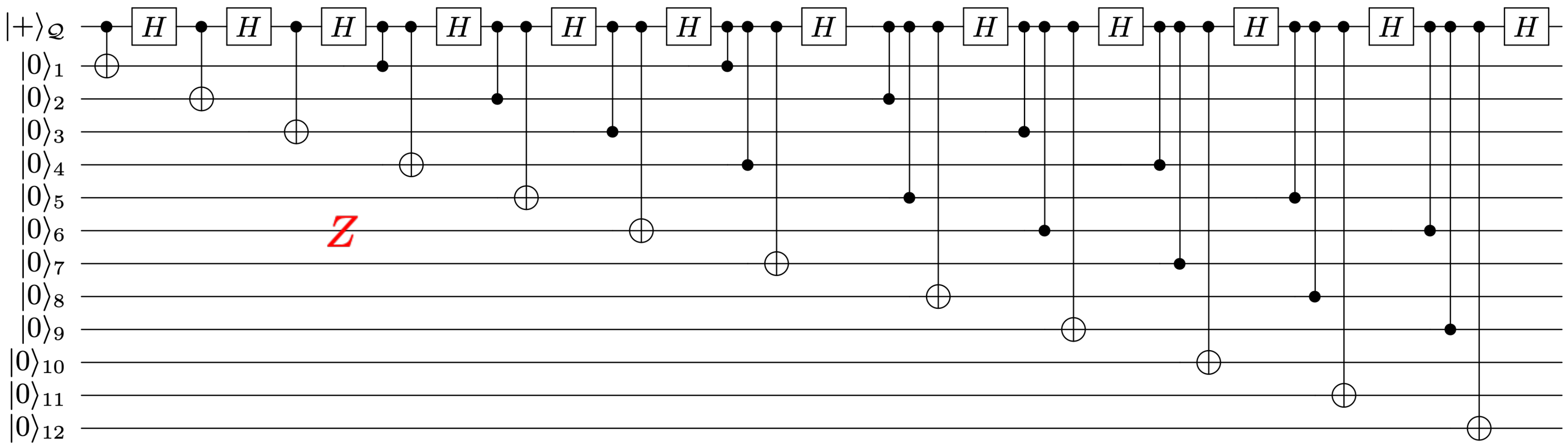

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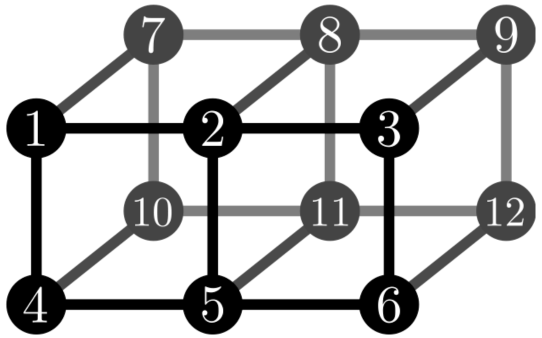

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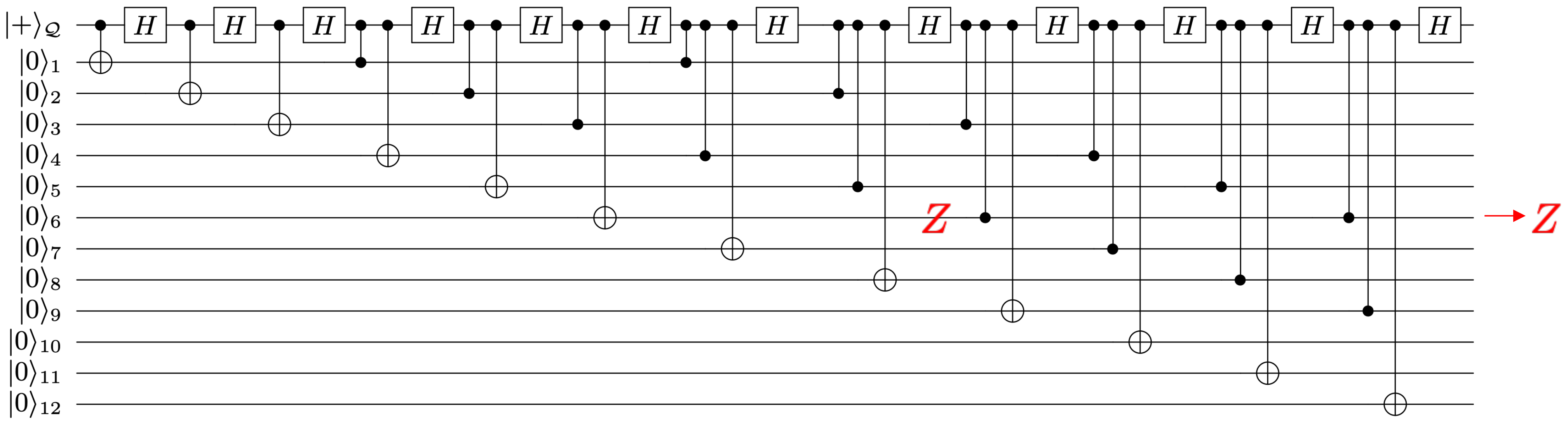

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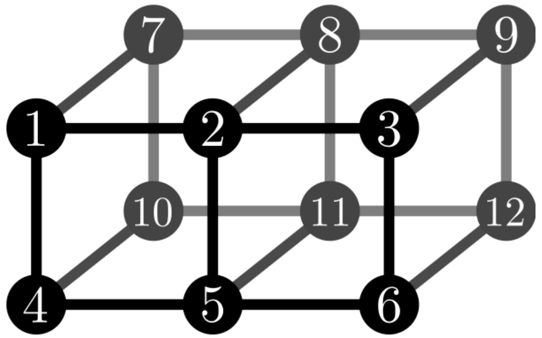

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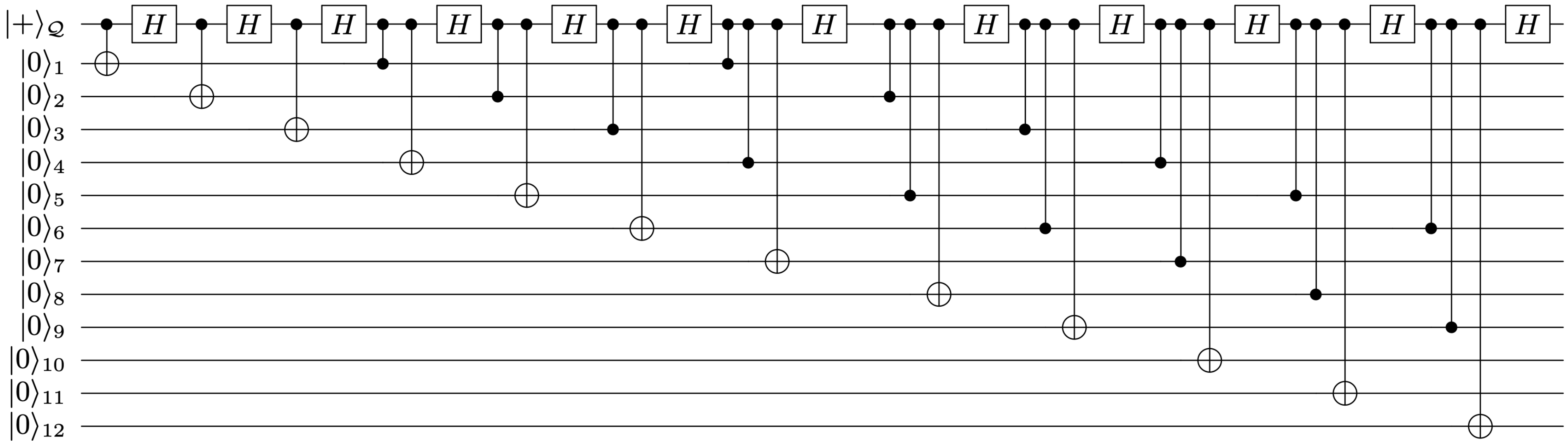

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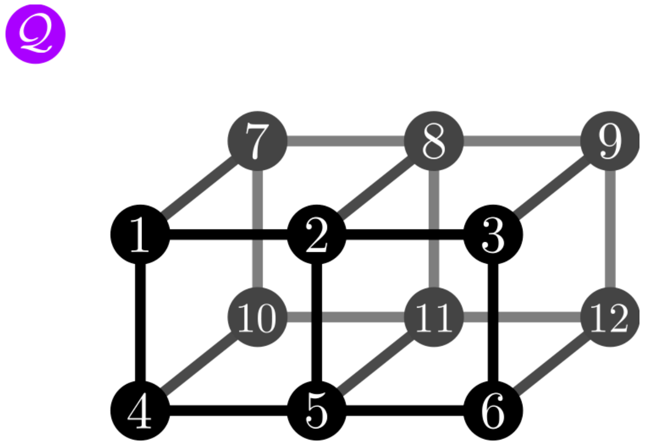

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recall stabilisers: $X_i \bigotimes_{j \in N(i)} Z_j$

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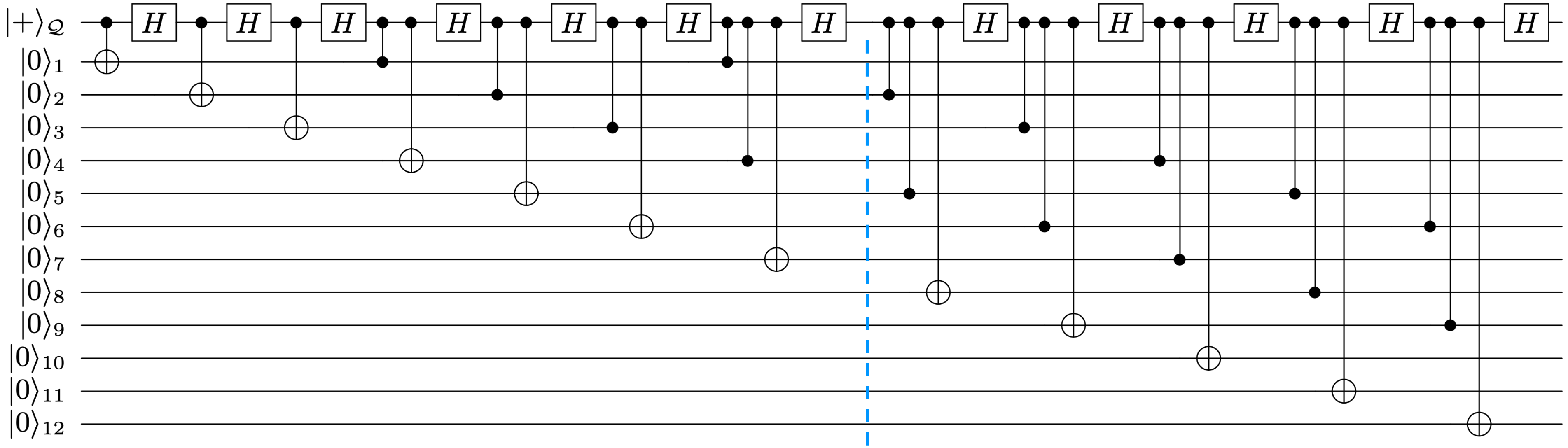

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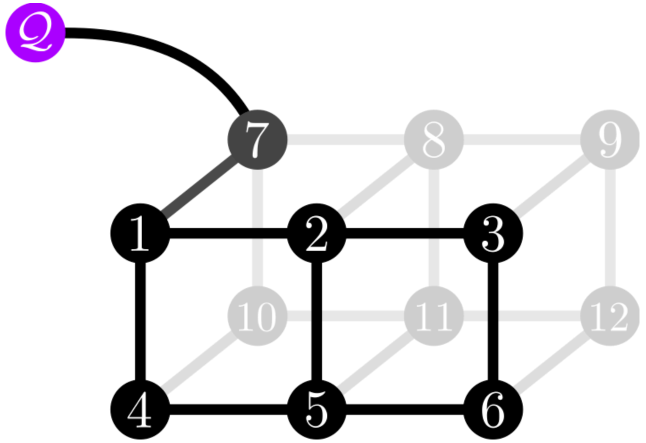

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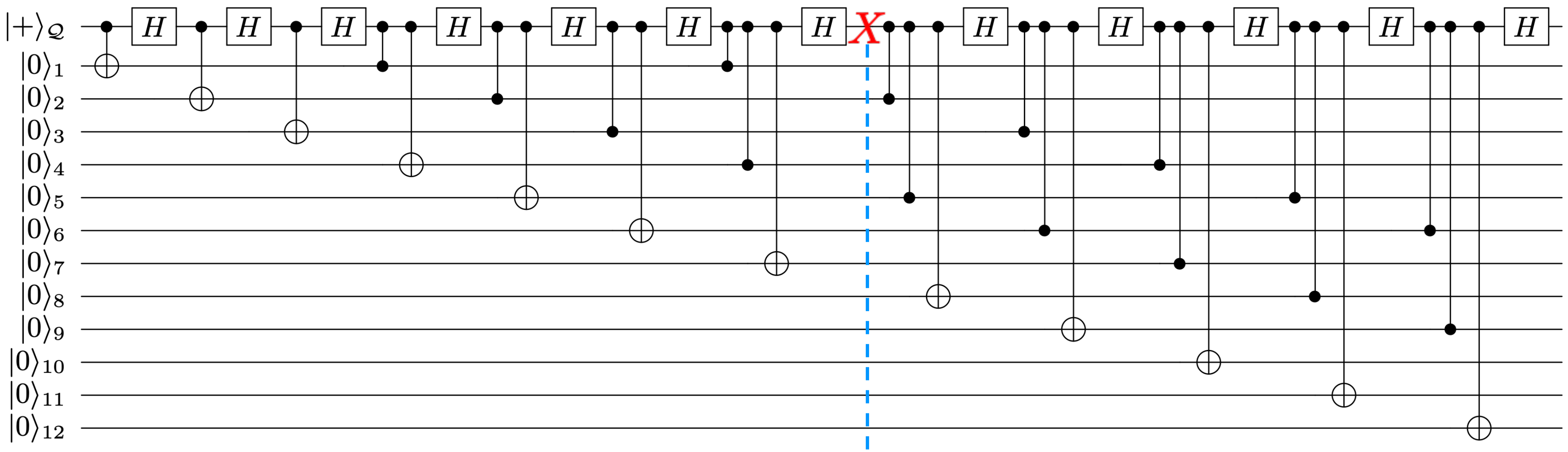

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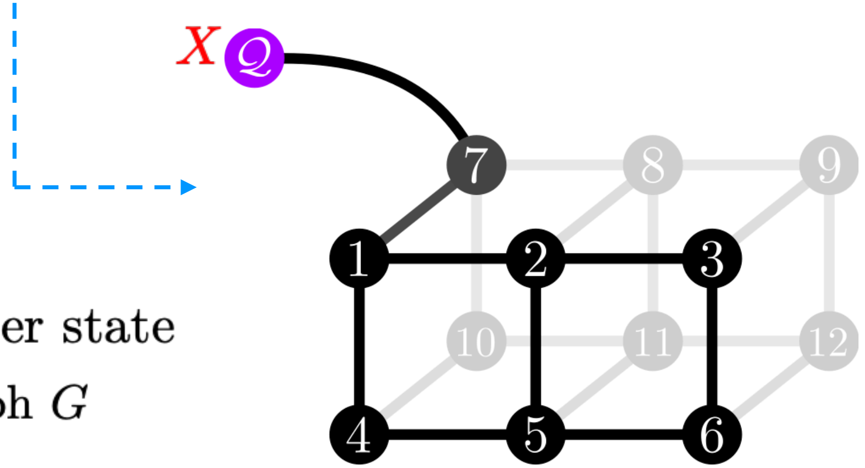

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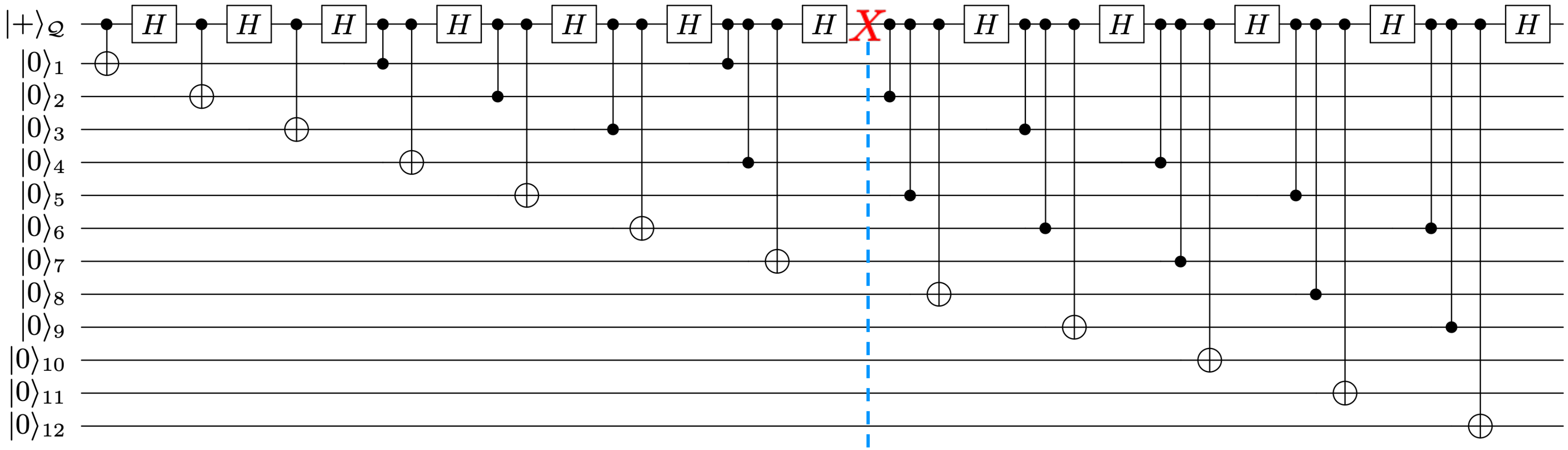

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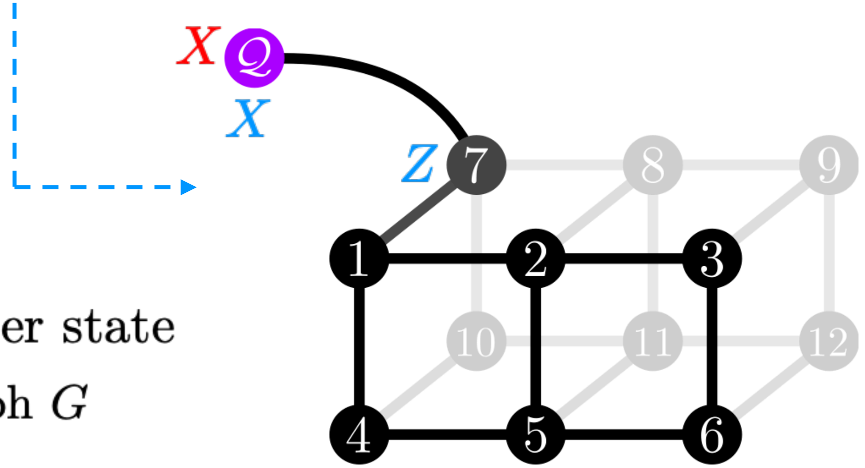

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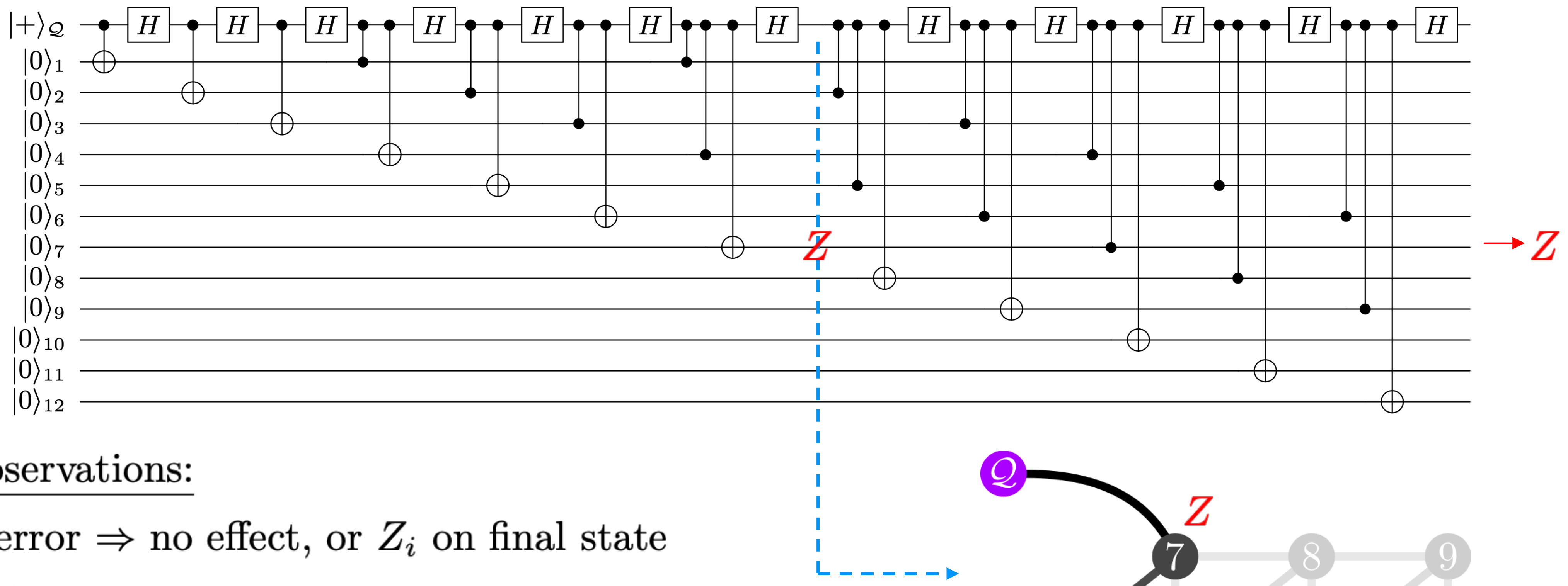

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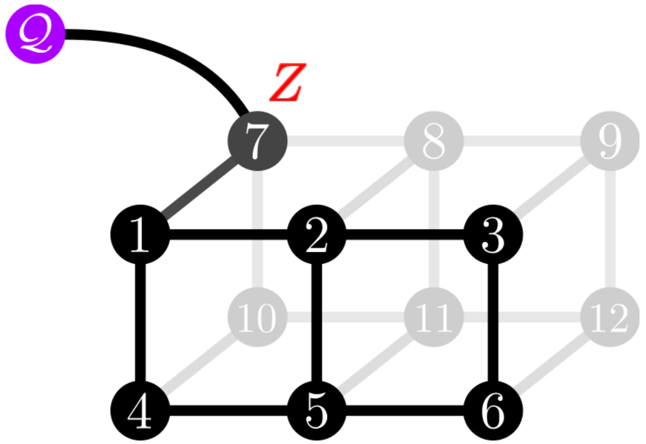

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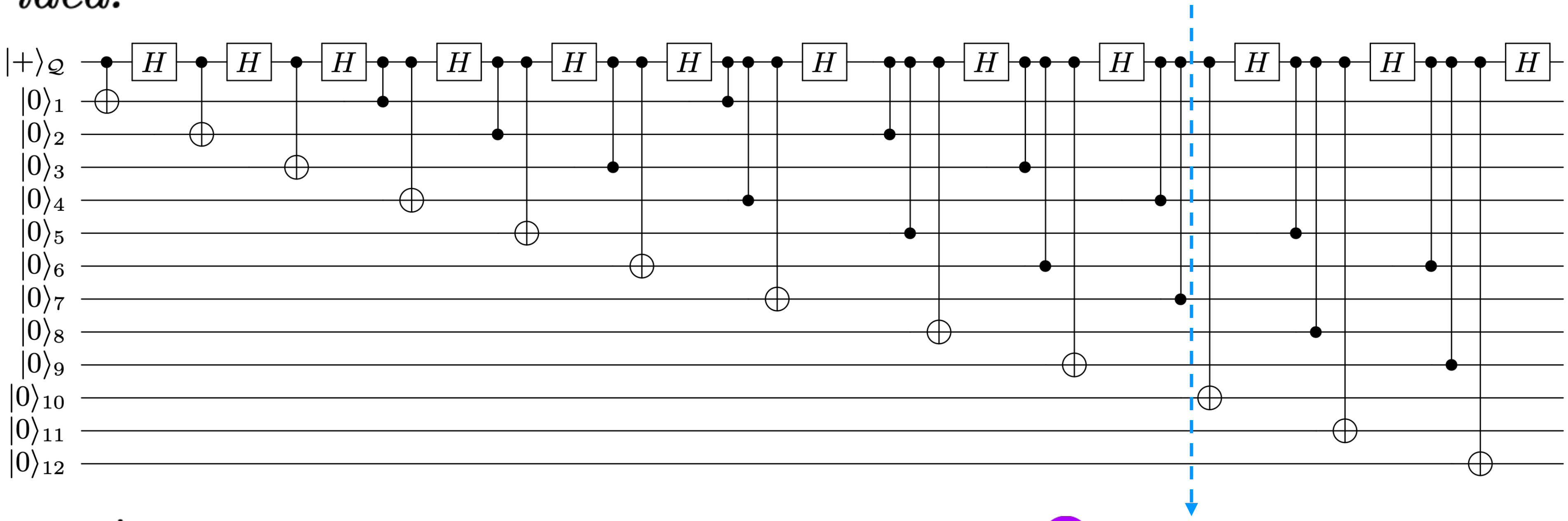

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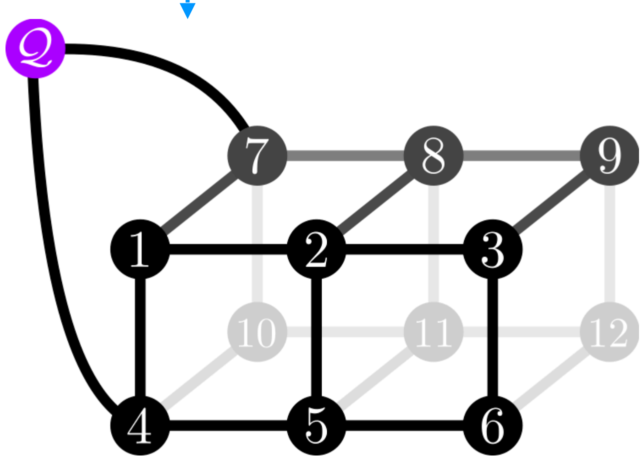

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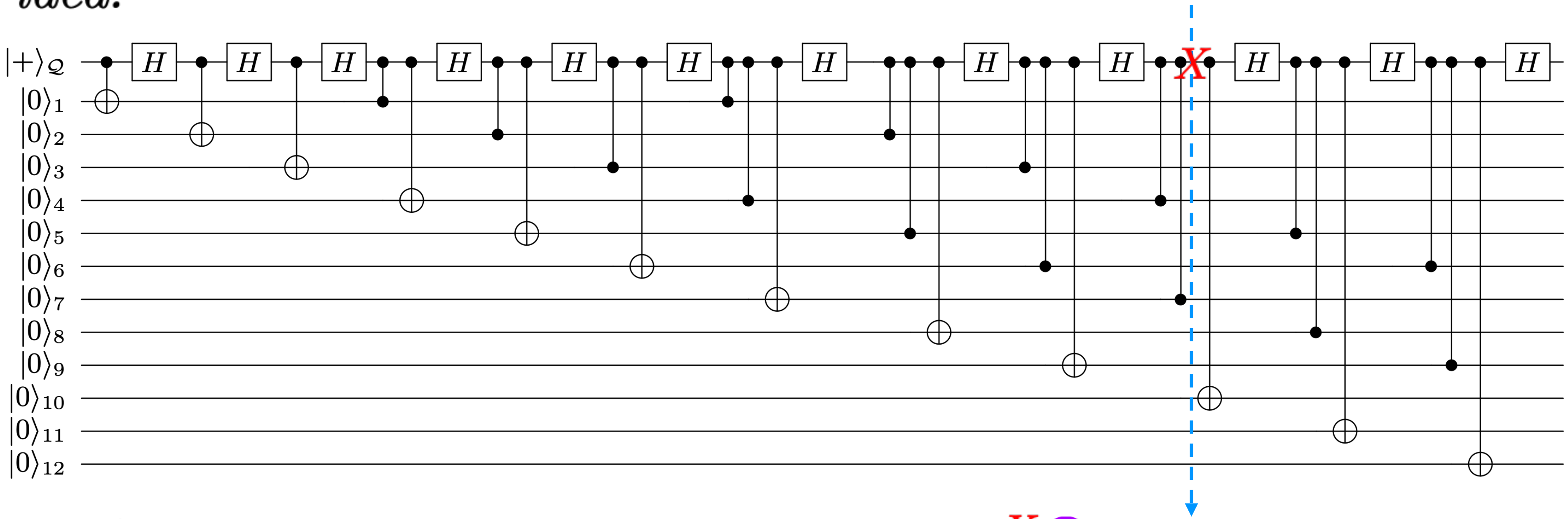

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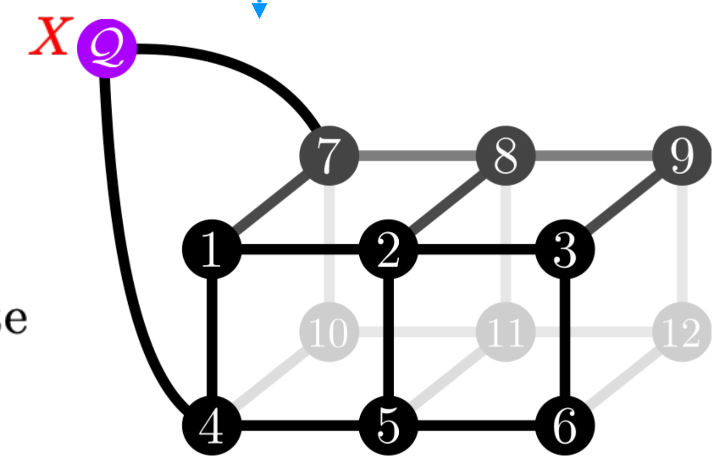

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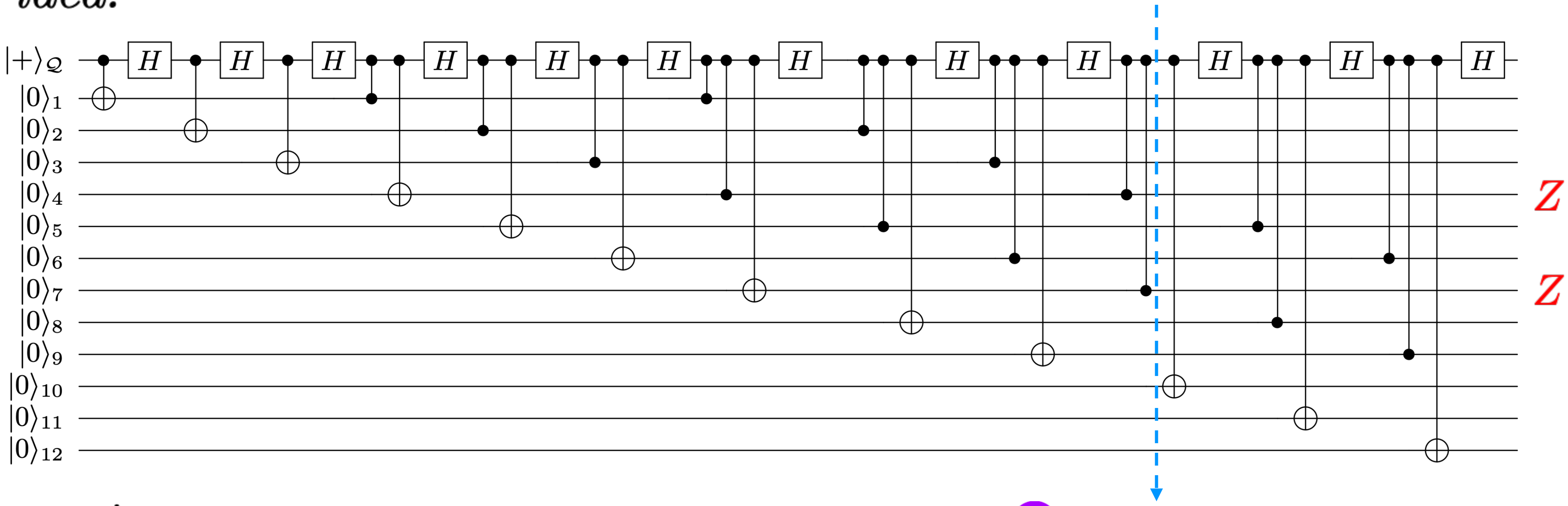

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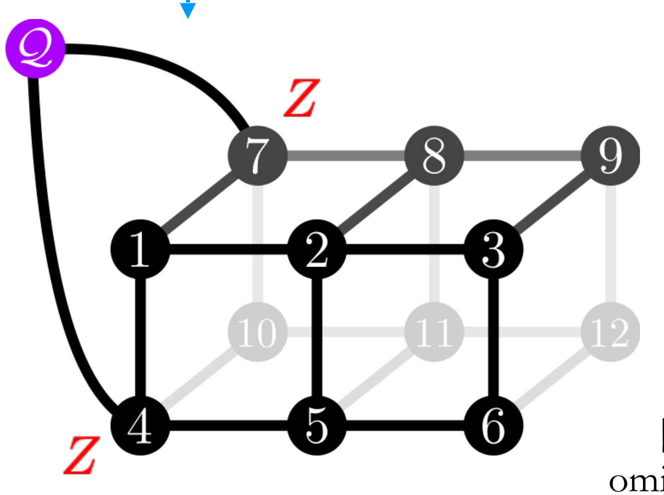

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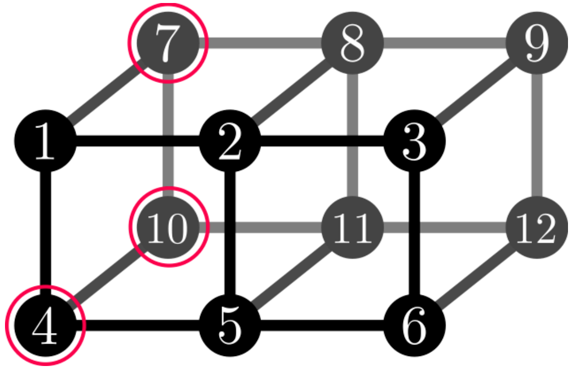


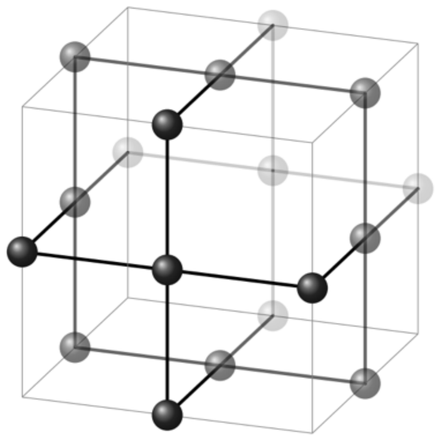
more generally, m single-qubit circuit-level errors $\Rightarrow$ m local errors on the prepared cluster state

potential concerns

1. propagation of circuit-level errors

   effective errors are actually local ($\Rightarrow$ weight $O(1)$ for bcc lattice)

potential concerns



0. cubic lattice $\to$ bcc lattice

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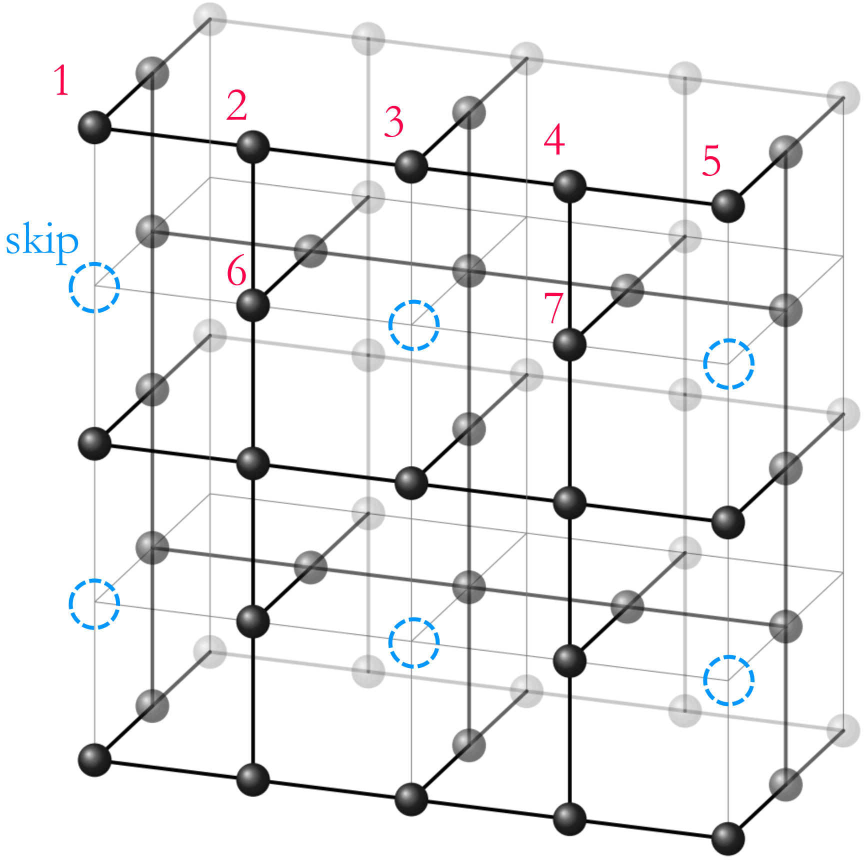
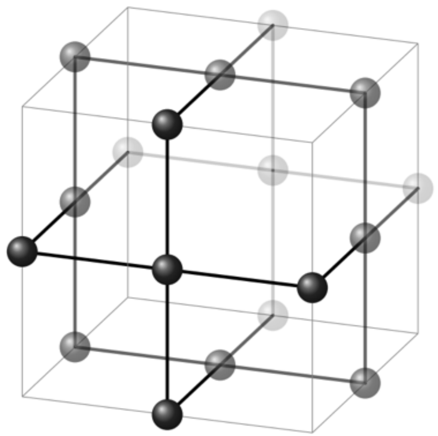
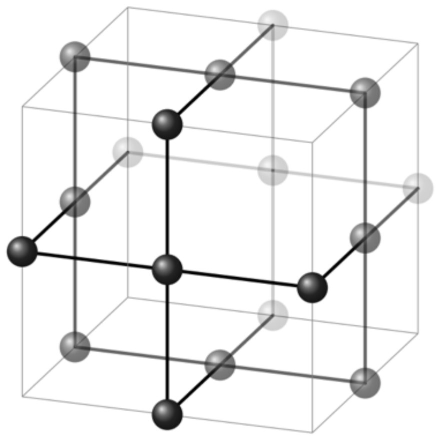
0. cubic lattice → bcc lattice





potential concerns



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   $G_{\text{bcc}} \subset G_{\text{cubic}}$, so omit qubits in $G_{\text{cubic}} \setminus G_{\text{bcc}}$

1. propagation of circuit-level errors

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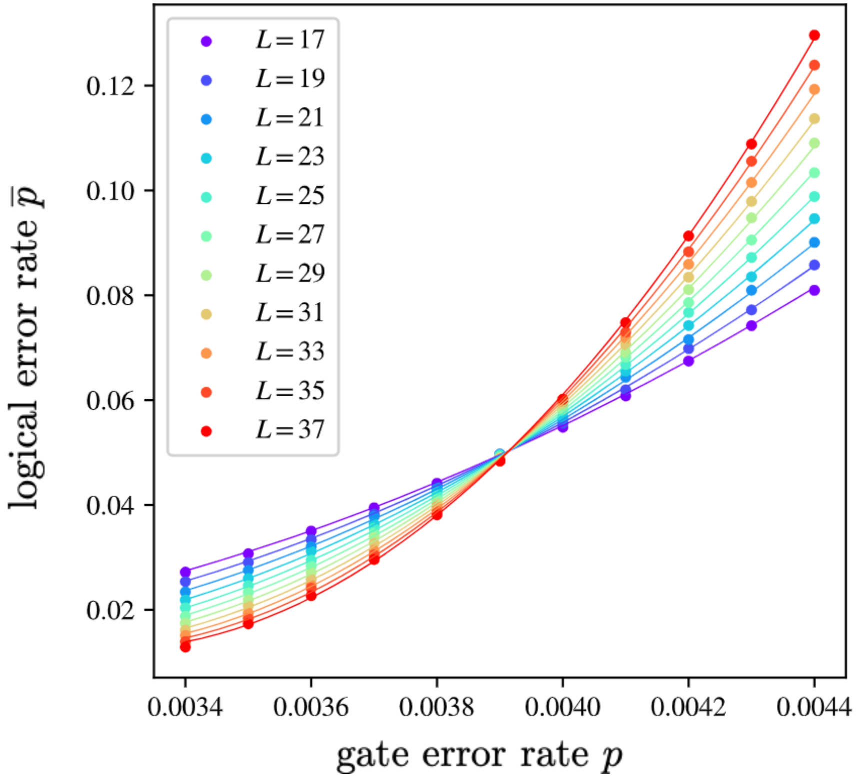
thresholds

- standard depolarising noise model for “gate errors” (error rate p):
  - single-qubit depolarising noise after each single-qubit operation (gate, measurement, state preparation)
  - two-qubit depolarising noise after each two-qubit gate

thresholds

- standard depolarising noise model for "gate errors" (error rate p)
- standard MWPM decoder

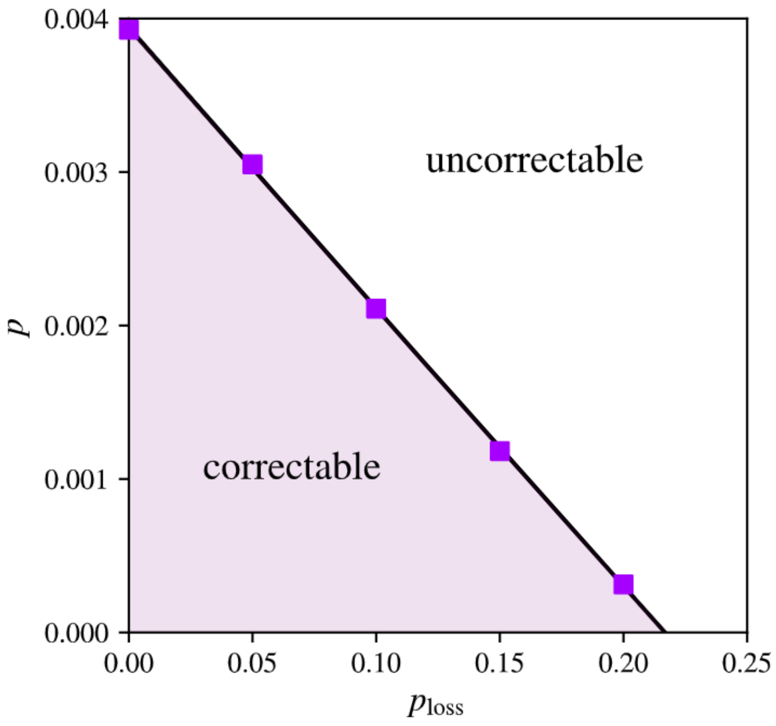


$\rightarrow \quad p_{\text{th}} \approx 0.39\%$

[Raussendorf *et al.*]: $\approx 0.58\%$
(cZ circuit)

thresholds

- standard depolarising noise model for "gate errors" (error rate p) + each qubit lost with probability p_{loss}
- generalised MWPM decoder of [Barrett & Stace '10]



potential concerns

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1. propagation of circuit-level errors

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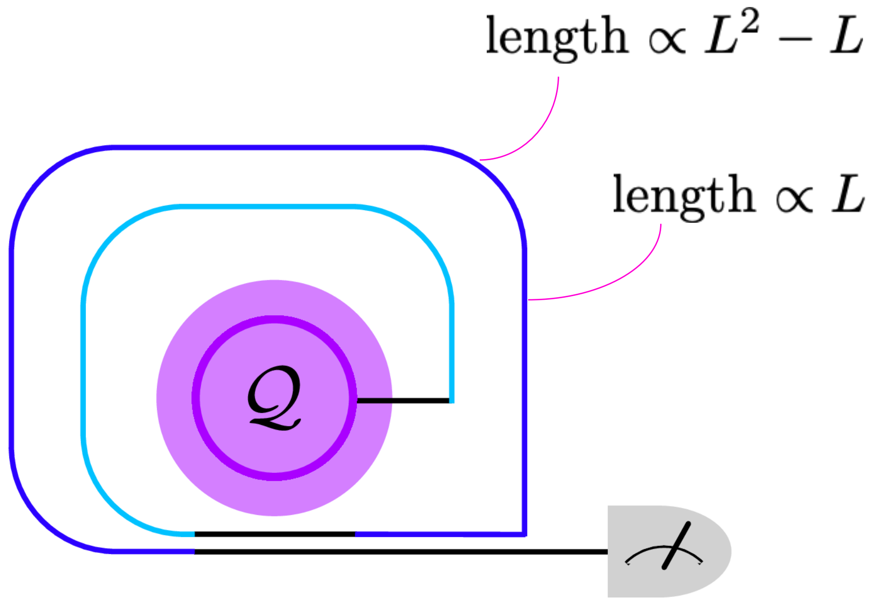
2. noisy delay lines $\rightarrow$ errors on idle qubits

   total delay line length $\propto L^2$



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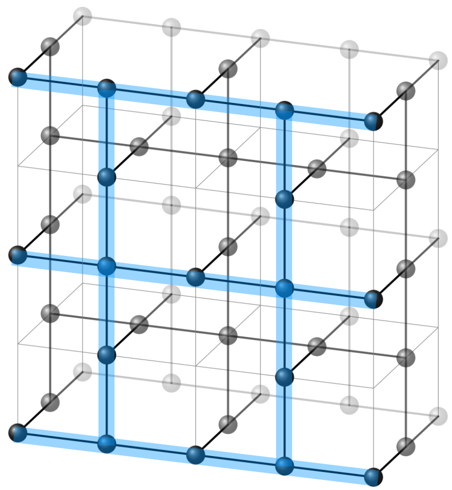
   total delay line length $\propto L^2$

   logical error probability $\sim \exp(-\sqrt{\text{delay line error rate}})$

delay line errors



$\eta =$ delay line error rate
total delay line error probability $\approx \eta L^2$

code distance $\propto L$

↳ for each η and gate error rate p, $\exists$ an optimal logical error rate $\overline{p}_*$

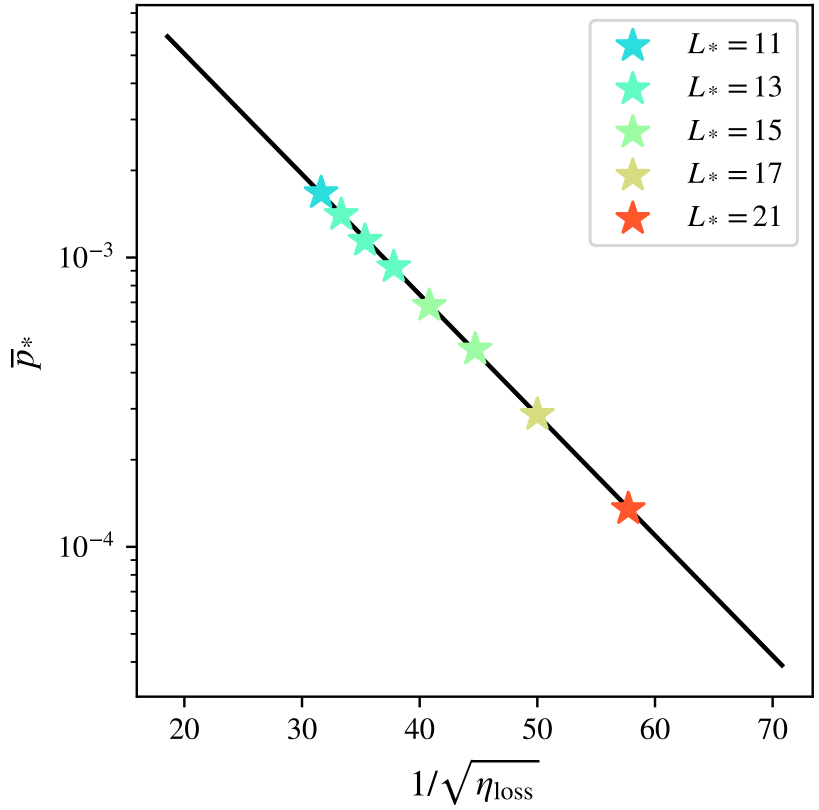
for fixed gate error rate p, expect $\overline{p}_*$ to scale with η as

$$\overline{p}_* \propto \exp(-c\sqrt{\eta})$$

delay line errors

e.g.,

- fix gate error $p = 10^{-3}$
- suppose $\eta = \eta_{\text{loss}}$



→ "break-even point" (at which $\bar{p}_* = p = 10^{-3}$) occurs at $\eta_{\text{loss}} = 7.4 \times 10^{-4}$

optical fibres: $\eta_{\text{loss}} \approx 9.6 \times 10^{-4}$
(assuming 17 ns between photons)

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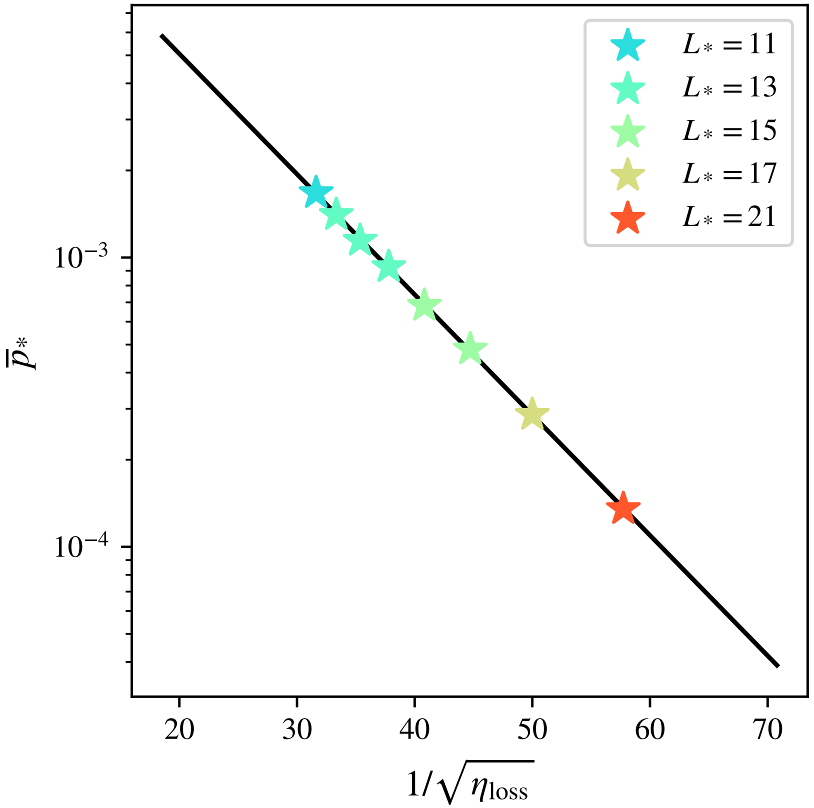

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dephasing

"break-even point" occurs at $\eta_{\text{dephasing}} = 6.5 \times 10^{-5}$

phononic waveguide: $\eta_{\text{dephasing}} \approx 4.8 \times 10^{-4}$ (160 ns between phonons)

delay line errors

e.g.,

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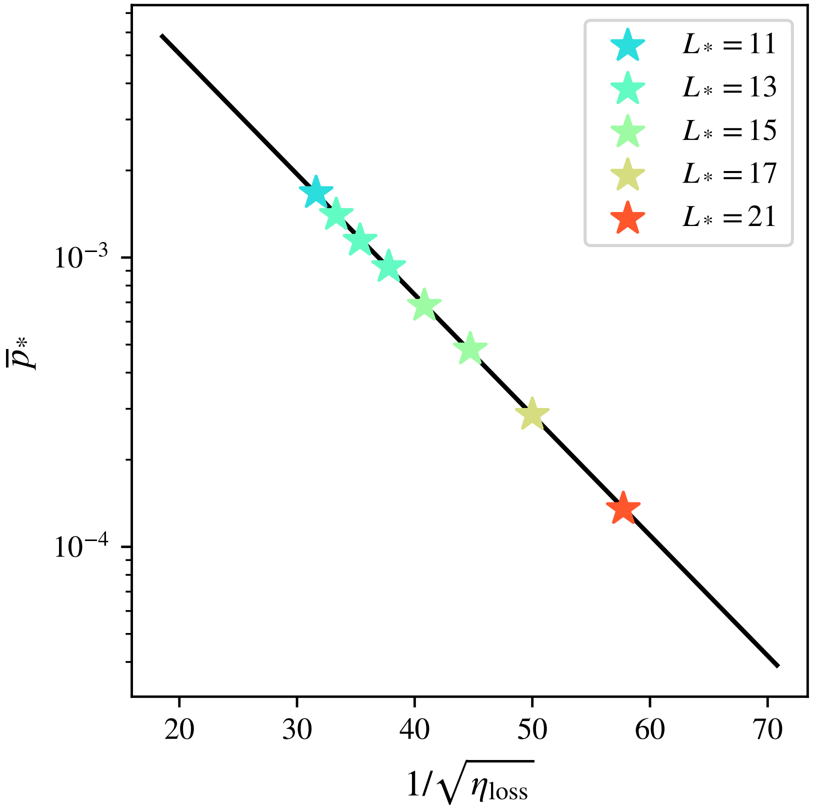

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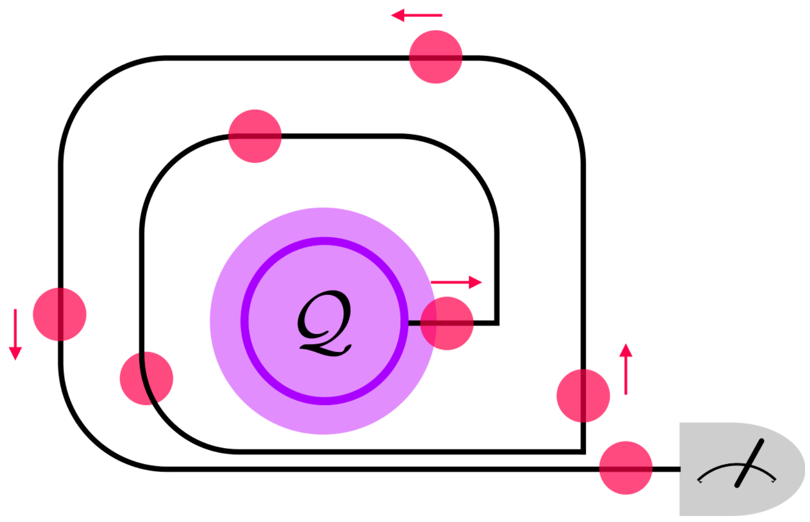
$\eta_{\text{loss}} = 1.4 \times 10^{-4} \;\rightarrow\; \bar{p}_* = 10^{-5}$ $(L_* \approx 30)$

$\eta_{\text{loss}} = 2.4 \times 10^{-5} \;\rightarrow\; \bar{p}_* = 10^{-10}$ $(L_* \approx 75)$

$\eta_{\text{loss}} = 9.5 \times 10^{-6} \;\rightarrow\; \bar{p}_* = 10^{-15}$ $(L_* \approx 115)$

summary



1. constant component overhead

2. local error propagation

    [open questions: can similar ideas be exploited in other contexts? better characterisation of these circuits?]

↳ 3. FTQC could potentially be achieved through incremental improvements to a small number of key components thanks for listening!

☺

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