The XZZX surface code

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We show that a variant of the surface code—the XZZX code—offers remarkable performance for fault-tolerant quantum computation. The error threshold of this code matches what can be achieved with random codes (hashing) for *every* single-qubit Pauli noise channel; it is the first explicit code shown to have this universal property. We present numerical evidence that the threshold even exceeds this hashing bound for an experimentally relevant range of noise parameters. Focusing on the common situation where qubit dephasing is the dominant noise, we show that this code has a practical, high-performance decoder and surpasses all previously known thresholds in the realistic setting where syndrome measurements are unreliable. We go on to demonstrate the favorable sub-threshold resource scaling that can be obtained by specializing a code to exploit structure in the noise. We show that it is possible to maintain all of these advantages when we perform fault-tolerant quantum computation. We finally suggest some small-scale experiments that could exploit noise bias to reduce qubit overhead in two-dimensional architectures. The complete version of this paper can be found at arXiv:2009.07851.

Background

A large-scale quantum computer must be able to reliably process data encoded in a nearly noiseless quantum system. To build such a quantum computer using physical qubits that experience errors from noise and faulty control, we require an architecture that operates fault-tolerantly [1, 18, 20, 29], using quantum error correction to repair errors that occur throughout the computation. We do not know the best way to realize a scalable quantum computer, nor do we have good bounds on the optimal performance that is achievable with a quantum fault-tolerant architecture.

The discovery of the surface code [5, 7, 19] heralded very high thresholds compared with other families of codes [29]. Moreover, it was shown quickly that it could be decoded efficiently using minimum-weight perfect matching [9, 21], and that the thresholds obtained with this decoder were near optimal [7] for commonly studied noise models such as independent and identically distributed bit-flip noise. Furthermore, this decoder generalized readily to the fault-tolerant case where measurements were unreliable [7, 36], with near-optimal thresholds [22, 26] also obtained with respect to a phenomenological noise model. It was shown [28] that a threshold near to 1% is obtained with respect to the gate error model using the minimum-weight perfect-matching algorithm where the circuit to measure the stabilizers is simulated, and errors are introduced with probability p by each circuit element. Beyond this original work, ongoing progress [11–13] has not demonstrated significant improvements in the threshold.

In this paper, we introduce and analyze a variant of the surface code that dramatically improves over this state of the art. Our main result is a highly efficient fault-tolerant architecture design that exploits the common structures in the noise experienced by physical qubits. Our central tool is a tailored version of the surface code [5, 7, 19] where the stabilizer checks are given by the product XZZX of Pauli operators across each face on a square lattice [37]. This seemingly innocuous local change of basis offers a number of significant advantages over its more conventional counterpart for structured noise models that deviate from depolarizing noise.

Overcoming the hashing bound

We first consider preserving a logical qubit in a quantum memory using this XZZX code. While some 2D codes have been shown to have high error thresholds for certain types of biased noise [24, 33], we find that the XZZX code gives exceptional thresholds for *all* single-qubit Pauli noise channels, matching what is known to be achievable with random coding [4], [38, Theorem 24.6.2]. It is

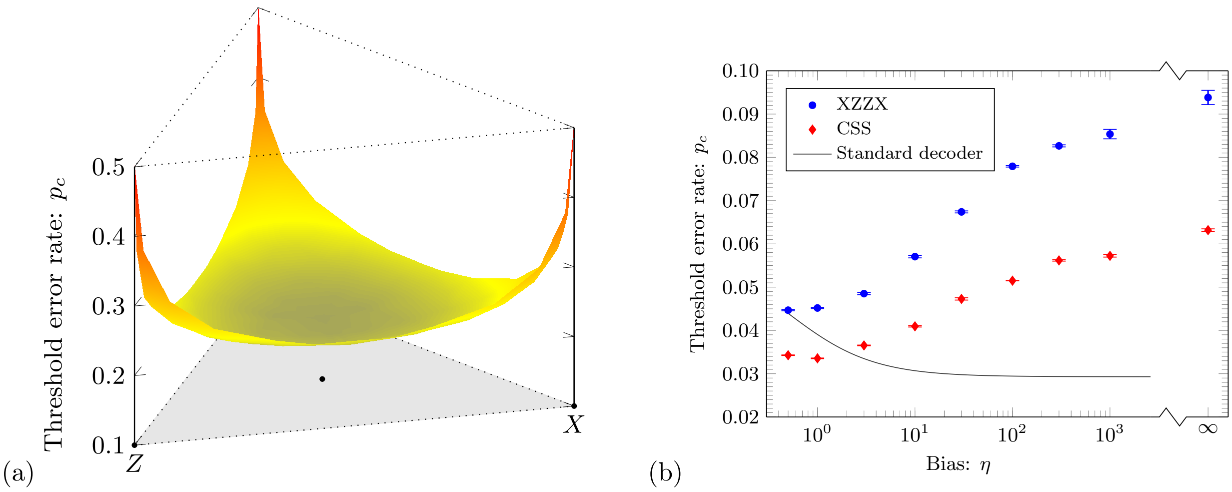

FIG. 1: (a) Optimal code-capacity threshold estimates for surface code variants over all single-qubit Pauli channels. Threshold estimates are found using approximate maximum-likelihood decoding for the XZZX code. The gray triangle represents a parametrization of all single-qubit Pauli channels, where the center corresponds to depolarizing noise, the labeled vertices correspond to pure X and Z noise, and the third vertex corresponds to pure Y noise. For the XZZX code, estimates closely match or exceed the hashing bound (not shown) for *all* single-qubit Pauli channels. (b) Phenomenological threshold error rates using our matching decoder for the XZZX code as a function of noise bias η, where the measurement error rate q is defined in the main text. The results found using our matching decoder for the XZZX code experiencing Pauli-Z biased noise (blue) are compared with the results found using the matching decoder presented in Ref. [34] experiencing Pauli-Y biased noise for the CSS surface code (red). Equivalent results to the red points are obtained with Pauli-Z biased noise using the tailored code of Ref. [33]. The XZZX code significantly outperforms the CSS code for all noise biases with both noise models. The gray line shows the threshold found using a conventional matching decoder for the CSS surface code for the phenomenological noise model where bit-flip and dephasing errors are decoded independently as in Ref. [36].

particularly striking that the XZZX code can match the threshold performance of a random code, for any single-qubit Pauli error model, while retaining the practical benefits of local stabilizers and an efficient decoder. Intriguingly, for noise that is strongly biased towards X or Z, we have numerical evidence to suggest that the XZZX threshold *exceeds* this random coding (hashing) bound, meaning it can correct errors when the noise entropy per qubit exceeds one bit. Thus, this code could potentially provide a practical demonstration of the superadditivity of coherent information [3, 8, 10, 30, 31].

An important lesson we learn from the example we present is that thresholds can be substantially improved by specializing the quantum error-correcting code to the noise the system experiences. Indeed, using the results of Ref. [27], it is reasonable to anticipate that highly biased noise will remain physically relevant even when using circuits that perform basis-changing gates such as CNOT without having to use the special methods of Ref. [2].

Low-overhead quantum computation

For a fault-tolerant architecture to be practical, it will need to correct for physically relevant errors with only a modest overhead. That is, quantum error correction can be used to create near-perfect logical qubits if the rate of relevant errors on the physical qubits is below some threshold, and a good architecture should have a sufficiently high threshold to be achievable in practice. These fault-tolerant designs should also be efficient, using a reasonable number of physical qubits to achieve the desired logical error rate. The most common architecture for fault-tolerant quantum computing is based on the surface code [12]. It offers thresholds against depolarizing noise that are already high, and encouraging recent results have shown that its performance against more structured noise can be considerably improved by tailoring the code to the noise model [32–35, 39]. While the surface code has already demonstrated promising thresholds, its overheads are daunting [12, 14]. For fault-tolerant quantum computing to become practical, we need to design architectures that provide high thresholds against relevant noise models while minimizing overheads through efficiencies in physical qubits and logic gates.

We show that the high thresholds demonstrated with the XZZX code persist with efficient, practical decoders, by using a generalization of a matching decoder in the regime where dephasing noise is dominant. In the fault-tolerant setting when stabilizer measurements are unreliable, we obtain thresholds in the biased-noise regime that surpass all previously known thresholds.

Once we have qubits and operations that perform below the threshold error rate, the practicality of scalable quantum computation is determined by the overhead, i.e., the number of physical qubits we need to obtain a target logical failure rate. Along with offering high thresholds against structured noise, we show that architectures based on the XZZX code require very low overhead to achieve a given target logical failure rate. We consider a biased noise model, where dephasing errors occur more frequently than other errors, by a factor η. In general, for large system sizes and low error rates p with noise bias η, the logical failure rate scales like $O((p/\sqrt{\eta})^{d/2})$, where d is the distance of the code. This improves the logical failure rate by a factor of $\sim \eta^{-d/4}$ meaning we can achieve a target logical failure rate using considerably fewer qubits at large bias. We also show that near-term devices, i.e., small sized systems with error rates near to threshold, the logical failure rate has a quadratically improved scaling with code distance, as $O(p^{d^2/2})$. This scaling has been demonstrated at infinite bias with a tailored surface code in prior work [35]. Thus, we should expect to achieve low logical failure rates using a modest number of physical qubits for experimentally plausible values of the noise bias where, say $10 \lesssim \eta \lesssim 1000$ [15, 23].

Finally, we consider fault-tolerant quantum computation with biased noise [2, 16, 27], and we show that the advantages of the XZZX code persist in this context. We show how to implement low-overhead fault-tolerant Clifford gates by taking advantage of the noise structure as the XZZX code undergoes measurement-based deformations [6, 17, 25]. With an appropriate lattice orientation, noise with bias η is shown to yield a reduction in the required number of physical qubits by a factor of $\sim \log \eta$ in a large-scale quantum computation. These advantages already manifest at code sizes attainable using present-day quantum devices.

Why QIP?

This paper will appeal to the broad QIP audience for several reasons. First, it offers a surprising connection to quantum Shannon theory by providing an example of a practical code family that seems to exceed the hashing bound for a significant regime of the parameter space we examine. Moreover, its thresholds approximately meet the hashing bound "universally", for every single-qubit Pauli channel. Second, it gives a huge "best yet" parameter improvement to the phenomenological noise threshold for a quantum memory. In the practical range of noise biases (around $10 \lesssim \eta \lesssim 1000$ [15, 23]), the threshold is around 6%–8%, a remarkable improvement over numbers that have been essentially stagnant for a decade. Third, the code itself is an almost shockingly trivial modification of existing mainstream code ideas. This means that the ideas are easily accessible to the broad audience, and it offers the tantalizing promise that inspired audience members can find similarly low-hanging fruit by using the simple design principles. Lastly, by showing that the XZZX code reduces the overhead for fault-tolerant quantum computation by a factor of $\sim \log \eta$ and the logical failure rate by a factor of $\sim \eta^{d/4}$, these results are likely to be of practical relevance in the near future.

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