Secure Software Leasing and Implications for Quantum Copy-Protection and Obfuscation

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Joint with
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Merge of

"Impossibility of Quantum Virtual Black-Box Obfuscation of Classical Circuits"
Alagic, Brakerski, Dulek, Schaffner
Link: https://arxiv.org/pdf/2005.06432

and

"Secure Software Leasing"
Ananth, La Placa
Link: https://arxiv.org/abs/2005.05289

Unclonable Cryptographic Primitives

Use no-cloning property of quantum computing
to obtain exciting cryptographic primitives!

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- Quantum Money [Weisner'83,...]
- Quantum Copy-Protection [Aar'09]
- Certifiable Deletion Encryption [BI'19]
- Revocable Timed-Release Encryption [Unr'13]
- Unclonable Encryption [Got'02,BL'19]
- One-Shot Signatures and Signature Tokens [BS'16,AGKZ'20]
- Quantum Lightning [Zha'17]

Problem: Software Piracy

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Can we use quantum no-cloning theorem to solve this?

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- **Security**: The user shouldn't be able to re-distribute the software to other people.

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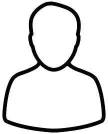
Quantum copy-protection: A solution for preventing software piracy.

- **Functionality**: The vendor can copy-protect the software and send the copy-protected version to the user.
- **Security**: The user shouldn't be able to re-distribute the software to other people.
  (uncloneability!)

Quantum Copy-Protection [Aaronson CCC'09]

Microsoft

(Eval, ρ_C) →



Goal: Copy protect circuit C

User

Functionality: $\forall x, \text{Eval}(\rho_C, x) = C(x)$

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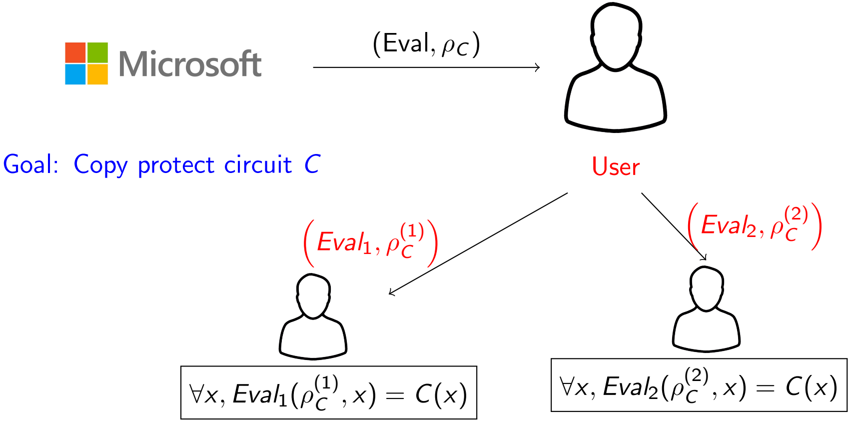

User can succeed only with very small (negligible) probability.

(For simplicity: consider product states and only two copies.
Adversary can output entangled states and multiple copies.)

Implications of Quantum Copy-Protection

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- Public-Key Quantum money [ALLZZ'20]
- Unclonable encryption [BL'19]
- Unclonable Decryption Keys [GZ'20]

Quantum Copy-Protection

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- Unlearnable functions in classical oracle model
  [Aaronson-Liu-Liu-Zhang-Zhandry arXiv'20]

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- Heuristic construction for point functions in the plain model
[Aaronson CCC'09]

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(open since [Aaronson CCC'09])

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Our work: NO (conditionally)

Our Result

Theorem.

Based on quantum hardness of learning with errors (QLWE), there does not exist quantum copy-protection for all unlearnable functions.

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Learning with errors is heavily used in cryptography.

It is conjectured to be secure against QPT algorithms

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- *Given any copy-protected state that computes this function, we can create new copies of this function.*

Implications to Program Obfuscation

Program Obfuscation

A compiler: $C \to \widehat{C}$

- $\widehat{C} \equiv C$ and,
- $\widehat{C}$ hides the implementation details of C.

Obfuscation has powerful implications in crypto and beyond.

- Secure multiparty computation
- Functional encryption
- Delegation
- Instantiating oracles.
- Differential privacy lower bounds
- Hardness of finding Nash
- $\cdots$

Virtual Black-box Property

For any QPT adversary $\mathcal{A}$, there exists a QPT simulator Sim, with oracle access to C, such that:

$$\{\mathcal{A}(\widehat{C})\} \approx \{\mathsf{Sim}^{\mathcal{O}(C)}(1^{|C|})\}$$

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Prior work: ruled out qVBB only for quantum circuits into reusable obfuscated states [AF'16]

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However, in some settings, weaker notions suffice.

Copy-protection: given (Eval, ρ_C), adversary cannot produce $(Eval_1, \rho_C^{(1)})$ and $(Eval_2, \rho_C^{(2)})$ such that:

$$Eval_1(\rho_C^{(1)}, \cdot) \text{ computes } C \text{ and,}$$
$$Eval_2(\rho_C^{(2)}, \cdot) \text{ computes } C.$$

In some scenarios, adversary does not get to choose its own evaluation algorithms.

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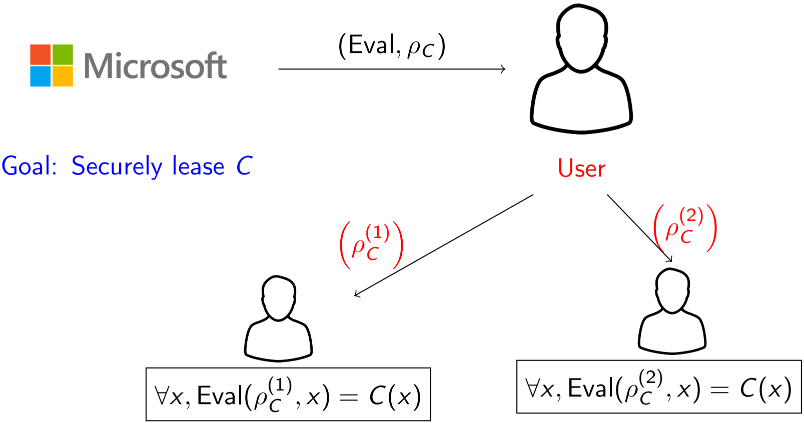
Example: Software X runs only on a specific OS.

Adversary can create "open source" version of X.

But we want to prevent them from creating new copies of X that run on the same OS.

Our Notion: Secure Software Leasing

Secure Software Leasing



User can succeed only with very small (negligible) probability.

(For simplicity: product states. Adversary can output entangled states.)

Adversary receives (ρ_C, Eval).

Infinite-term Lessor Security: only with negl. probability, it can produce $(\rho_C^{(1)})$ and $(\rho_C^{(2)})$ such that,

$$\text{Eval}(\rho_C^{(1)}, \cdot) \equiv C \text{ and,}$$
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Finite-term Lessor Security: adversary has to return back ρ_C. After returning back the state, only with negligible probability, it can produce ρ'_C such that,

$$\mathsf{Eval}(\rho'_C, \cdot) \equiv C$$

Result

Theorem.

*Based on cryptographic assumptions,
there exists infinite-term SSL for a subclass of unlearnable functions.*

Class of functions: compute-and-compare.
($f_a(x)$: take as input x, computes on x to obtain a' and outputs 1 if and only if $a' = a$.)

Subsequent Work

- Finite-term SSL for a subclass of unlearnable functions from QLWE [KNY'20]
- Infinite-term SSL for a different class of unlearnable functions [ALLZZ'20]
- Information-theoeretic SSL in the random oracle model [CMP'20]

Summary of Results

1. Conditional impossibility result of quantum copy-protection
2. Conditional impossibility result of quantum VBB obfuscation.
3. Construction of a weaker notion of copy-protection, called SSL.

Impossibility of Quantum Copy-Protection

Tool: Quantum Fully Homomorphic Encryption (QFHE)

Given encryption of ρ, quantum circuit C,

can efficiently recover encryption of $C(\rho)$.

First Attempt:
Barak et al.'s technique [BGIRSVY CRYPTO'01]

Using QFHE, we construct a class of unlearnable circuits that cannot be copy-protected

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Proof of quantum unlearnability:
Adversary method [Ambainis STOC'00]

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To copy $C_{a,b}$, we need to recover b...
.. but b is encrypted.

Idea: use special-purpose program obfuscation to recover b.

This notion can be based on QLWE.

Class of Circuits

$C_{a,b}$: on input x,

- If $x = 0$, output $(\mathrm{Enc}(a),\ \mathcal{O}(G))$
- If $x = a$, output b.
- On all other inputs, output 0.

Description of G:

On input $\mathrm{Enc}(b)$, output a, b
(implicity the function G has the decryption key hardwired inside it.)

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Using $\mathrm{Enc}(a), a, b$, can create as many copies of $C_{a,b}$ as we want!

Requirement of QFHE: evaluation of arbitrary depth quantum circuits

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Current constructions of QFHE based on circular-secure QLWE [Mahadev FOCS'18, Brakerski CRYPTO'18]

Removing Circular Security

Replace QFHE with leveled QFHE.
(Leveled QFHE: evaluation of fixed-depth quantum circuits.)

Idea

Instead of producing the public-key and the ciphertext in one shot,

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Refer to [Alagic-Brakerski-Dulek-Schaffner'20] for more details.

Summary and Future Questions

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- For contrived class of unlearnable functions, copy-protection is impossible.
- Weaker notions of copy-protection for a non-trivial class of unlearnable functions can be constructed.
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Future Questions:

- Other variants of copy-protection.
- Provably secure constructions copy-protection for any non-trivial class of circuits (such as point functions).
- Constructing weaker variants of quantum obfuscation.
  For example: quantum indistinguishability obfuscation.

Thanks! [1]

[1] Some of the slides were prepared by Rolando L. La Placa