Optimal State Transfer and Entanglement Generation in Power-law Interacting Systems

(arXiv:2010.02930)

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Joint work with Abhinav Deshpande, Andrew Guo, Andrew Lucas, and Alexey Gorshkov

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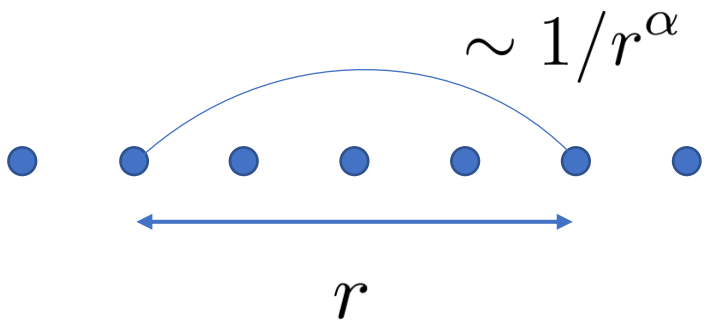
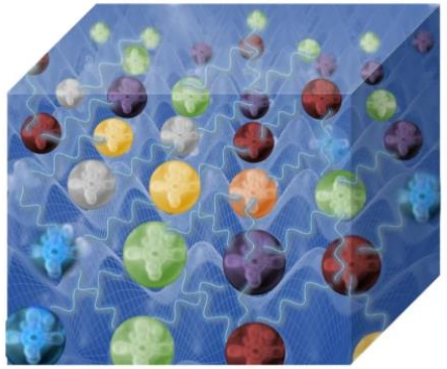
Most physical interactions decay with distance, e.g. as a power law



Coulomb

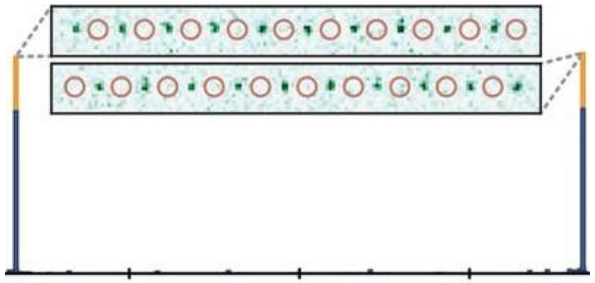
e.g. Magnetic atoms in optical lattice



Lepoutre et al, Nat Commun 10, 1714 (2019)

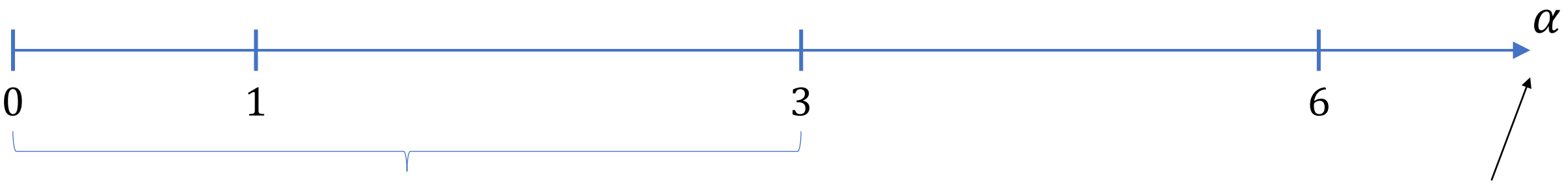
dipole-dipole

e.g. Rydberg-Rydberg interaction



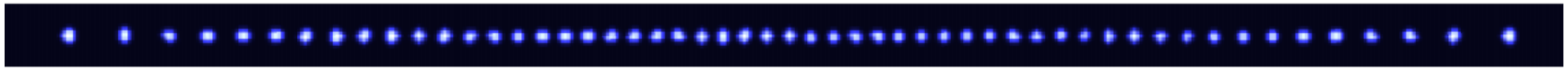
Omran et al, Science 365, 570 (2019)

van der Waals



Trapped ions

Zhang et al, Nature 551, 601–604 (2017)



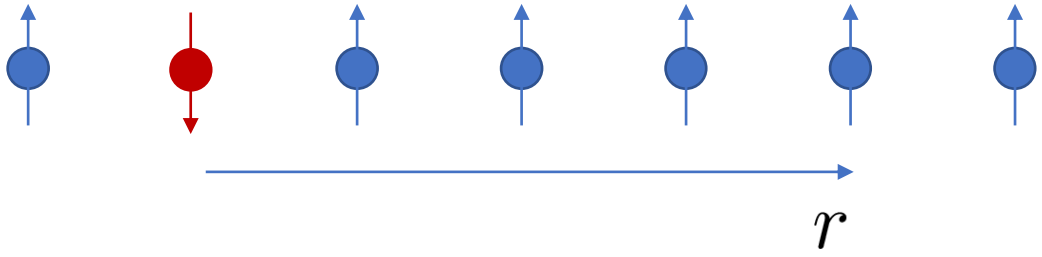
Nearest-neighbor

$(\alpha \to \infty)$

Decaying interactions

Locality:

It takes time for information to propagate in the system

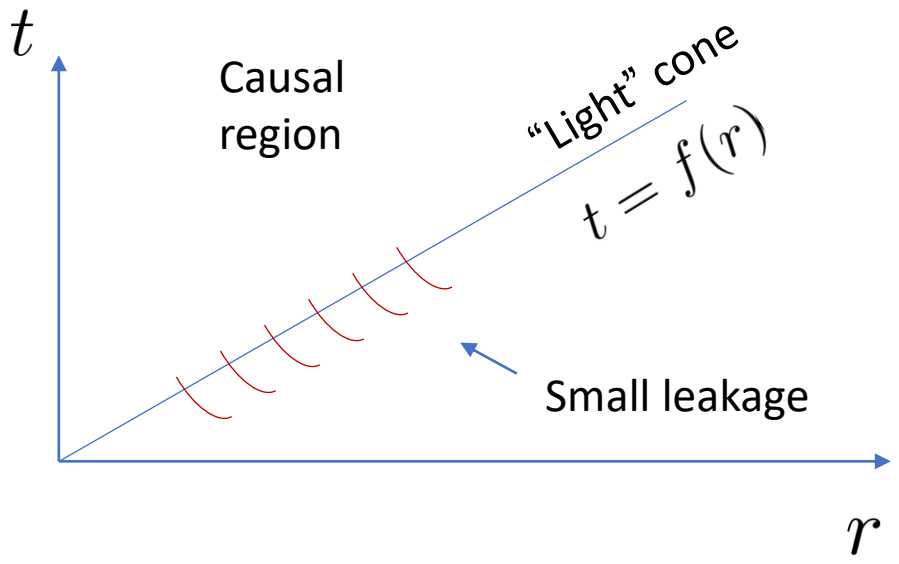


Larger distance r → Takes longer time to propagate

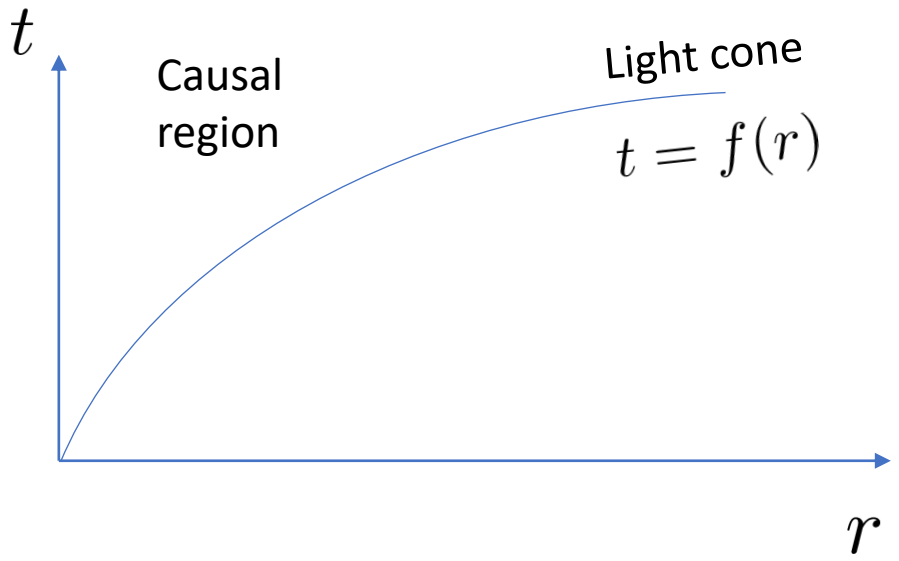
In general:

$$t \geq f(r)$$

Some function

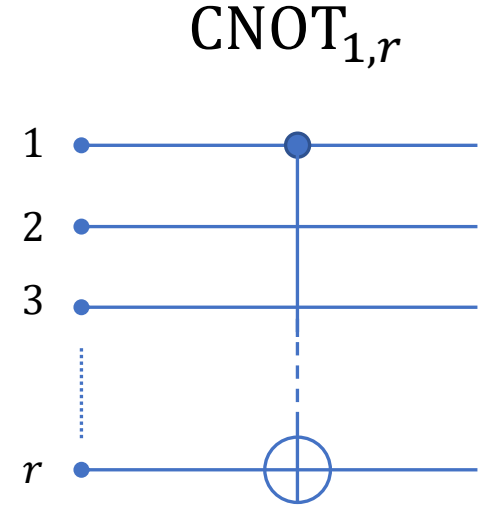


Longer-range interactions



Implications of locality?

Suppose we want to apply quantum gates between distant particles



- With nearest-neighbor interactions:

⇒ Take time $t \sim r$ (time unit)

- With power-law ($1/r^{\alpha}$) interactions:

Can we do faster than $t \sim r$?

Transfer quantum states

Generate entanglement

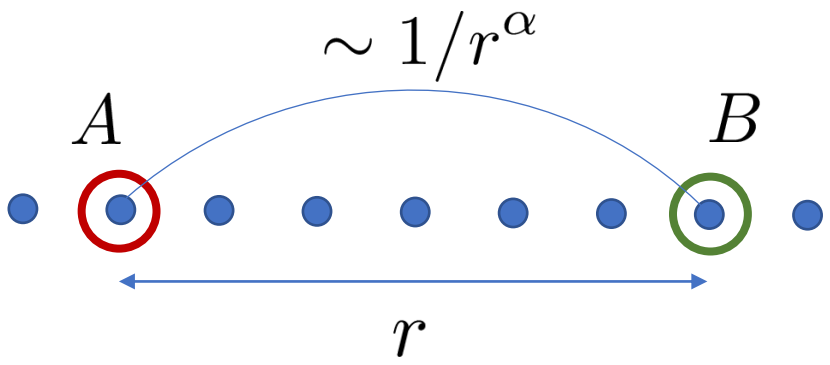
Scramble information

…

Outline

- Theoretical limits: Lieb-Robinson bounds
- Optimal state transfer and entanglement generation
  - Key implications of the protocol

Setup:



- Power-law interacting Hamiltonian $H(t) = \sum_{i,j} h_{ij}(t)$ s.t. $\|h_{ij}(t)\| \leq 1/r_{ij}^{\alpha} \quad \forall t$
- Two operators A, B initially supported on sites separated by distance r

Lieb-Robinson bounds:

$$\| [A(t), B(0)] \| \leq ?$$

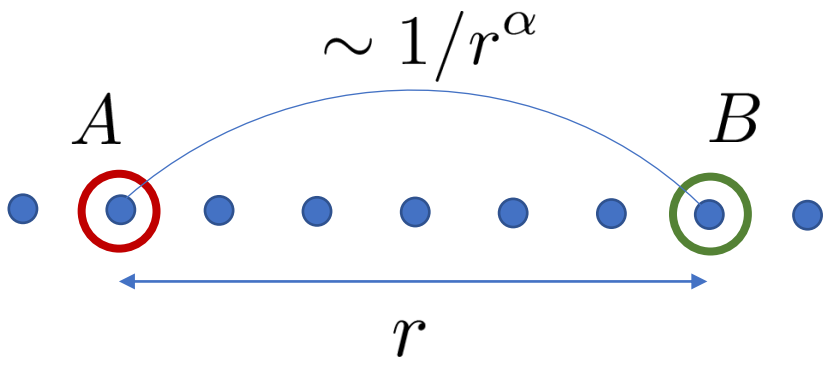
$e^{iHt} A e^{-iHt}$

$$t = 0 \Rightarrow [A, B] = 0$$

$$t > 0 \Rightarrow [A(t), B] \neq 0$$

Because $A(t)$ spreads to B

Setup:



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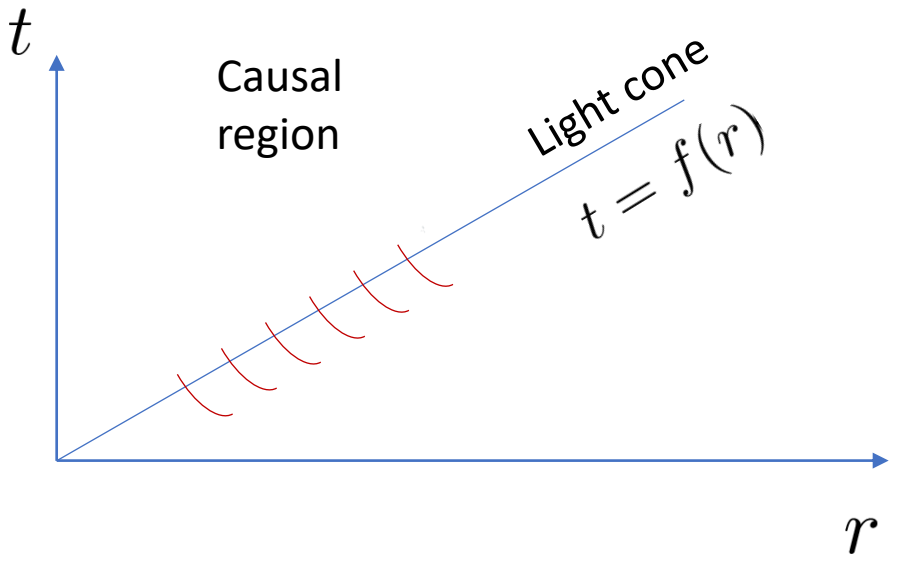
- Two operators A, B initially supported on sites separated by distance r

E.g.: Nearest-neighbor $(\alpha \to \infty)$

Lieb & Robinson, CMP '72

$$\| [A(t), B(0)] \| \lesssim e^{vt-r}$$

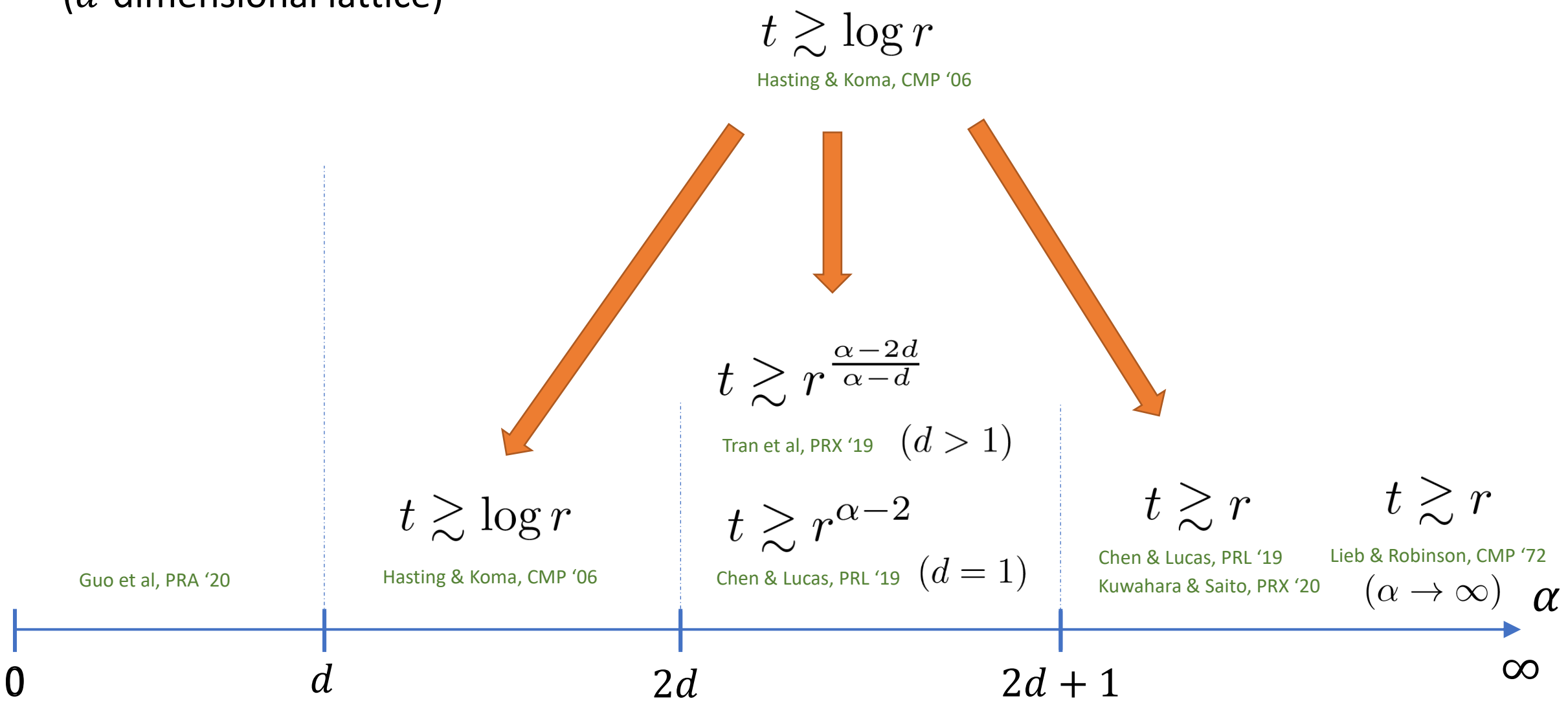
⇒ Linear light cone: $t \gtrsim r$



Theoretical limits: Lieb-Robinson bounds

Related works: Hazzard et al, 2013; Hauke & Tagliacozzo, 2013; Schachenmayer et al, 2013; Knap et al, 2013; Juenemann et al, 2013; Eisert et al, 2013; Hazzard et al, 2014; Gong et al, 2014; Foss-Feig et al, 2015; Storch et al, 2015; Rajabpour et al, 2014, 2015, ...

(d-dimensional lattice)



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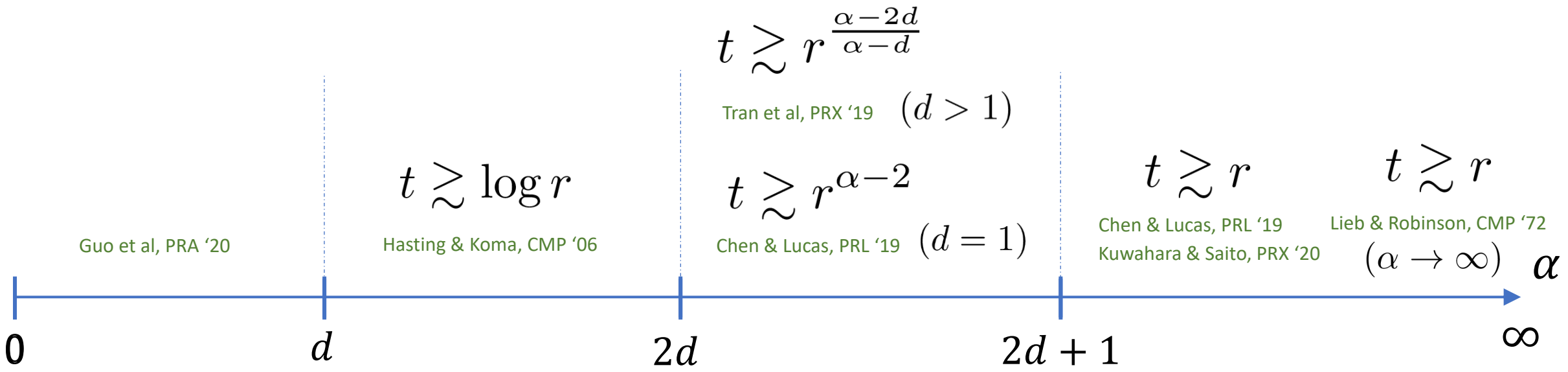
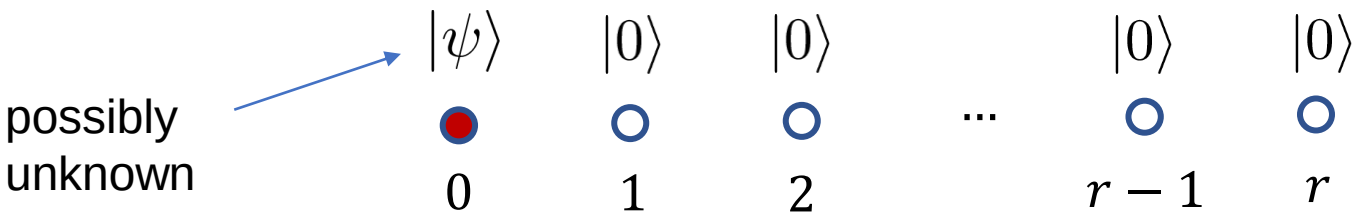
Are the bounds tight?

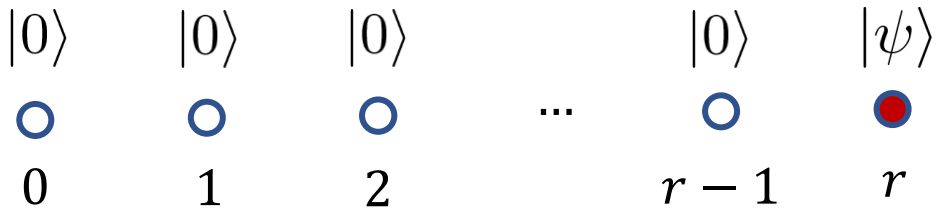


Quantum state transfer

Initial state:

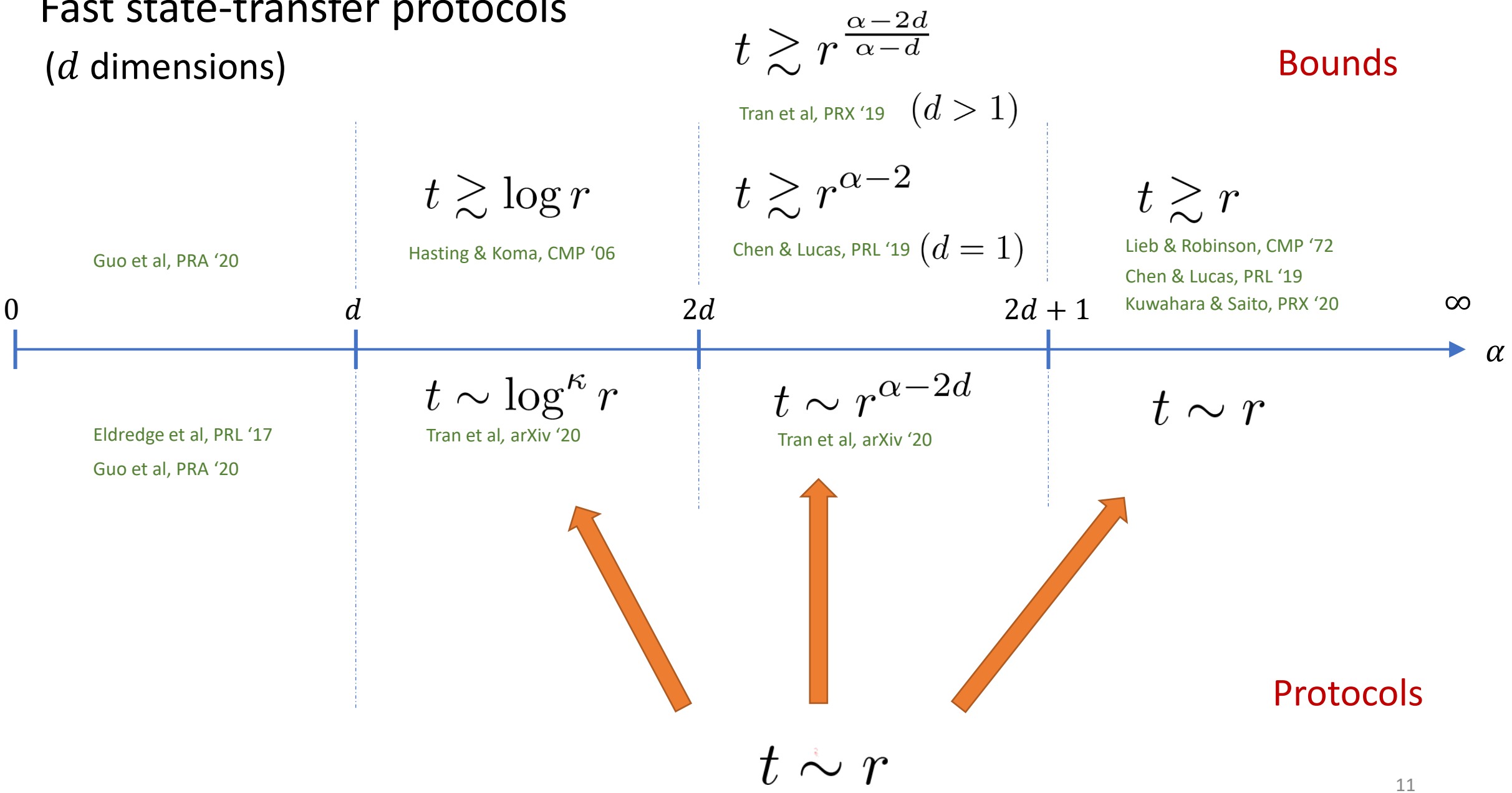


Target state:



Fast state-transfer protocols

(d dimensions)



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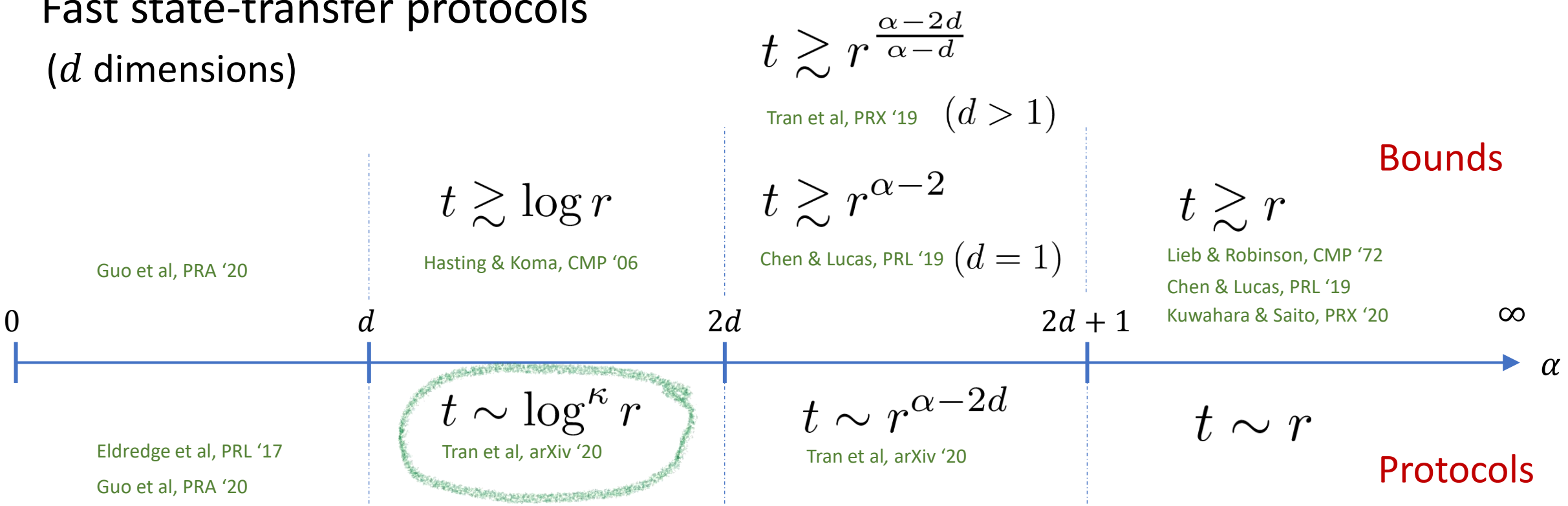
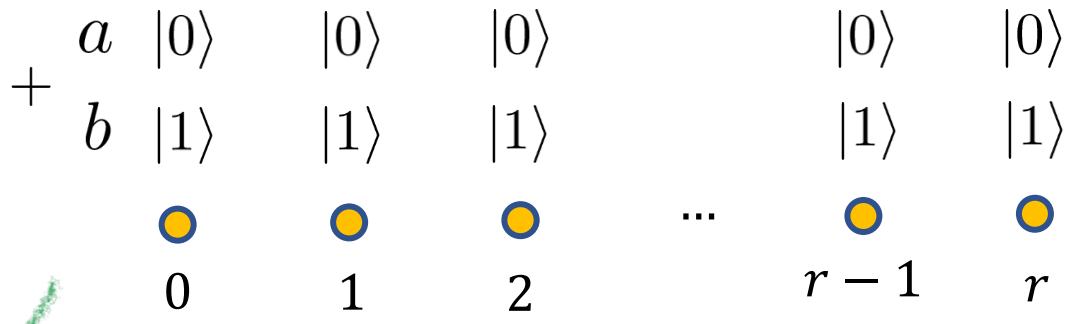


Optimal quantum state transfer

Initial state:

$a|0\rangle + b|1\rangle$

Strategy:
Encode information into r-qubit Greenberger–Horne–Zeilinger (GHZ) state



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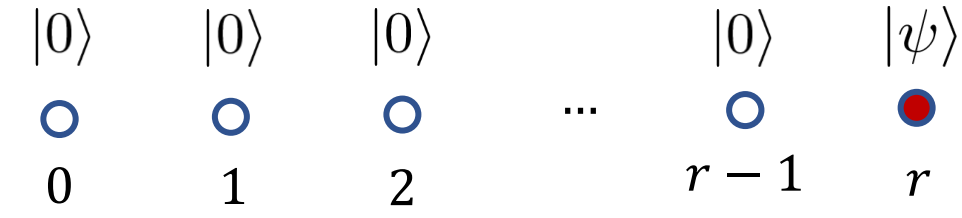
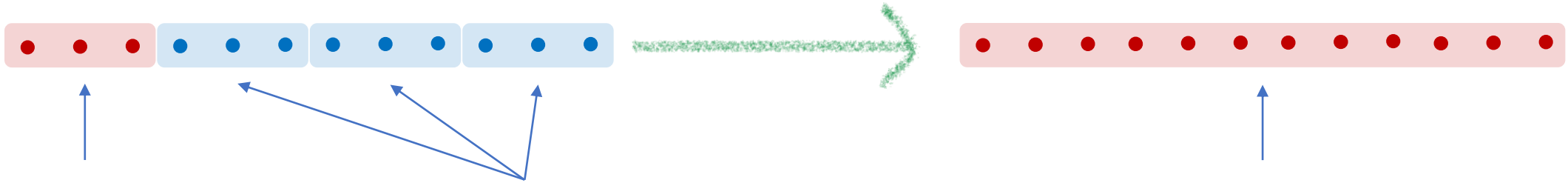


Decode the information into the target qubit

Encoding information into a GHZ state

Suppose we can “merge” several GHZ states into one big GHZ state



Generalized GHZ state with information a, b

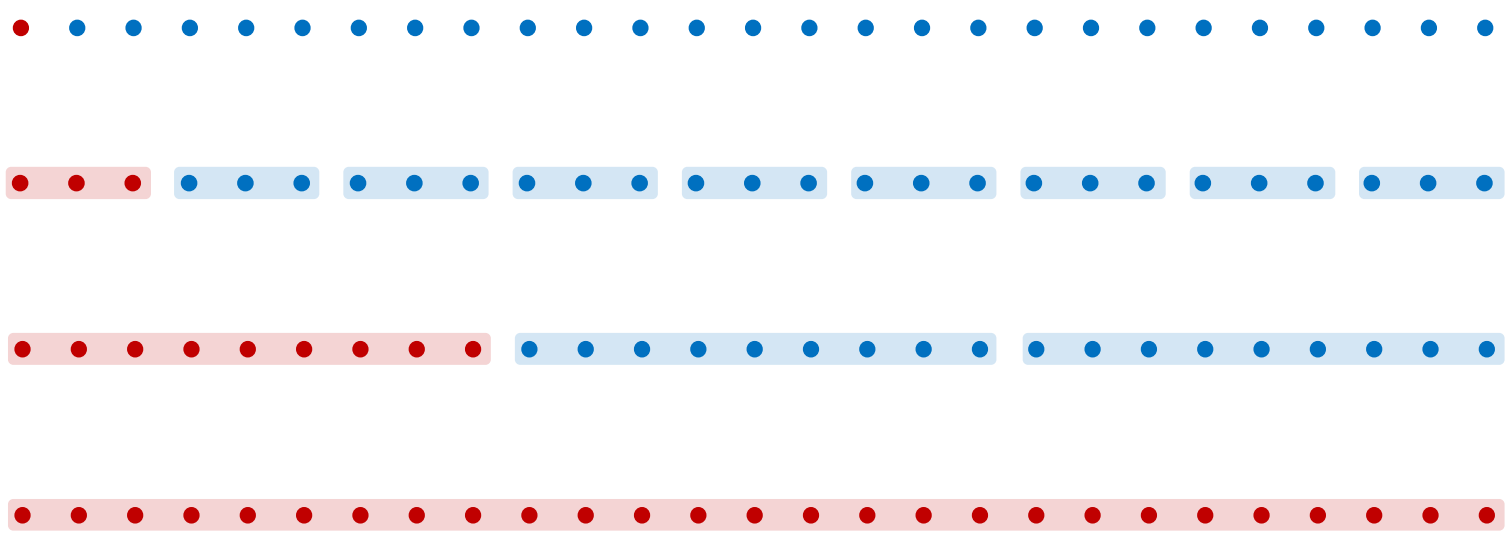
Symmetric GHZ states

A larger generalized GHZ state with information a, b

Prepare “one-qubit GHZ states”:

- $\bullet \longrightarrow |\psi\rangle = a|0\rangle + b|1\rangle$
- $\bullet \longrightarrow |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$

Our strategy: Repeatedly merge nearby GHZ states into larger and larger GHZ states with the encoded information



Optimal quantum state transfer

Initial state:

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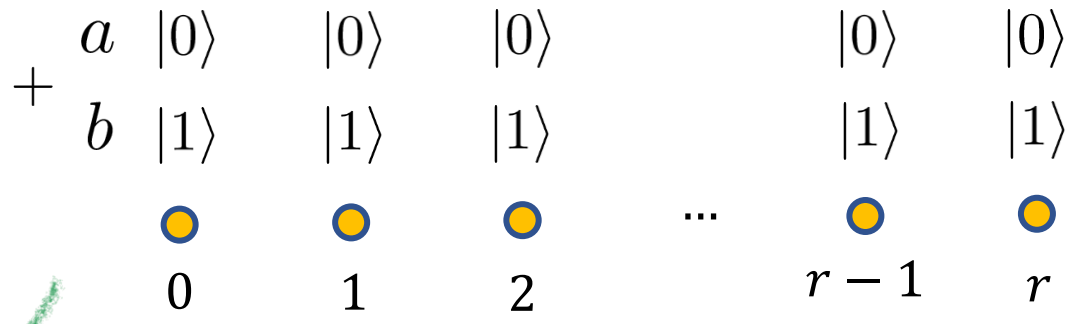
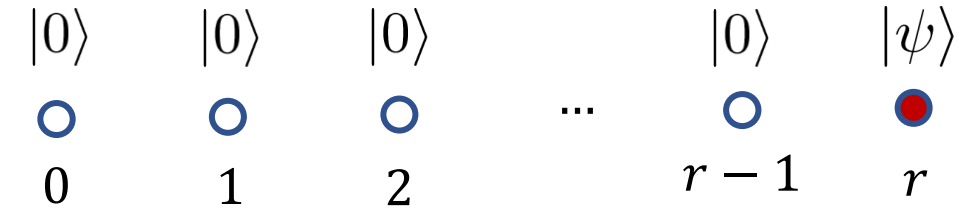
Strategy:
Encode information into r-qubit Greenberger–Horne–Zeilinger (GHZ) state

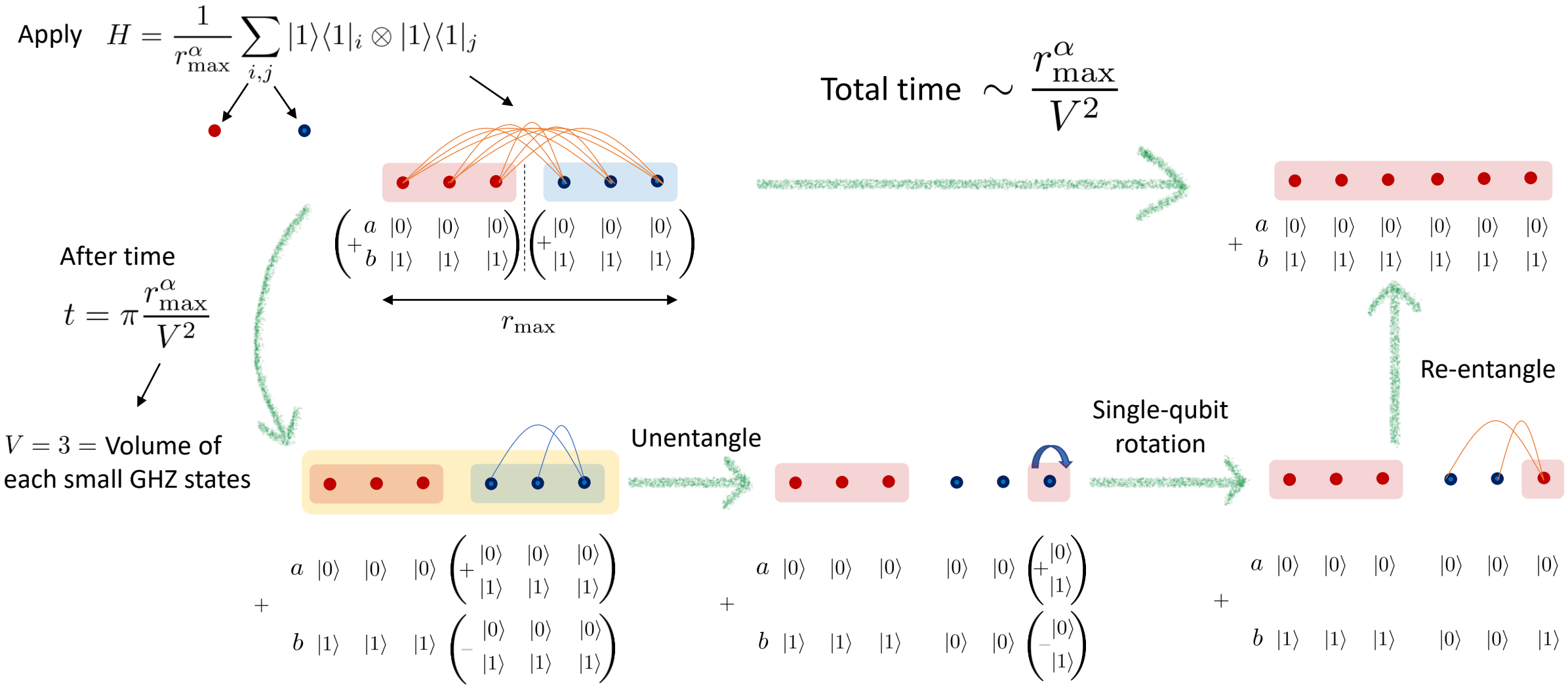


Target state:



Decode the information into the target qubit

Subroutine: merging GHZ states

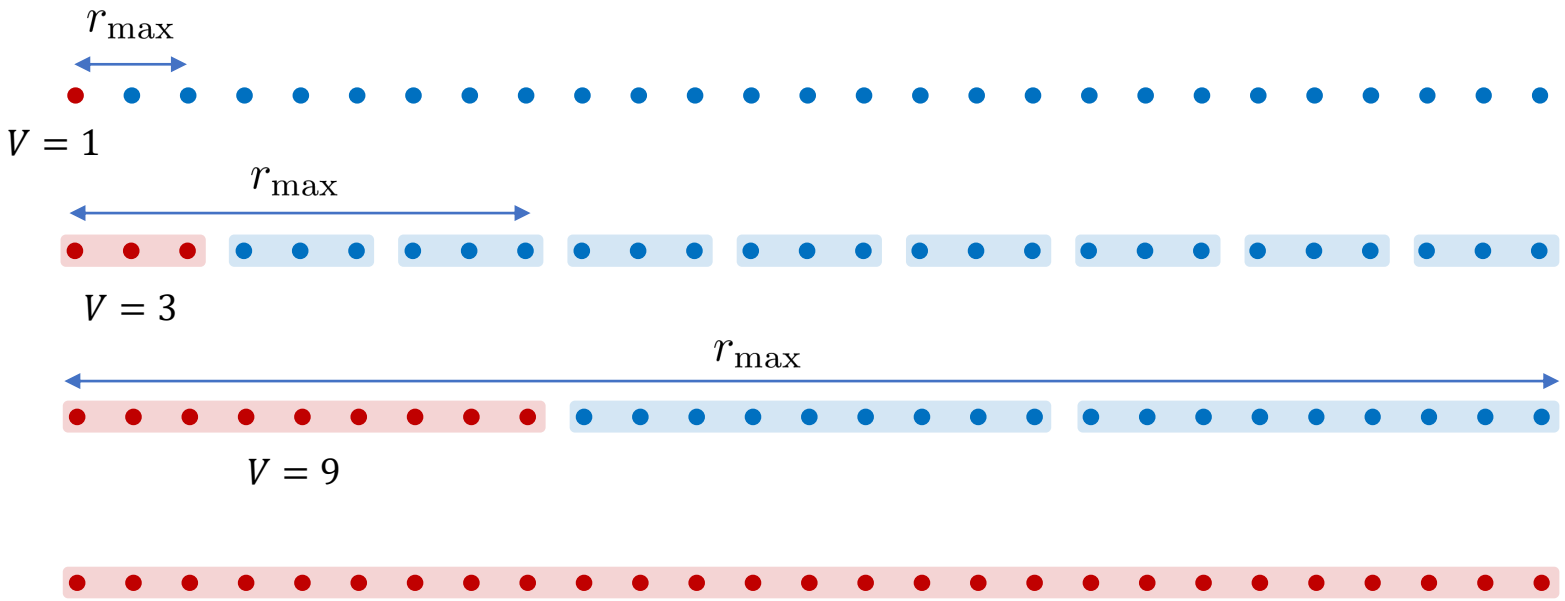


Encoding information into a GHZ state

Prepare “one-qubit GHZ states”:

- $\longrightarrow$ $|\psi\rangle = a|0\rangle + b|1\rangle$
- $\longrightarrow$ $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$

Time for implementing each iteration $\sim \dfrac{r_{\max}^{\alpha}}{V^2}$



Larger V $\Rightarrow$ Allow larger $r_{\max}$

$\Rightarrow$ Bigger and bigger GHZ states

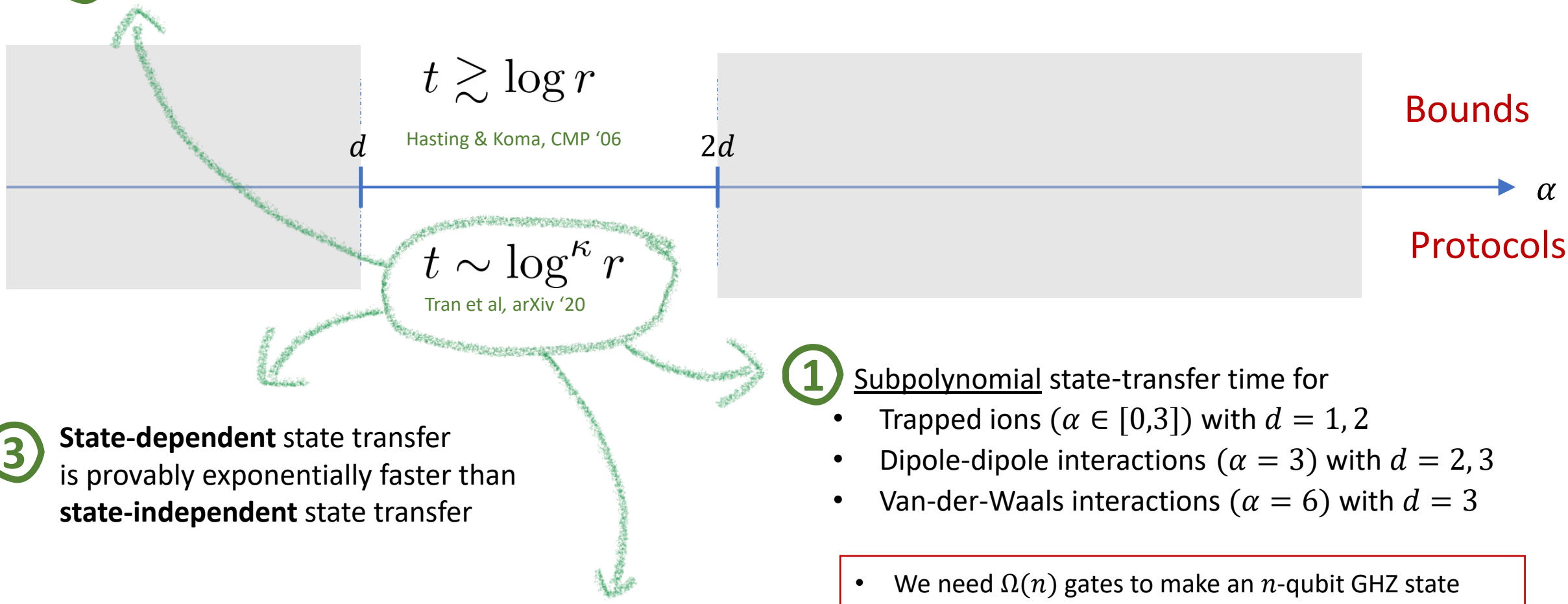
Total time (asymptotically):

$$t \sim \begin{cases} r^{\alpha-2d} & (2d < \alpha < 2d+1) \\ \log^{\kappa} r & (\alpha < 2d) \end{cases}$$

Strategy: Repeatedly merge nearby GHZ states into larger and larger GHZ states with the encoded information

More details in
Tran et al, arXiv:2010.02930 (2020)

② Saturates* the Lieb-Robinson bound



① <u>Subpolynomial</u> state-transfer time for

- Trapped ions ($\alpha \in [0,3]$) with $d = 1, 2$
- Dipole-dipole interactions ($\alpha = 3$) with $d = 2, 3$
- Van-der-Waals interactions ($\alpha = 6$) with $d = 3$

③ **State-dependent** state transfer is provably exponentially faster than **state-independent** state transfer

④ A lower bound on the gate count of quantum simulation algorithms

- We need $\Omega(n)$ gates to make an n-qubit GHZ state
- Simulating $e^{-i\,H\,t}$ for $t = t_*$ must also take $\Omega(n)$ gates
- Meanwhile, Suzuki-Trotter takes $O(n^{2+o(1)})$
- Either ∃ better simulation algorithms

  or making GHZ states is strictly easier than simulation

Acknowledgments

Abhinav Deshpande
(UMD)

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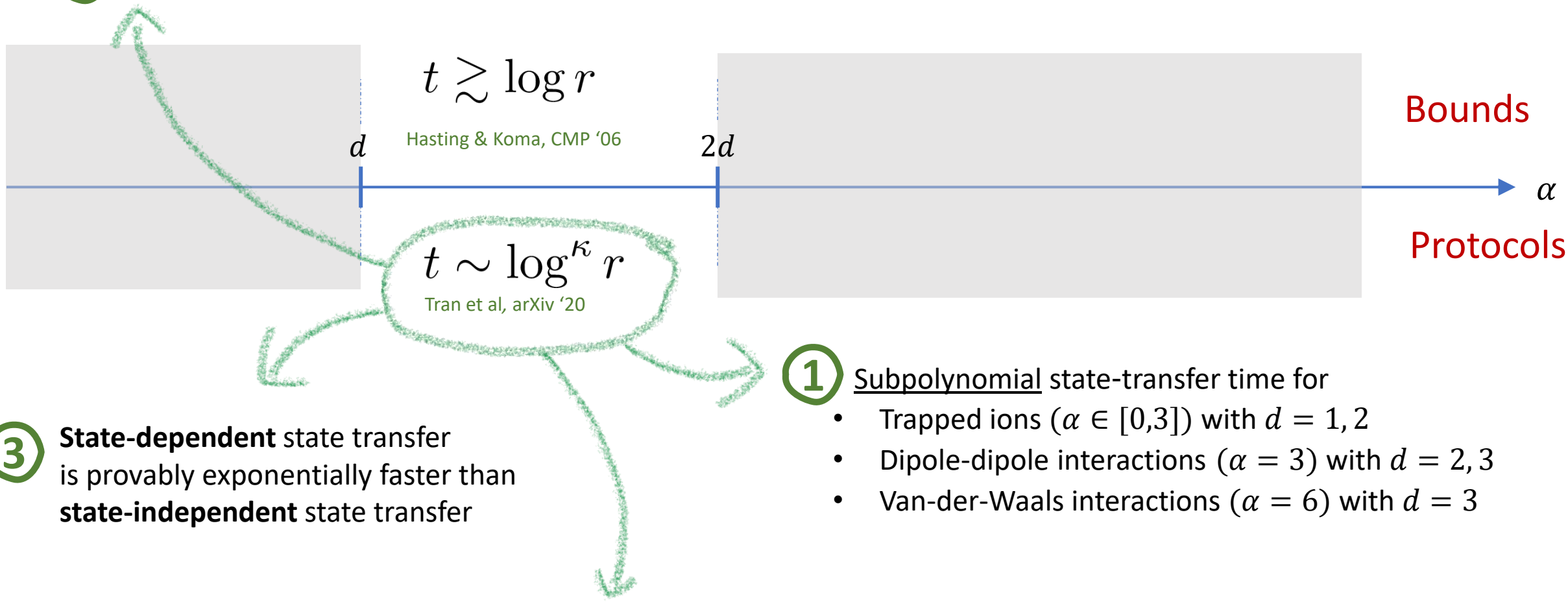
2 Saturates* the Lieb-Robinson bound



1 Subpolynomial state-transfer time for

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4 A lower bound on the gate count of quantum simulation algorithms

Thank you!