



Fusion-based quantum computation

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February 2, 2021

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[arXiv:2101.09310](https://arxiv.org/abs/2101.09310)

Fusion-based quantum computation (FBQC):

A general framework for fault-tolerant quantum computation with only:

Resource States:

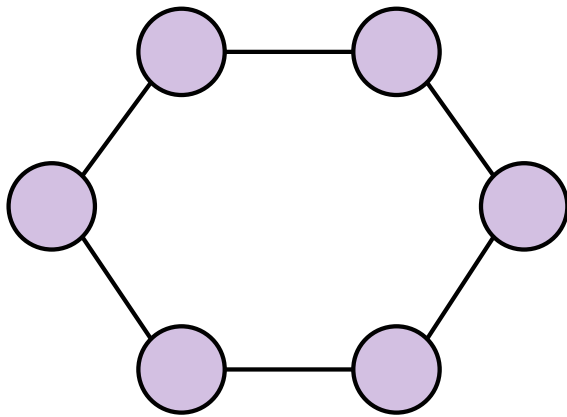
- Small constant sized entangled states

Fusions:

- Destructive entangling measurements

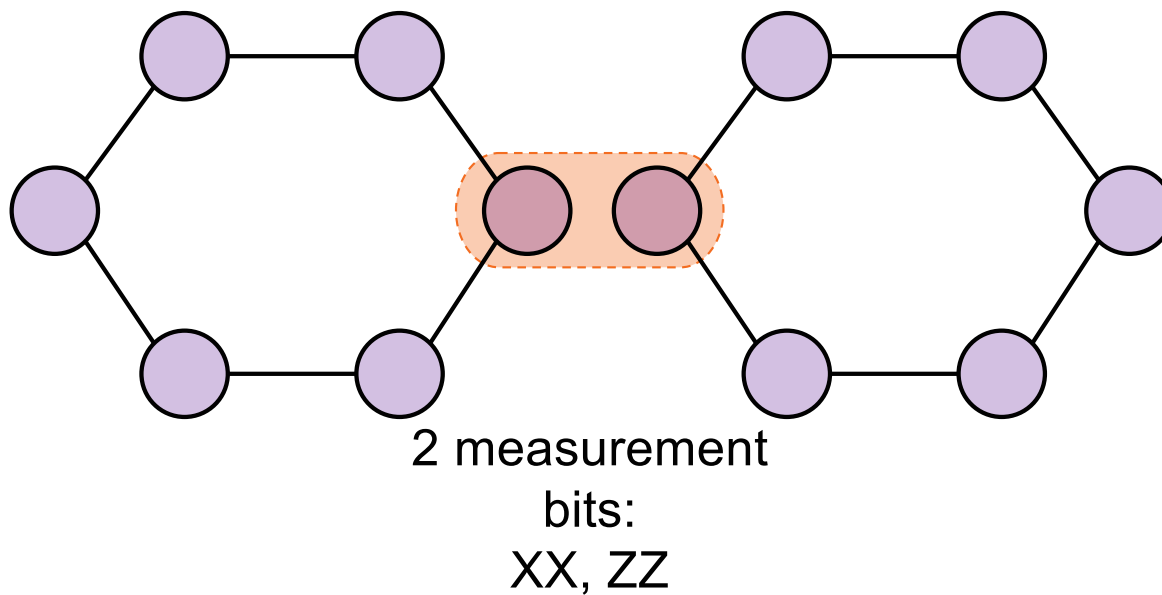
} Easy for
photons

Overview of FBQC: Resource State



- Constant size stabilizer states
- Low errors

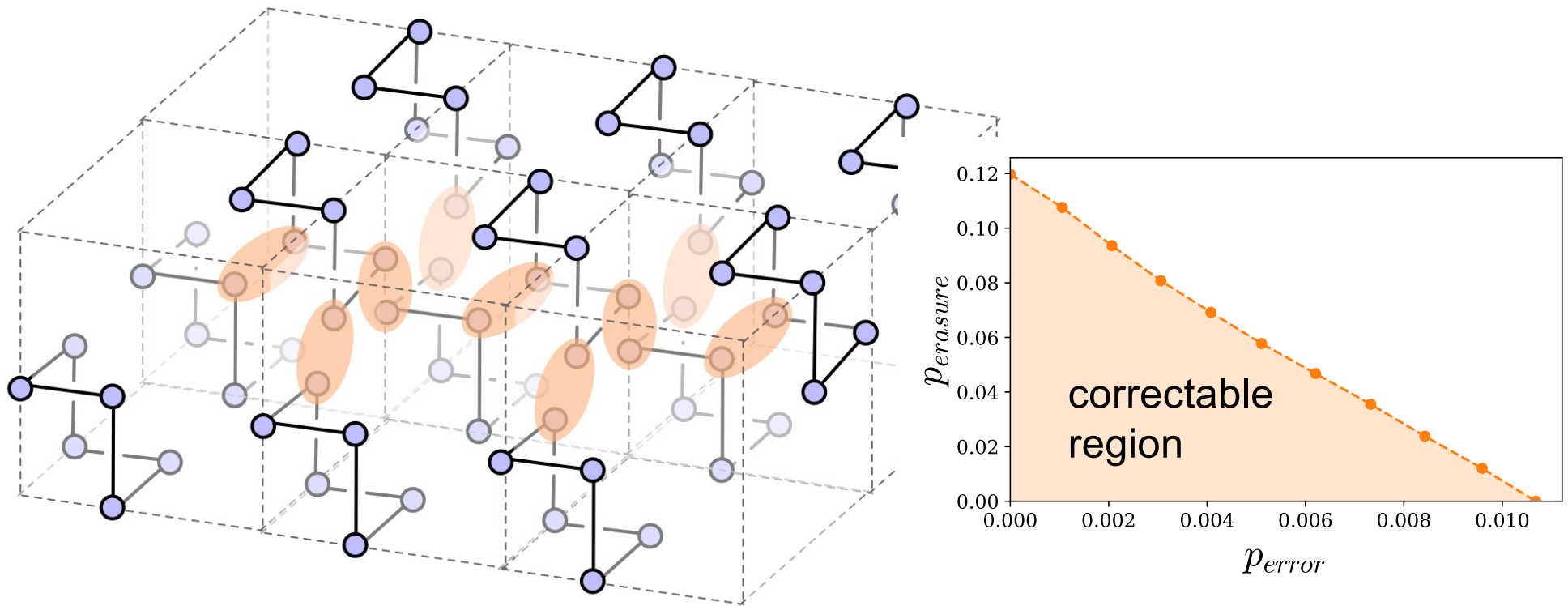
Overview of FBQC: Fusion



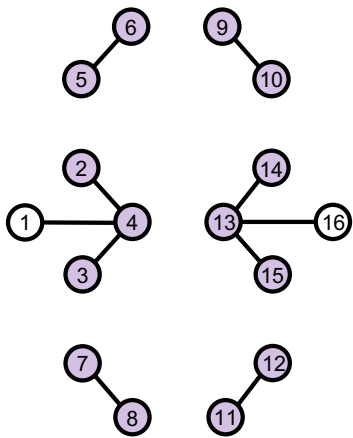
A measurement bit can be

- Erased ($p_{erasure}$)
- Flipped (p_{error})

Overview of FBQC: Fusion Network

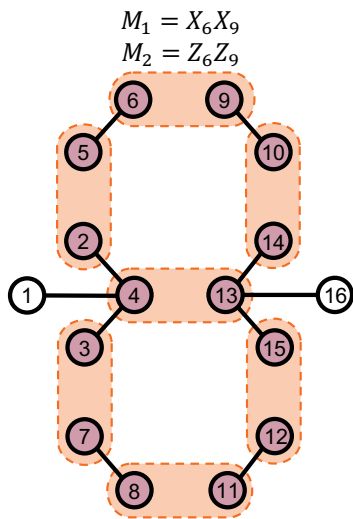


Fault-tolerance in the fusion based framework: a toy example



Resource States, R

$$\left\langle \left(\begin{array}{c} X_1 Z_4, \\ X_2 Z_4, \\ X_3 Z_4, \\ X_4 Z_1 Z_2 Z_3, \end{array} \right), \left(\begin{array}{c} X_{14} Z_{13}, \\ X_{15} Z_{13}, \\ X_{16} Z_{13}, \\ X_{13} Z_{14} Z_{15} Z_{16} \end{array} \right), (X_5 Z_6), (X_7 Z_8), (X_9 Z_{10}), (X_{11} Z_{12}), (X_{10} Z_9), (X_{12} Z_{11}) \right\rangle$$



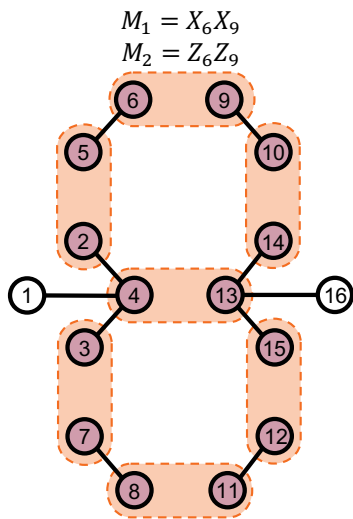
Resource States, R

$$\left\langle \begin{pmatrix} X_1 Z_4, \\ X_2 Z_4, \\ X_3 Z_4, \\ X_4 Z_1 Z_2 Z_3, \end{pmatrix}, \begin{pmatrix} X_{14} Z_{13}, \\ X_{15} Z_{13}, \\ X_{16} Z_{13}, \\ X_{13} Z_{14} Z_{15} Z_{16} \end{pmatrix}, (X_5 Z_6), (X_7 Z_8), (X_9 Z_{10}), (X_{11} Z_{12}), (X_8 Z_7), (X_{10} Z_9), (X_{12} Z_{11}) \right\rangle$$

Fusion, F

$$\left\langle \begin{pmatrix} X_2 X_5 \\ Z_2 Z_5 \end{pmatrix}, \begin{pmatrix} X_3 X_7 \\ Z_3 Z_7 \end{pmatrix}, \begin{pmatrix} X_6 X_9 \\ Z_6 Z_9 \end{pmatrix}, \begin{pmatrix} X_8 X_{11} \\ Z_8 Z_{11} \end{pmatrix}, \begin{pmatrix} X_4 X_{13} \\ Z_4 Z_{13} \end{pmatrix}, \begin{pmatrix} X_{10} X_{14} \\ Z_{10} Z_{14} \end{pmatrix}, \begin{pmatrix} X_{12} X_{15} \\ Z_{12} Z_{15} \end{pmatrix}, -\mathbf{1} \right\rangle$$

Fault-tolerance in the fusion based framework: a toy example

Resource States, R

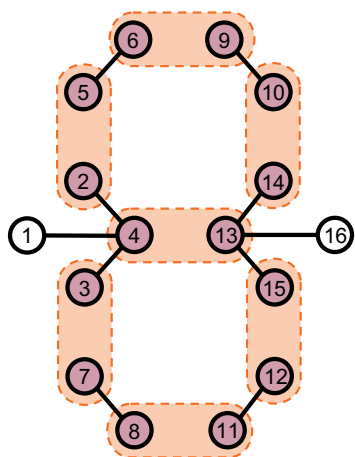
$$\left\langle \begin{pmatrix} X_1 Z_4, \\ X_2 Z_4, \\ X_3 Z_4, \\ X_4 Z_1 Z_2 Z_3, \end{pmatrix}, \begin{pmatrix} X_{14} Z_{13}, \\ X_{15} Z_{13}, \\ X_{16} Z_{13}, \\ X_{13} Z_{14} Z_{15} Z_{16} \end{pmatrix}, (X_5 Z_6), (X_7 Z_8), (X_9 Z_{10}), (X_{11} Z_{12}), (X_8 Z_7), (X_{10} Z_9), (X_{12} Z_{11}) \right\rangle$$

Fusion, F

$$\left\langle \begin{pmatrix} X_2 X_5 \\ Z_2 Z_5 \end{pmatrix}, \begin{pmatrix} X_3 X_7 \\ Z_3 Z_7 \end{pmatrix}, \begin{pmatrix} X_6 X_9 \\ Z_6 Z_9 \end{pmatrix}, \begin{pmatrix} X_8 X_{11} \\ Z_8 Z_{11} \end{pmatrix}, \begin{pmatrix} X_4 X_{13} \\ Z_4 Z_{13} \end{pmatrix}, \begin{pmatrix} X_{10} X_{14} \\ Z_{10} Z_{14} \end{pmatrix}, \begin{pmatrix} X_{12} X_{15} \\ Z_{12} Z_{15} \end{pmatrix}, -\mathbf{1} \right\rangle$$

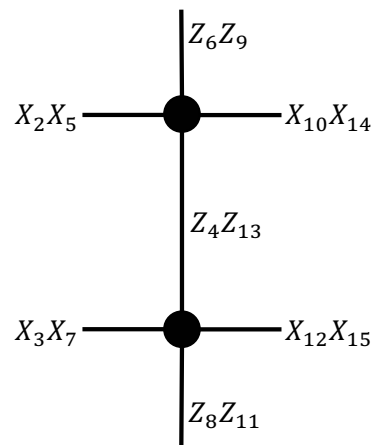
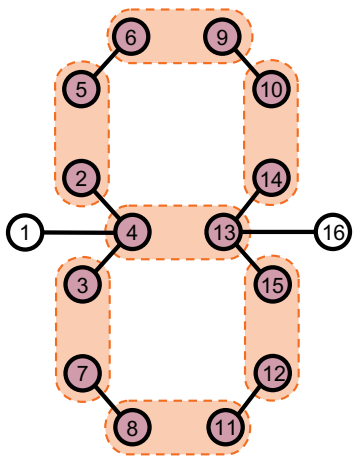
$$C = R \cap F$$

Fault-tolerance in the fusion based framework: a toy example



Resource States, R	
$\left\langle \left(\begin{pmatrix} X_1 Z_4, \\ X_2 Z_4, \\ X_3 Z_4, \\ X_4 Z_1 Z_2 Z_3, \end{pmatrix}, \begin{pmatrix} X_{14} Z_{13}, \\ X_{15} Z_{13}, \\ X_{16} Z_{13}, \\ X_{13} Z_{14} Z_{15} Z_{16} \end{pmatrix}, \begin{pmatrix} X_5 Z_6, \\ X_6 Z_5 \end{pmatrix}, \begin{pmatrix} X_7 Z_8, \\ X_8 Z_7 \end{pmatrix}, \begin{pmatrix} X_9 Z_{10}, \\ X_{10} Z_9 \end{pmatrix}, \begin{pmatrix} X_{11} Z_{12}, \\ X_{12} Z_{11} \end{pmatrix} \right) \right\rangle$	
Fusion, F	
$\langle (X_2 X_5), (X_3 X_7), (X_6 X_9), (X_8 X_{11}), (X_4 X_{13}), (X_{10} X_{14}), (X_{12} X_{15}), -\mathbf{1} \rangle$	
Checks, C	$\langle (X_2 X_5)(Z_6 Z_9)(X_{10} X_{14})(Z_4 Z_{13}), (X_3 X_7)(Z_8 Z_{11})(X_{12} X_{15})(Z_{13} Z_4) \rangle$

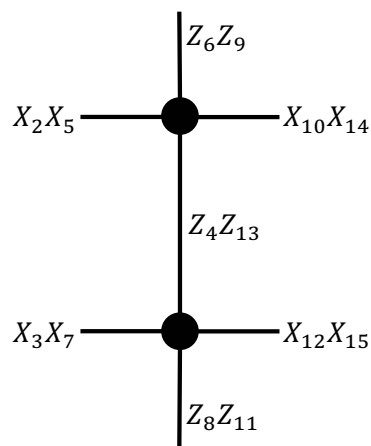
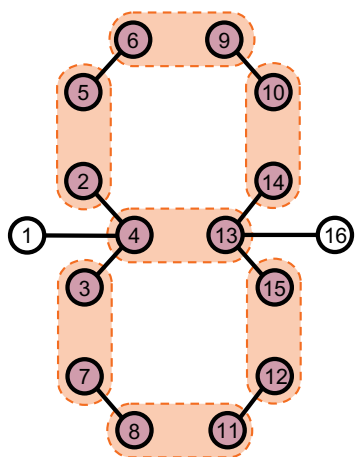
Fault-tolerance in the fusion based framework: a toy example



Syndrome graph

Resource States, R	
$\left\langle \left(\begin{pmatrix} X_1 Z_4, \\ X_2 Z_4, \\ X_3 Z_4, \\ X_4 Z_1 Z_2 Z_3, \end{pmatrix}, \begin{pmatrix} X_{14} Z_{13}, \\ X_{15} Z_{13}, \\ X_{16} Z_{13}, \\ X_{13} Z_{14} Z_{15} Z_{16} \end{pmatrix}, \begin{pmatrix} X_5 Z_6, \\ X_6 Z_5 \end{pmatrix}, \begin{pmatrix} X_7 Z_8, \\ X_8 Z_7 \end{pmatrix}, \begin{pmatrix} X_9 Z_{10}, \\ X_{10} Z_9 \end{pmatrix}, \begin{pmatrix} X_{11} Z_{12}, \\ X_{12} Z_{11} \end{pmatrix} \right) \right\rangle$	
Fusion, F	
$\left\langle \begin{pmatrix} X_2 X_5, \\ Z_2 Z_5 \end{pmatrix}, \begin{pmatrix} X_3 X_7, \\ Z_3 Z_7 \end{pmatrix}, \begin{pmatrix} X_6 X_9, \\ Z_6 Z_9 \end{pmatrix}, \begin{pmatrix} X_8 X_{11}, \\ Z_8 Z_{11} \end{pmatrix}, \begin{pmatrix} X_4 X_{13}, \\ Z_4 Z_{13} \end{pmatrix}, \begin{pmatrix} X_{10} X_{14}, \\ Z_{10} Z_{14}, \end{pmatrix}, \begin{pmatrix} X_{12} X_{15}, \\ Z_{12} Z_{15}, \end{pmatrix}, -\mathbf{1} \right\rangle$	
Checks, C	$\langle (X_2 X_5)(Z_6 Z_9)(X_{10} X_{14})(Z_4 Z_{13}), (X_3 X_7)(Z_8 Z_{11})(X_{12} X_{15})(Z_{13} Z_4) \rangle$

Fault-tolerance in the fusion based framework: a toy example

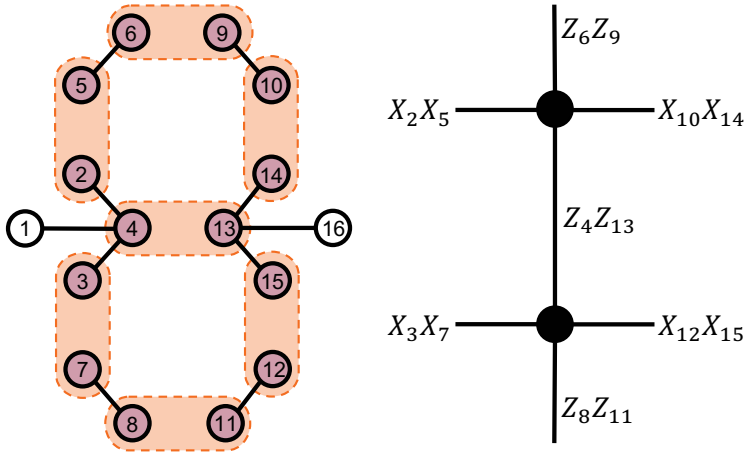


Resource States, R	
$\left\langle \left(\begin{pmatrix} X_1 Z_4, \\ X_2 Z_4, \\ X_3 Z_4, \\ X_4 Z_1 Z_2 Z_3, \end{pmatrix}, \begin{pmatrix} X_{14} Z_{13}, \\ X_{15} Z_{13}, \\ X_{16} Z_{13}, \\ X_{13} Z_{14} Z_{15} Z_{16} \end{pmatrix}, (X_5 Z_6), (X_7 Z_8), (X_9 Z_{10}), (X_{11} Z_{12}), (X_{12} Z_{11}) \right\rangle$	
Fusion, F	
$\left\langle (X_2 X_5), (X_3 X_7), (X_6 X_9), (X_8 X_{11}), (X_4 X_{13}), (X_{10} X_{14}), (X_{12} X_{15}), -\mathbf{1} \right\rangle$	
Checks, C	$\langle (X_2 X_5)(Z_6 Z_9)(X_{10} X_{14})(Z_4 Z_{13}), (X_3 X_7)(Z_8 Z_{11})(X_{12} X_{15})(Z_{13} Z_4) \rangle$

$$S = \mathcal{Z}_R(F)$$

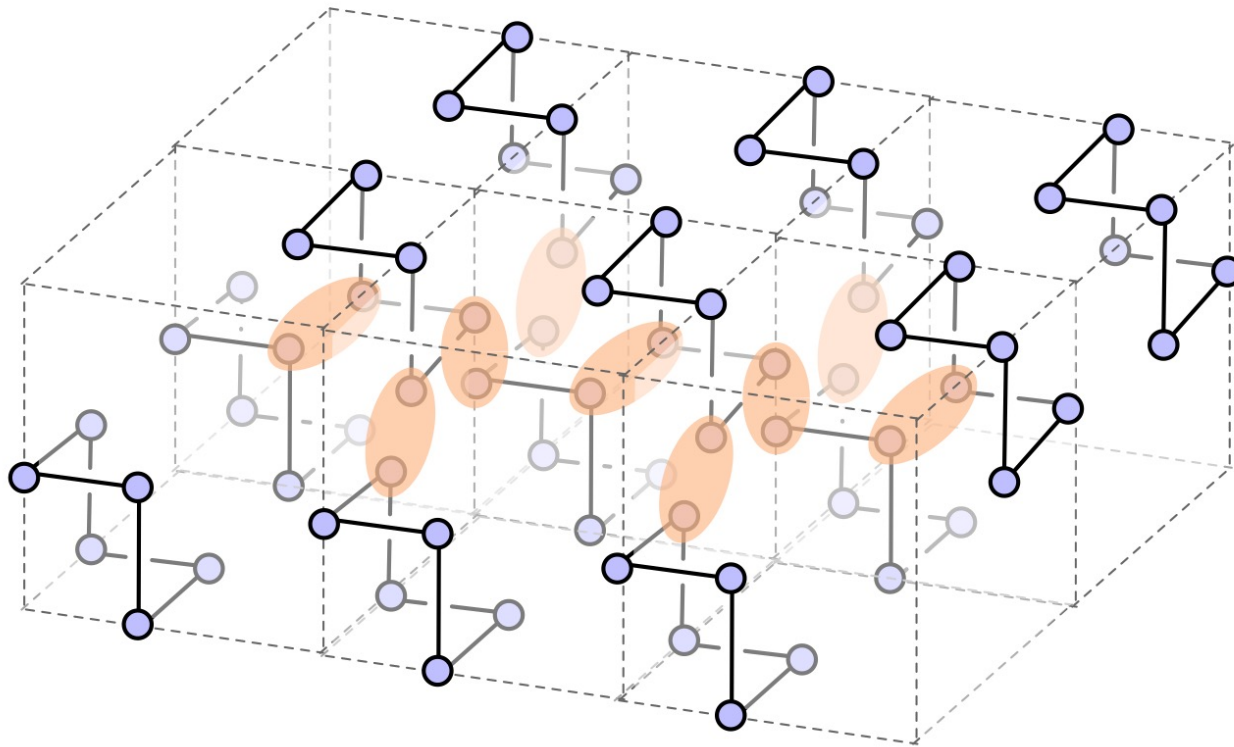
(centralizer of F in R)

Fault-tolerance in the fusion based framework: a toy example

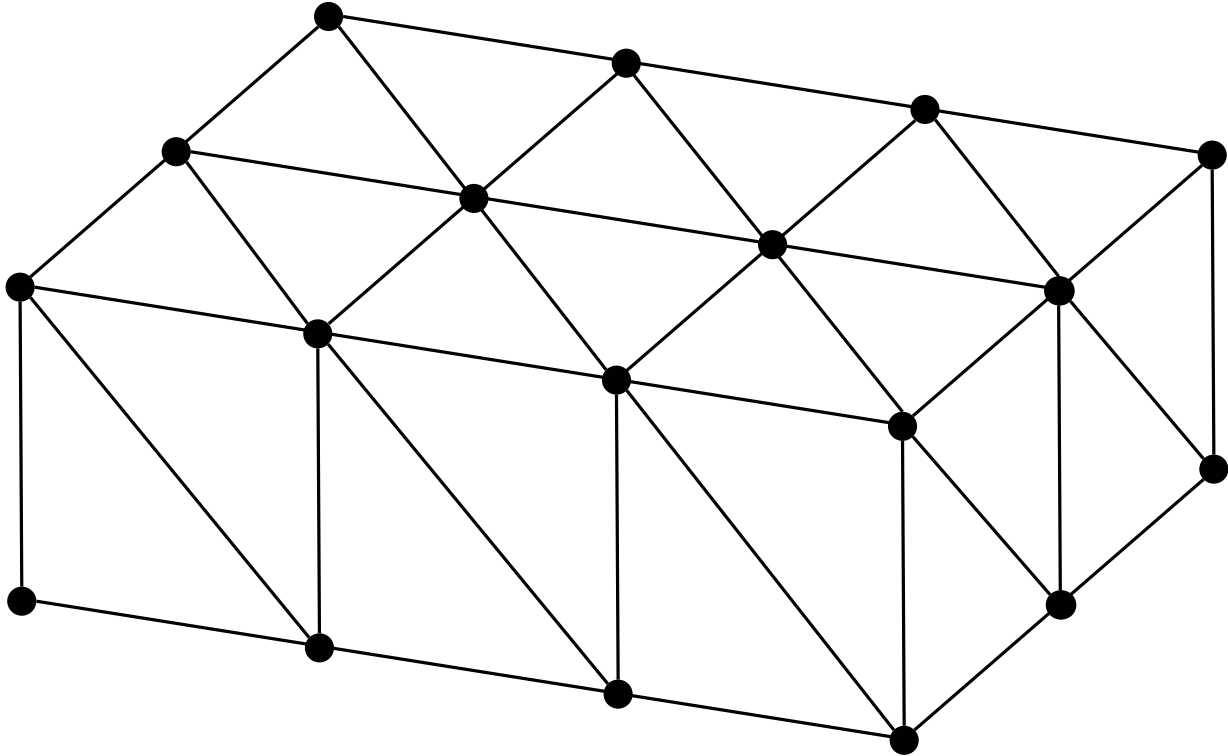


Resource States, R	
$\left\langle \left(\begin{pmatrix} X_1 Z_4, \\ X_2 Z_4, \\ X_3 Z_4, \\ X_4 Z_1 Z_2 Z_3, \end{pmatrix}, \begin{pmatrix} X_{14} Z_{13}, \\ X_{15} Z_{13}, \\ X_{16} Z_{13}, \\ X_{13} Z_{14} Z_{15} Z_{16} \end{pmatrix}, \begin{pmatrix} X_5 Z_6, \\ X_6 Z_5 \end{pmatrix}, \begin{pmatrix} X_7 Z_8, \\ X_8 Z_7 \end{pmatrix}, \begin{pmatrix} X_9 Z_{10}, \\ X_{10} Z_9 \end{pmatrix}, \begin{pmatrix} X_{11} Z_{12}, \\ X_{12} Z_{11} \end{pmatrix} \right) \right\rangle$	
Fusion, F	
$\left\langle \begin{pmatrix} X_2 X_5, \\ Z_2 Z_5 \end{pmatrix}, \begin{pmatrix} X_3 X_7, \\ Z_3 Z_7 \end{pmatrix}, \begin{pmatrix} X_6 X_9, \\ Z_6 Z_9 \end{pmatrix}, \begin{pmatrix} X_8 X_{11}, \\ Z_8 Z_{11} \end{pmatrix}, \begin{pmatrix} X_4 X_{13}, \\ Z_4 Z_{13} \end{pmatrix}, \begin{pmatrix} X_{10} X_{14}, \\ Z_{10} Z_{14} \end{pmatrix}, \begin{pmatrix} X_{12} X_{15}, \\ Z_{12} Z_{15} \end{pmatrix}, -\mathbf{1} \right\rangle$	
Checks, C	$\langle (X_2 X_5)(Z_6 Z_9)(X_{10} X_{14})(Z_4 Z_{13}), (X_3 X_7)(Z_8 Z_{11})(X_{12} X_{15})(Z_{13} Z_4) \rangle$
Surviving Stabilizer, S	$\langle C, X_1 (Z_4 Z_{13}) X_{16}, Z_1 (Z_2 Z_5)(X_6 X_9)(Z_{10} Z_{14})(Z_{12} Z_{15})(X_8 X_{11})(Z_3 Z_7)(Z_4 Z_{13}) Z_{16} \rangle$

Fault-tolerant fusion network



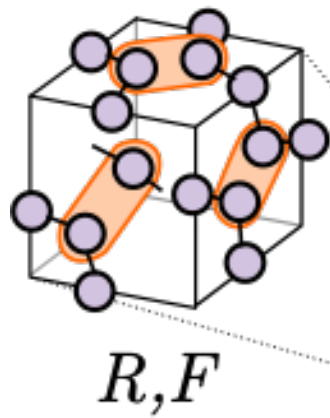
Fusion network: syndrome graph



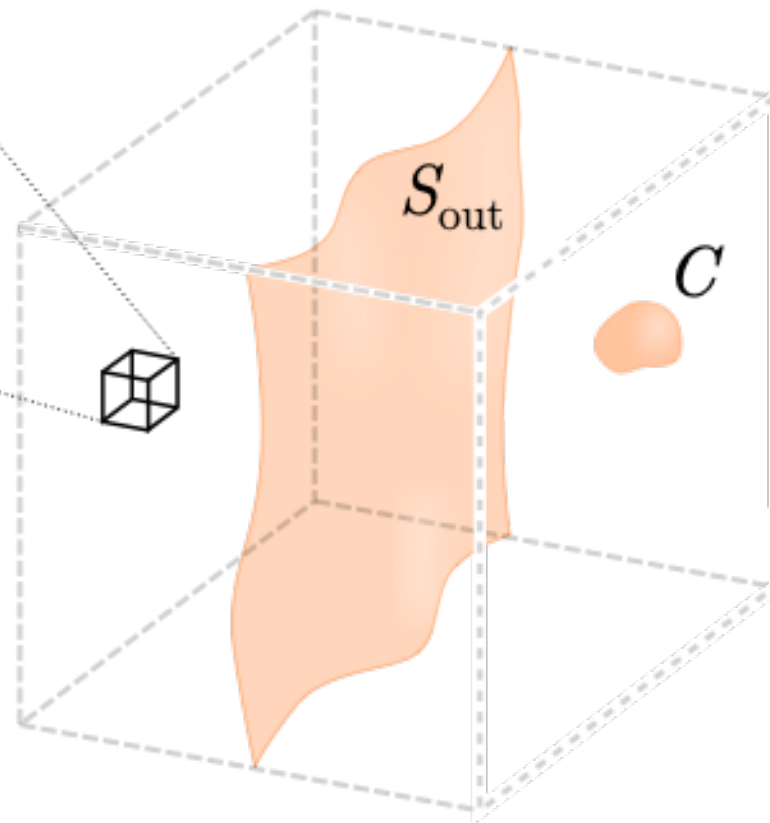
self-dual

Fusion network: surviving stabilizers

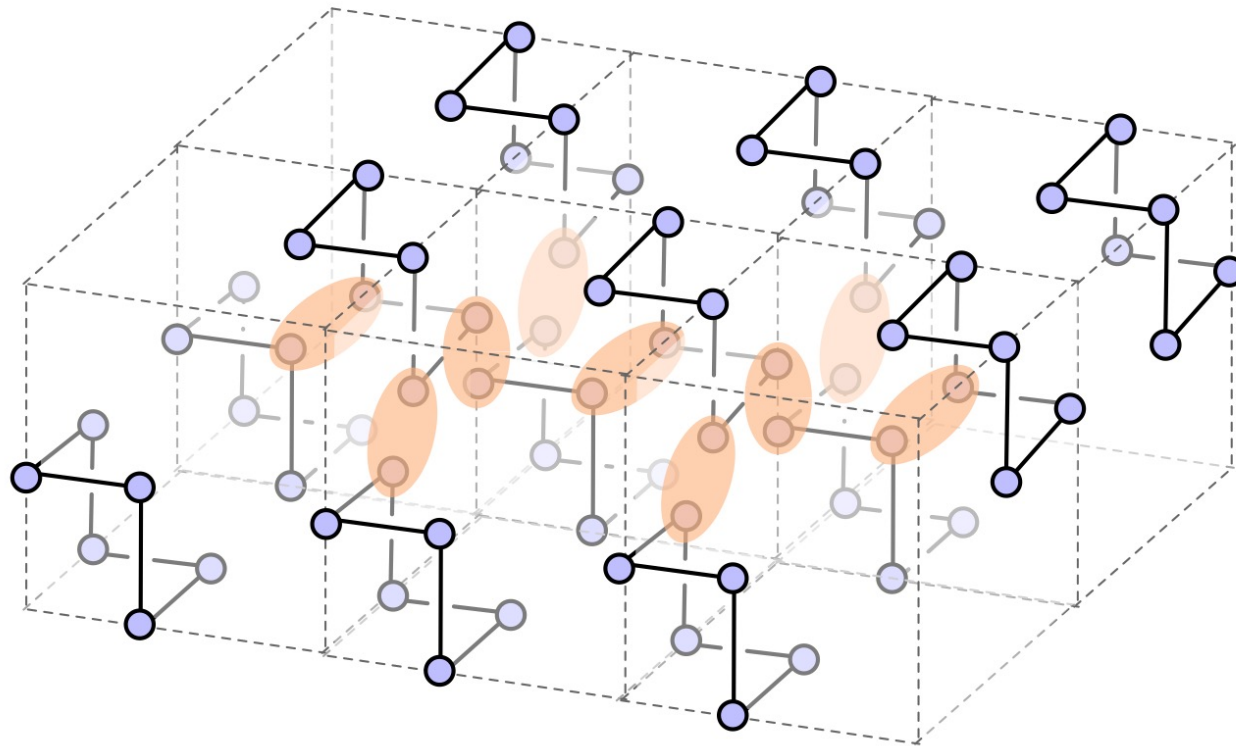
Fusion Network



Surviving Stabilizers

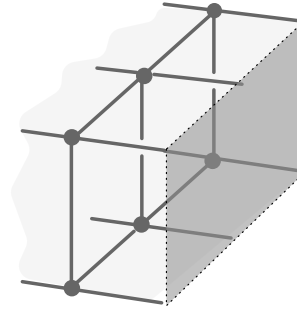
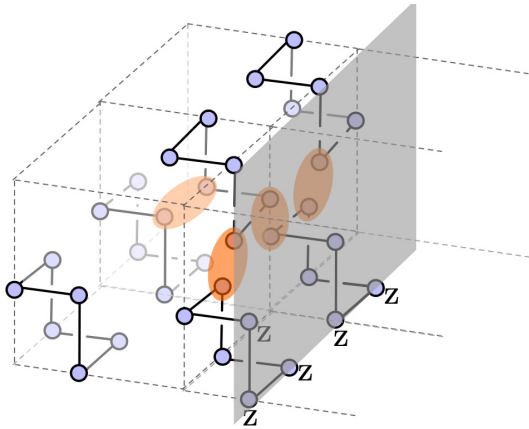


The Bulk

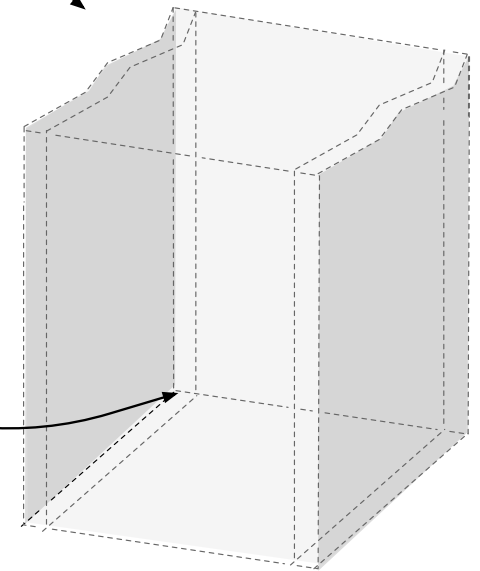


Logic

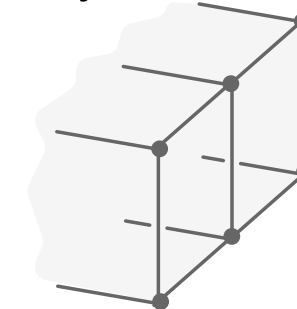
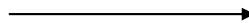
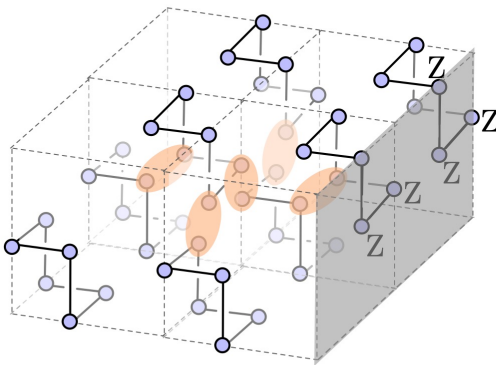
rough primal boundary



Identity gate



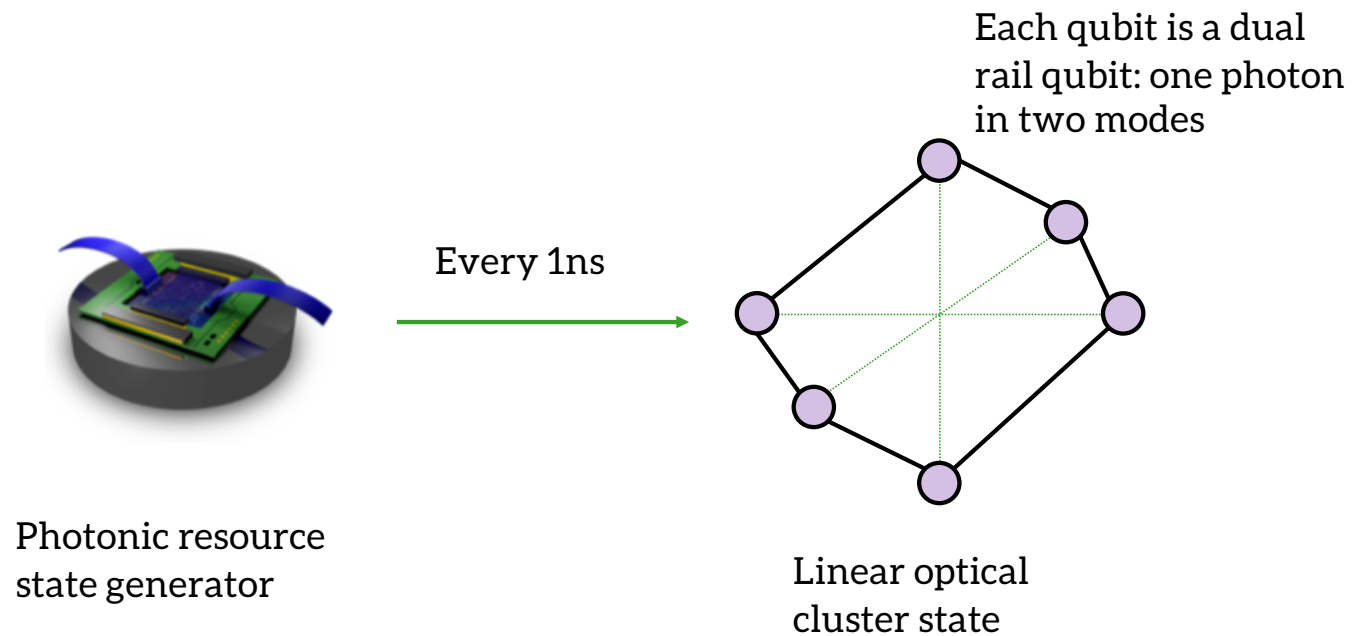
smooth primal boundary



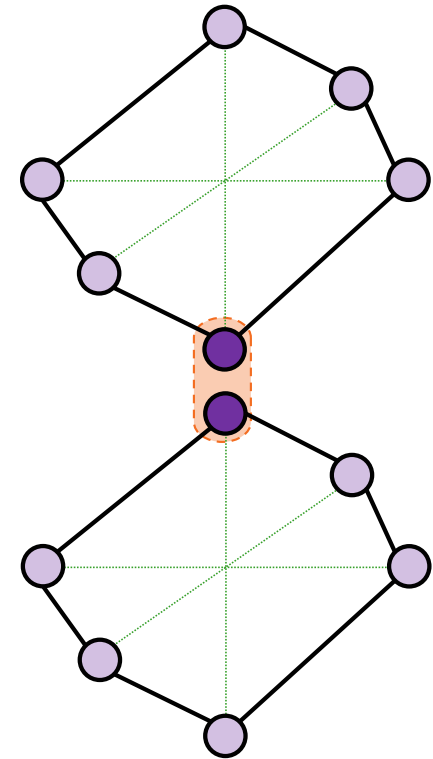
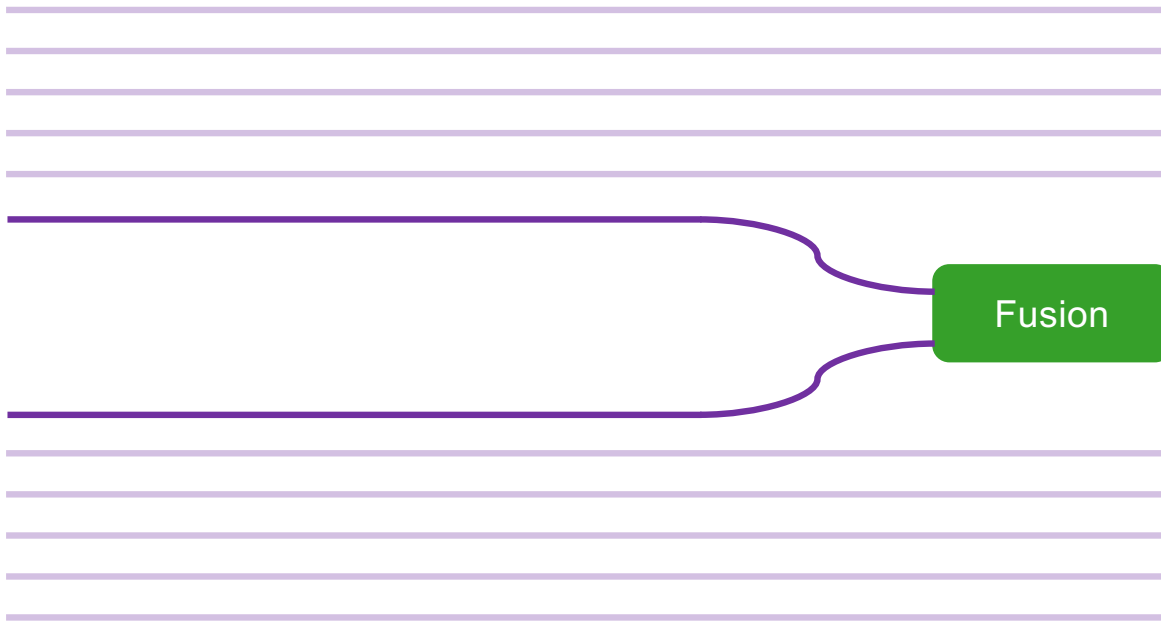
upcoming PsiQuantum publication

Fusion Networks in real space and time

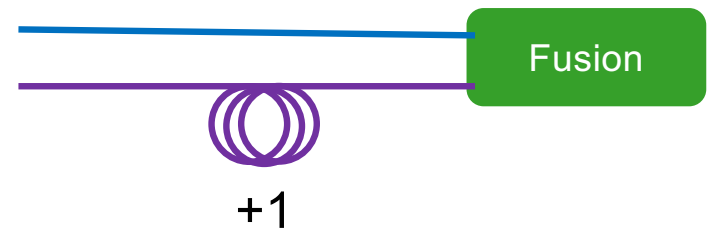
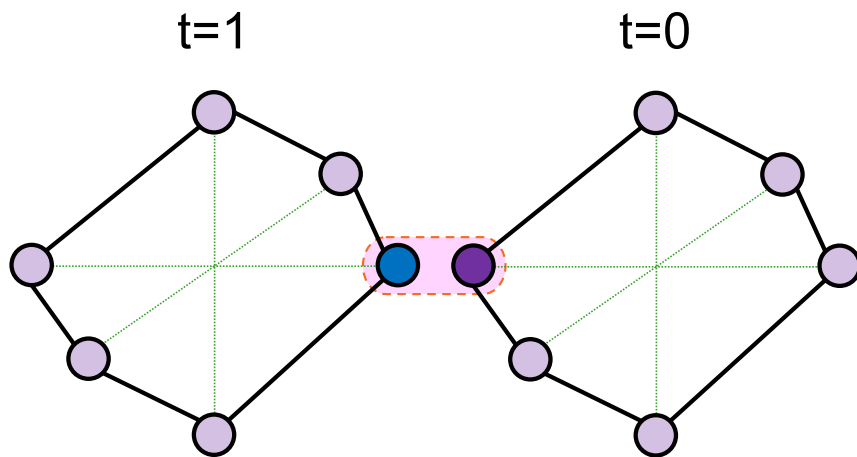
The physical layout



The physical layout: space-like fusions

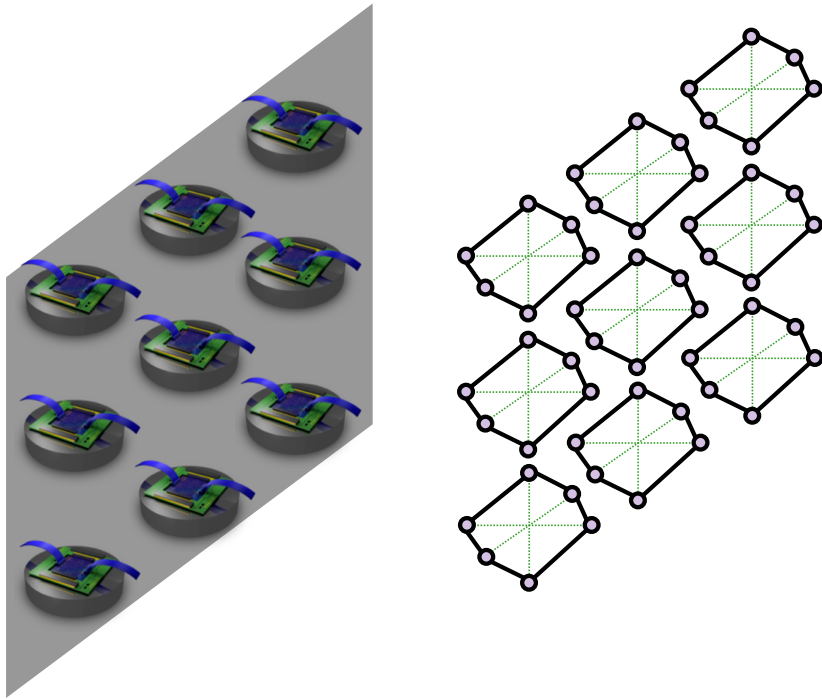


The physical layout: time-like fusions



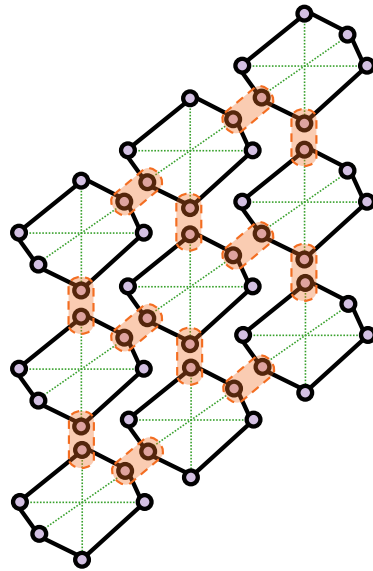
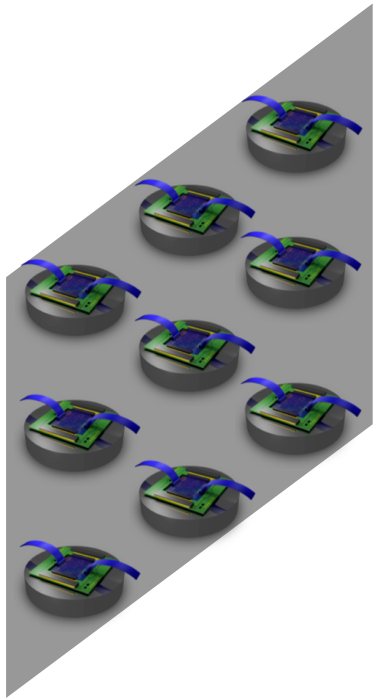
The physical layout

Resource state
generation

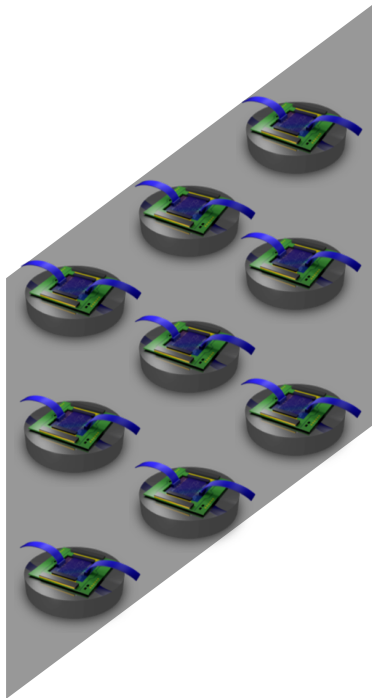


The physical layout

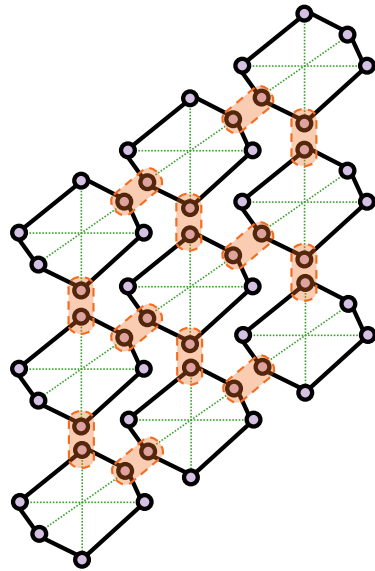
Space-like
fusions



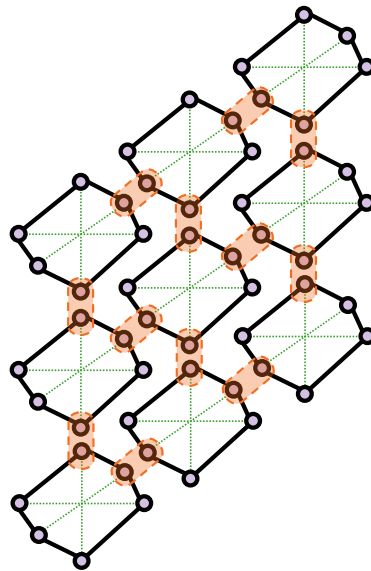
The physical layout



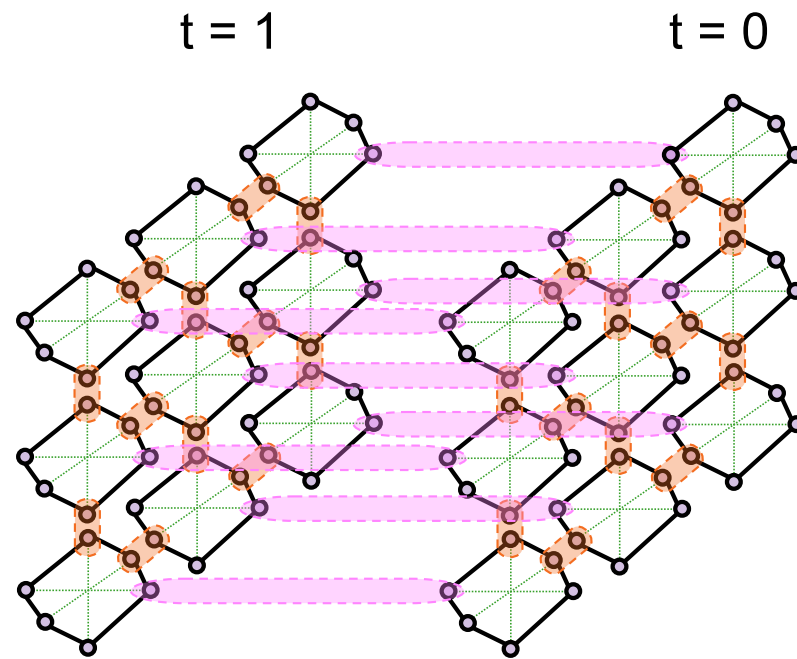
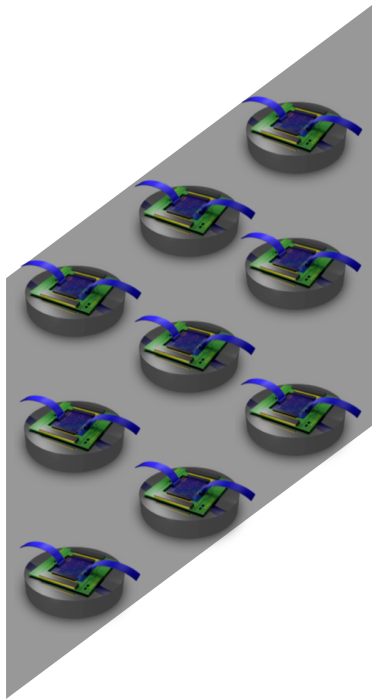
$t = 1$



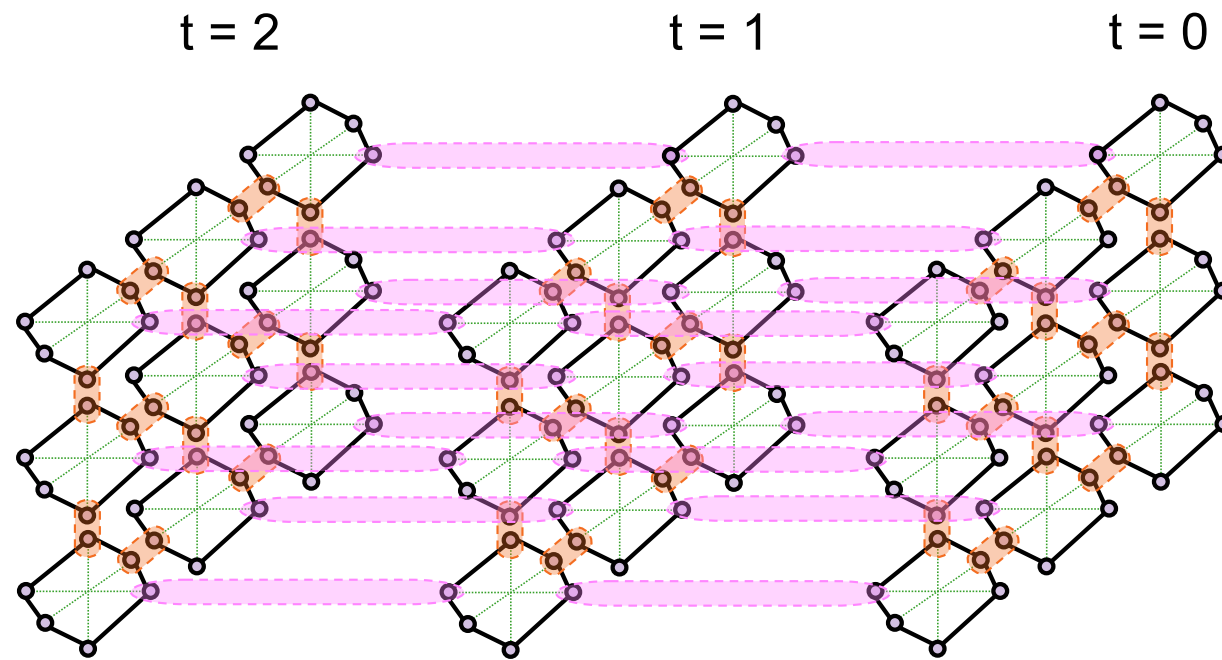
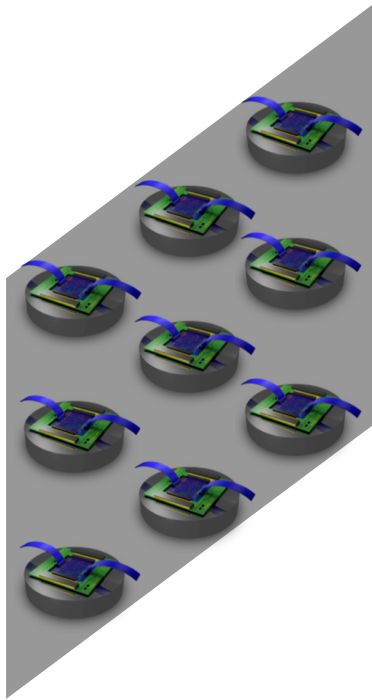
$t = 0$



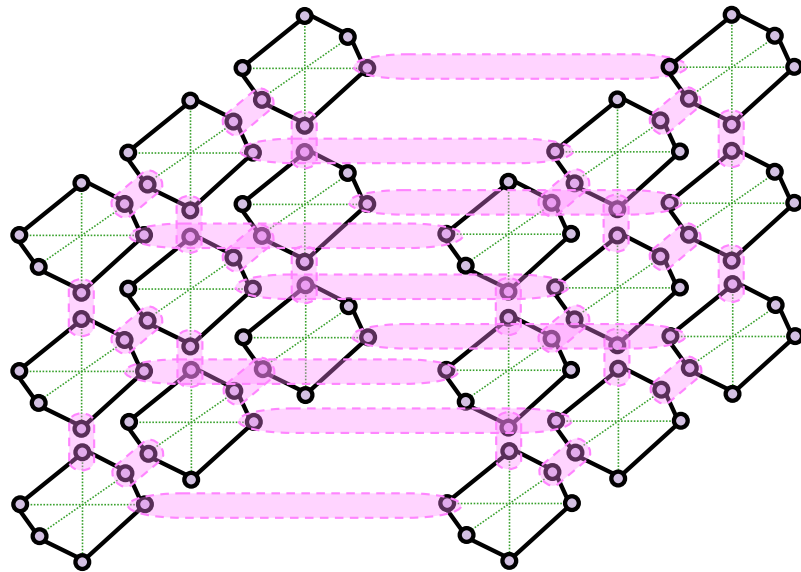
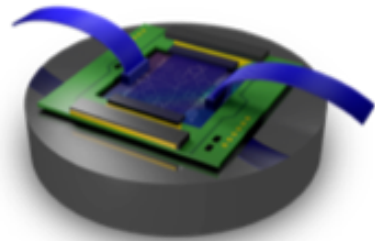
The physical layout



The physical layout



Reusing resources



upcoming PsiQuantum publication

Fusion Based Quantum Computing (FBQC)

- Fault-tolerance with natural hardware primitives
- Low depth modular structure
 - Constant lifetime of physical qubits
- Separation of logical and physical timescales
 - Operations in the bulk don't depend on gate being performed
- Better handling of linear optic fusion failure
 - Loss tolerance for linear optical quantum computing

[arXiv:2101.09310](https://arxiv.org/abs/2101.09310)