Quantum encryption with certified deletion

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Motivation

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"I deleted the ciphertext!"

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“I deleted the ciphertext!”

“How do I know?”

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- With a classical ciphertext, Bob cannot prove deletion to Alice
    - Bob can always make a copy of the ciphertext that can be decrypted once the key is received
- Therefore, we must consider a non-classical solution

A solution

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  - No-cloning theorem: there is no map that will create an identical copy of an arbitrary quantum state

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A solution

- A quantum ciphertext?
  - No-cloning theorem: there is no map that will create an identical copy of an arbitrary quantum state
- But what would a proof of deletion look like?
- Entropic uncertainty relations: measurement in one basis can cause loss of information about what the measurement outcome in another basis would have been
- Conjugate coding (Wiesner/BB84 states) and measurements will be integral to our scheme

Previous results

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- [Coiteux-Roy and Wolf 2019] “Proving Erasure”

Scheme: parameters

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- m: number of qubits used in the quantum encoding

Scheme: key generation

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- $\theta \leftarrow \{\theta \in \{0,1\}^m \mid \omega(\theta) = k\}$, where k is less than m.
  - Basis for encoding qubits
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- $r_{diag} \leftarrow \{0, 1\}^k$
    - Also called “check bits”
- $u \leftarrow \{0, 1\}^n$
- $H \leftarrow$ universal$_2$ family of hash functions
    - Domain: strings of length $m - k$; codomain: strings of length n

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- Ciphertext: r encoded in basis θ, with $msg \bigoplus x \bigoplus u$.

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- Measure qubits in basis θ to yield r, and hence r_{comp}
- Compute $H(r_{comp}) = x$.
- Compute $msg \bigoplus x \bigoplus u \bigoplus x \bigoplus u = msg$.

Scheme: delete

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- Measure qubits in the Hadamard basis and obtain a certificate of deletion $y \leftarrow \{0, 1\}^m$

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- Using θ, take the substring of the received string that corresponds to the diagonal positions of the qubits (call the result y').

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- Accept if $\omega\left(r_{diag} \bigoplus y'\right) < \delta k$.

Error tolerance

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- More details in the paper

Certified deletion security: Game 1

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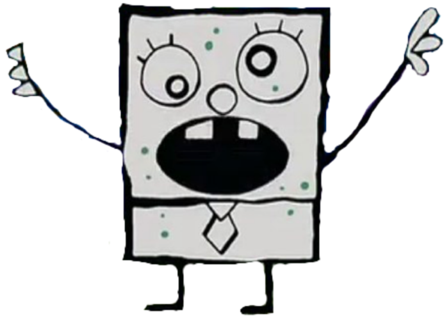
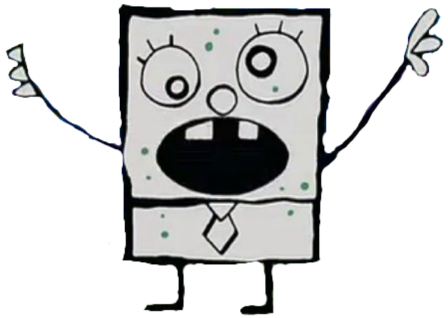


Certified deletion security: Game 1

msg_0



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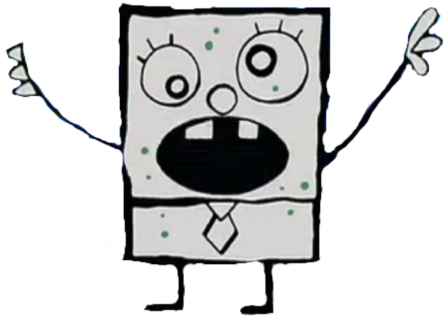

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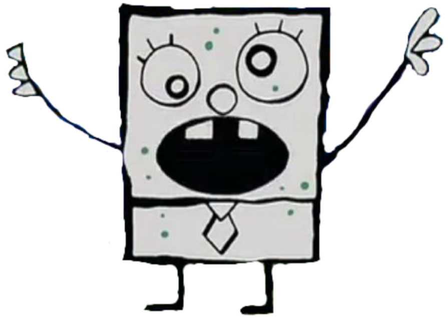
msg_0

$\theta,\ u,\ H,$
r_{diag}



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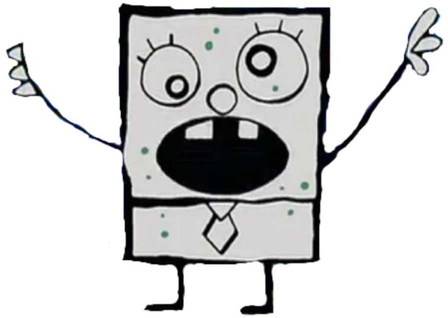


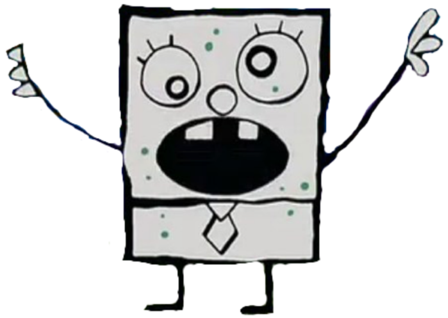
b

Certified deletion security: Game 1

msg_0

$\theta,\ u,\ H,$
r_{diag}
ciphertext

b



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msg_0

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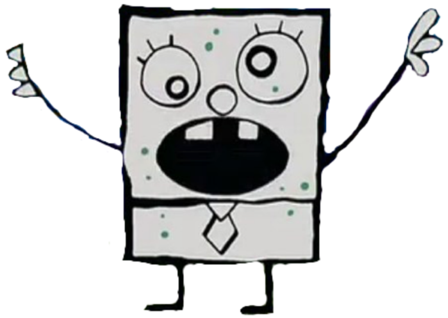


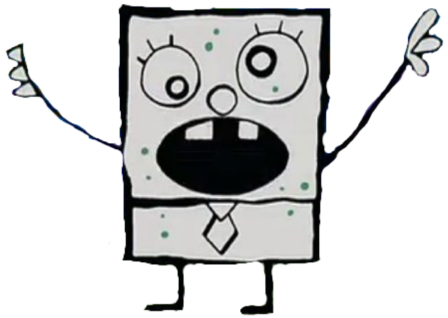
ciphertext

b

Certified deletion security: Game 1

msg_0

y



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ciphertext

b

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msg_0

y

$\theta,\ u,\ H,$
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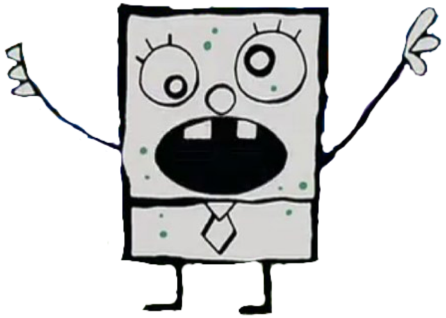

ciphertext

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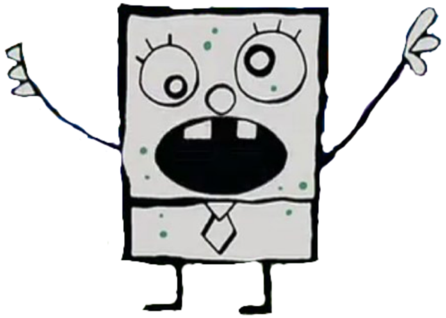
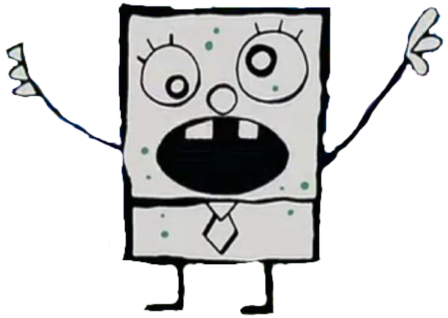


b ok

Certified deletion security: Game 1

msg_0

y



$\theta,\ u,\ H,$
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ciphertext

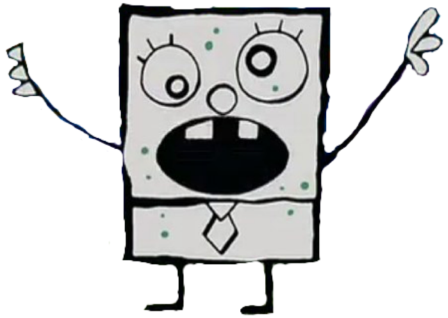
b ok

Certified deletion security: Game 1

msg_0

y

b'



$\theta,\ u,\ H,$
r_{diag}

ciphertext

b ok

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- Bob can be seen as having two goals:
  1. Determine whether his message was encrypted in the ciphertext
  2. Convince Alice that he deleted the ciphertext prior to receiving the key

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  1. Determine whether his message was encrypted in the ciphertext
  2. Convince Alice that he deleted the ciphertext prior to receiving the key
- Scheme is secure if the probabilities of the following two events are negligibly close:
  1. Verification passes and Bob's guess of b is 1, in the case that Alice encrypted the string of zeros
  2. Verification passes and Bob's guess of b is 1, in the case that Alice encrypted the candidate message.

Certified deletion security: Game 2

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- Game 1 is difficult to analyze

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- Game 1 is difficult to analyze
- We developed a Game 2 which is based on an entanglement-based series of interactions
- A reduction shows that statements about Game 2 can translate into statements about Game 1
  - We thereby achieve bounds relevant to our scheme

Certified deletion security: Game 2

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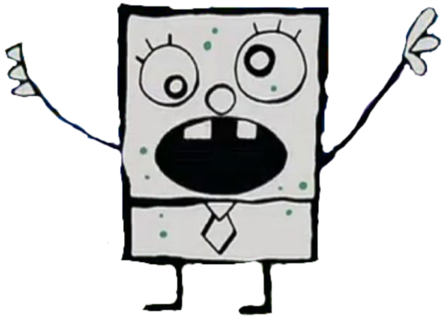

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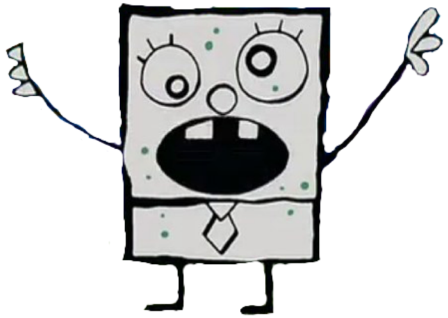

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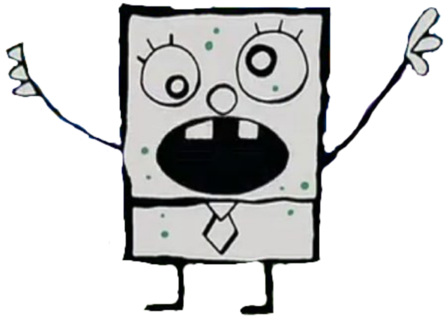
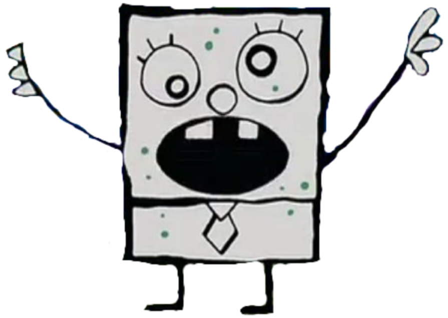


Certified deletion security: Game 2

msg_0

A
B
B'



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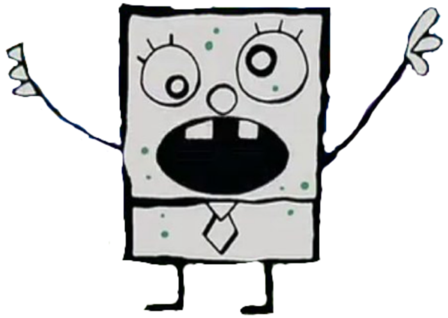

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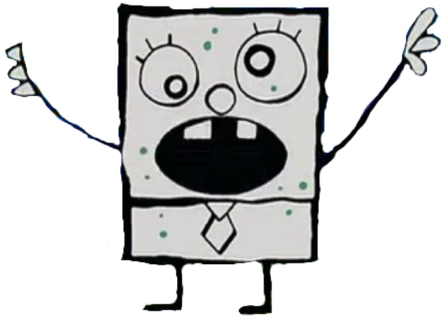
msg_0

A

B'



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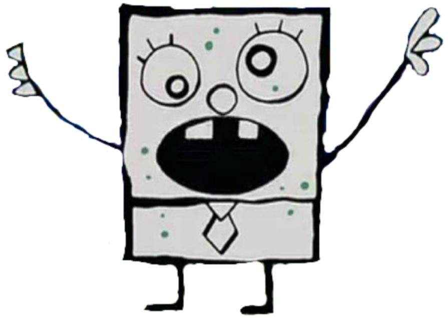

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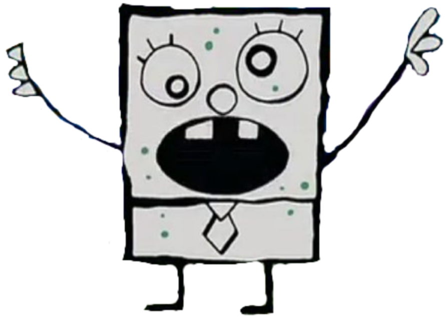

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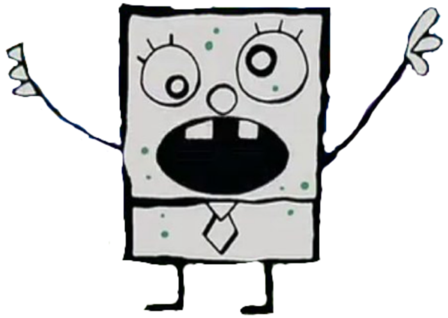
A
y

$\theta,\ u,\ H$

B'



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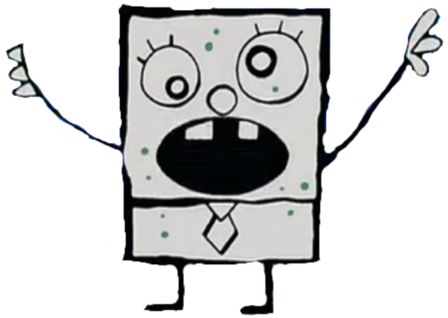


Certified deletion security: Game 2

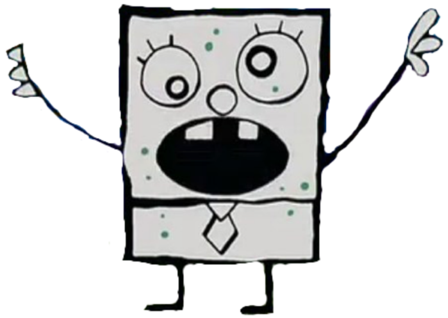
msg_0

y

$\theta,\ u,\ H$

r

B'



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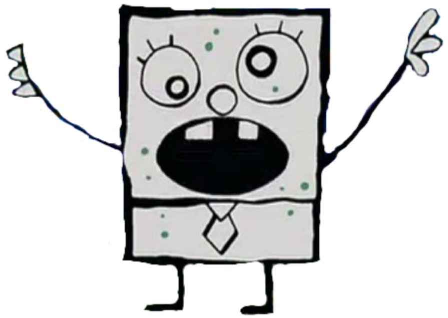

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msg_0

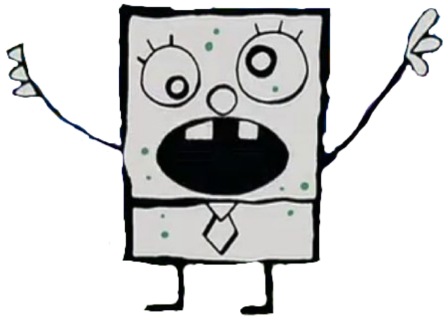
y

$\theta,\ u,\ H$

r_{comp}

r_{diag}

B'



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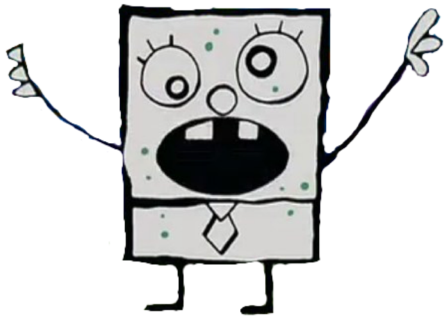
r_{comp} x

r_{diag}

B'



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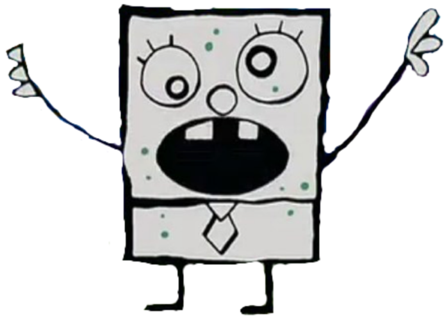

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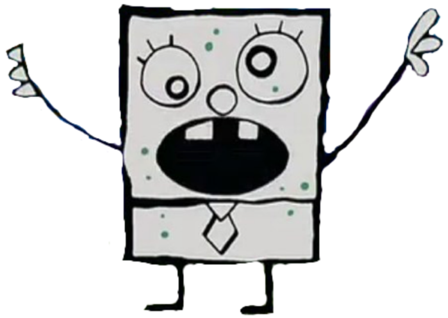

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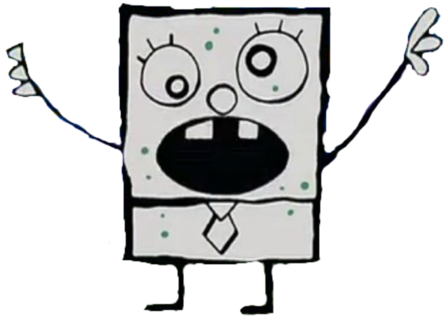
$\theta,\ u,\ H$

r_{comp} x

r_{diag}

b ok $msg \bigoplus x \bigoplus u \bigoplus x \bigoplus u$



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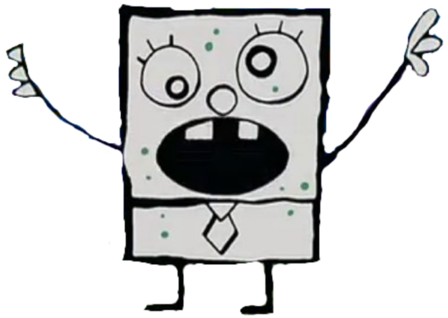

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B'

$\theta, \ u, \ H$

r_{comp} x

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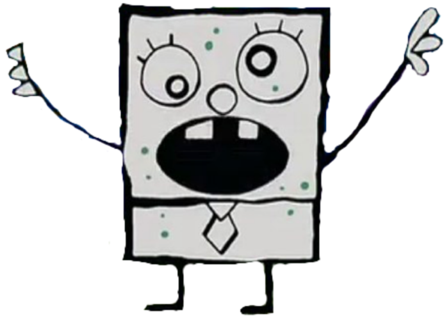
$msg \oplus x \oplus u \oplus x \oplus u$



Certified deletion security: similarity

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- Entanglement in Game 2 corresponds to Bob's measurement in Game 1
  - Measuring everything in the Hadamard basis in Game 1 is like fully entangling A and B in Game 2 – this will give him r_{diag}
  - Measuring everything in the computational basis in Game 1 is like fully entangling A and B' in Game 2, and then measuring B' in the computational basis – this will give him r_{comp}

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- We use one from work by Tomamichel (arXiv: 1203.2142)
- Here, it can be used to describe the information trade-off that Bob is making in Game 2 using smooth min- and max-entropies.
- Takeaway: if the verification test is passed: the information that Bob has access to about r_{comp} is low with high probability

Privacy amplification

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- Formalized using the Leftover Hashing Lemma from Renner
  - Lower bound on Bob's uncertainty about r_{comp} tells us how close x is to a uniformly random string from Bob's perspective
  - Bob is blocked from getting information about msg

Applications and Next Steps

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- Protection against data retention
  - EU regulation 2016/679

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- Protection against data retention
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- Everlasting security
  - Transform long-term computational assumption into a temporary one
- Homomorphic encryption

Thank you!