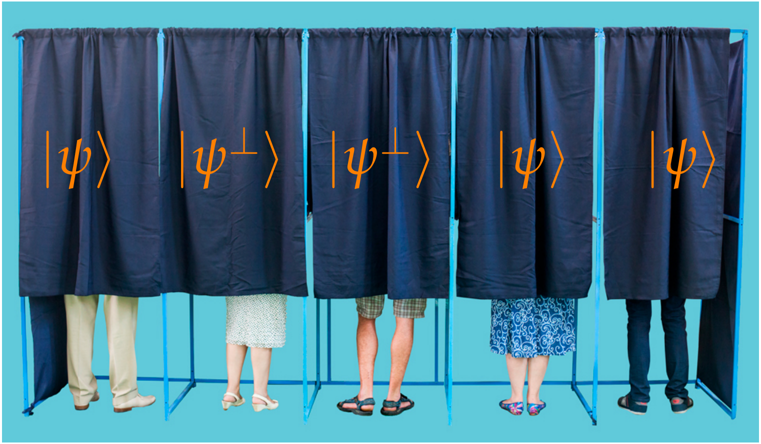
Quantum majority

and other Boolean functions with quantum inputs



Harry Buhrman

Laura Mančinska

Māris Ozols

Noah Linden

Ashley Montanaro

Manifesto

- All information is quantum…

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- …there is no classical information

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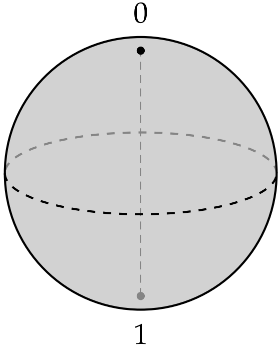

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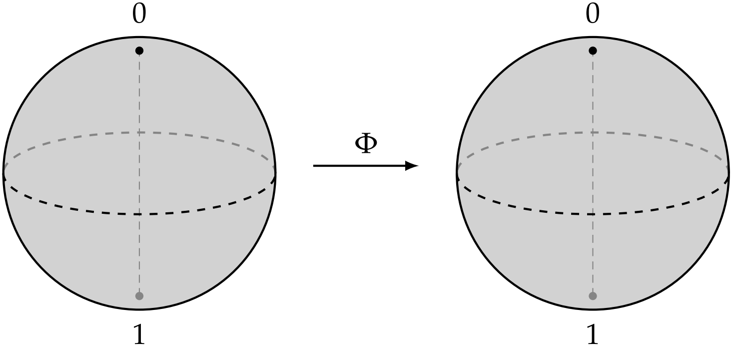
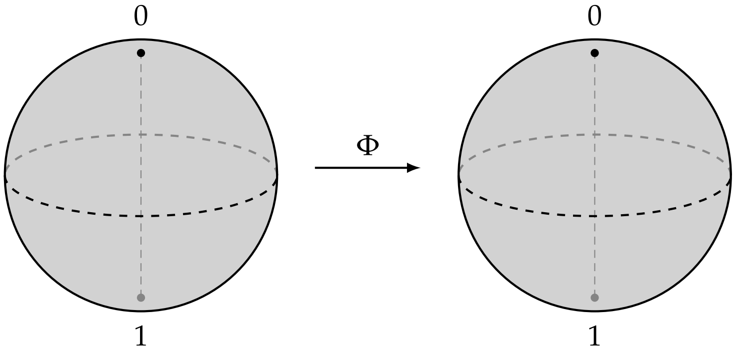
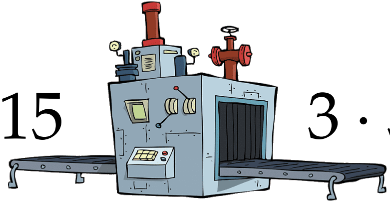


Manifesto

- All information is quantum…
- …there is no classical information
- Algorithm = CPTP map
- Classical computer science is dead!



Manifesto (lite)

15  $3 \cdot 5$

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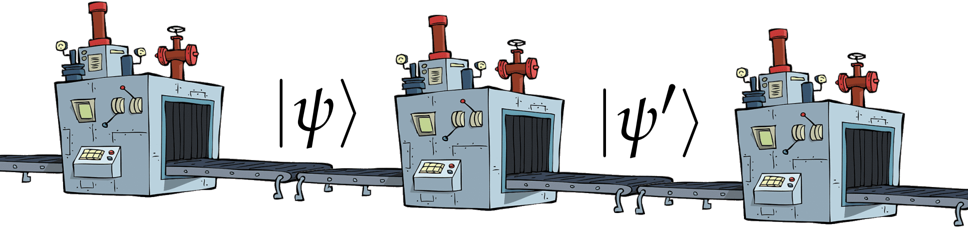

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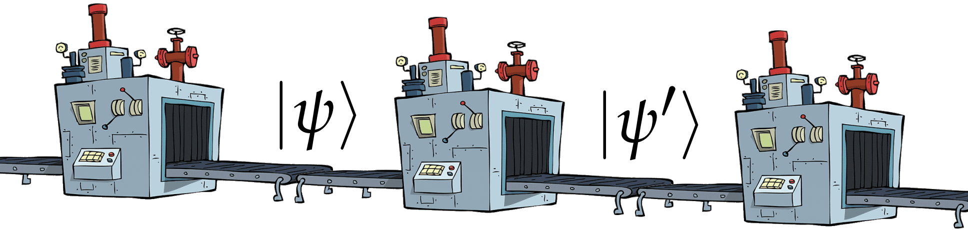
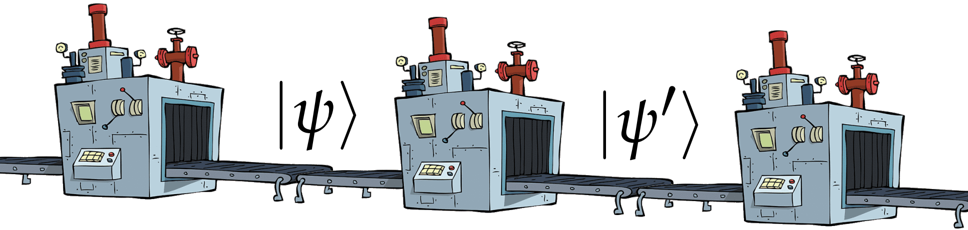


- quantum Fourier transform
- Grover iteration
- swap test
- …

Manifesto (lite)

New *quantum* primitives!



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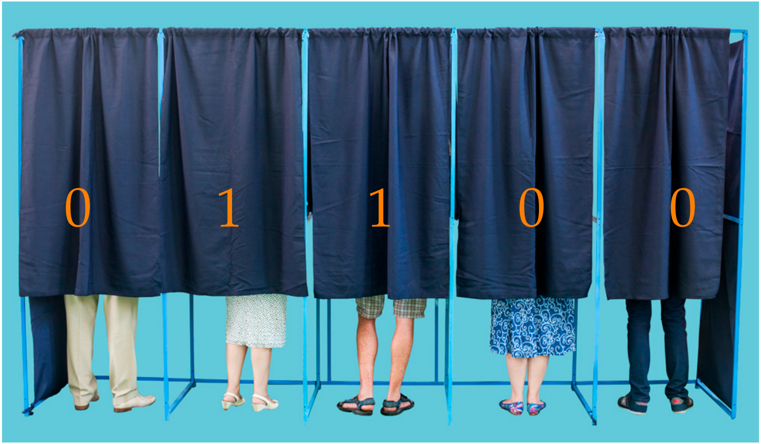
Majority vote



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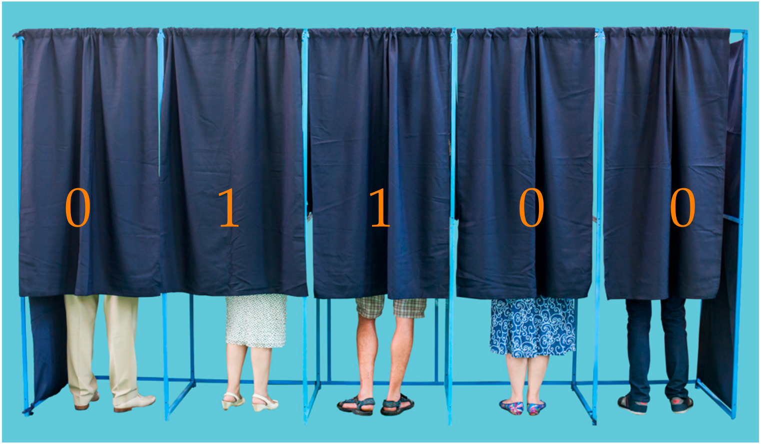
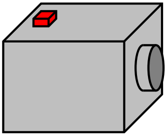


- success amplification
- error correction
- democracy

Quantum majority vote



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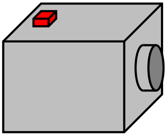
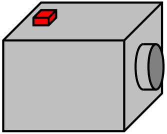


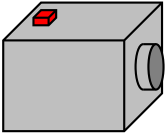
$|\psi\rangle$

Quantum majority vote



$|\psi\rangle|\psi\rangle$

Quantum majority vote



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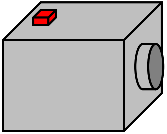
Quantum majority vote



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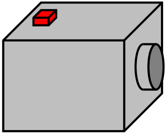

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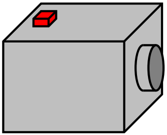

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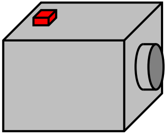

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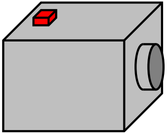
Computation in an unknown basis

Quantum majority vote



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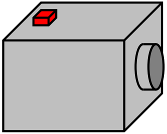

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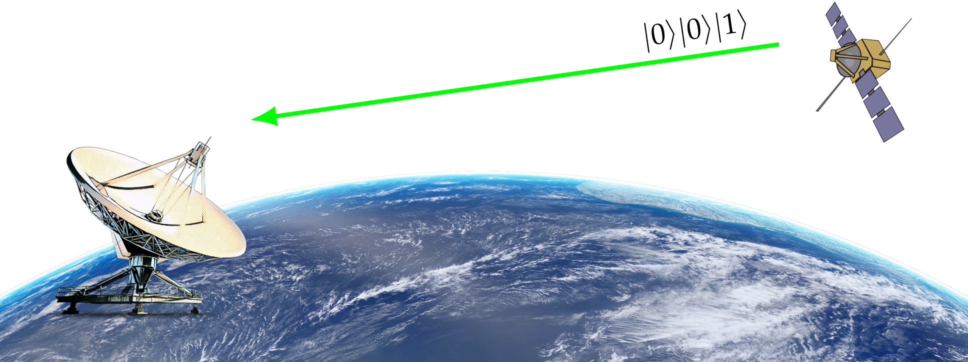

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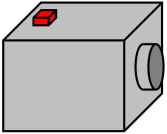

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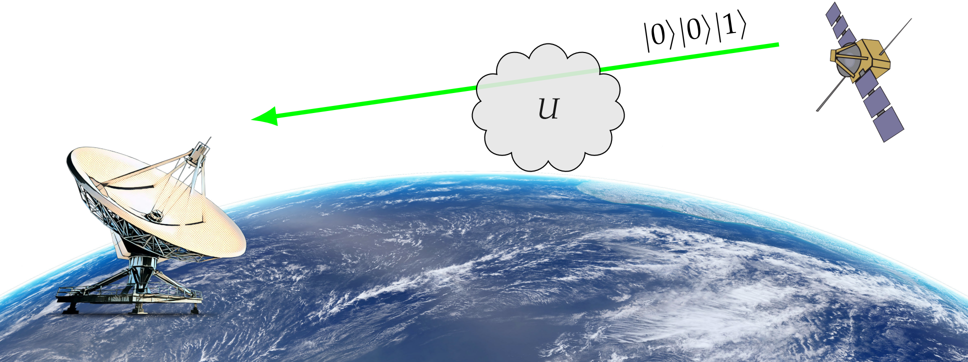

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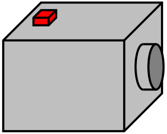

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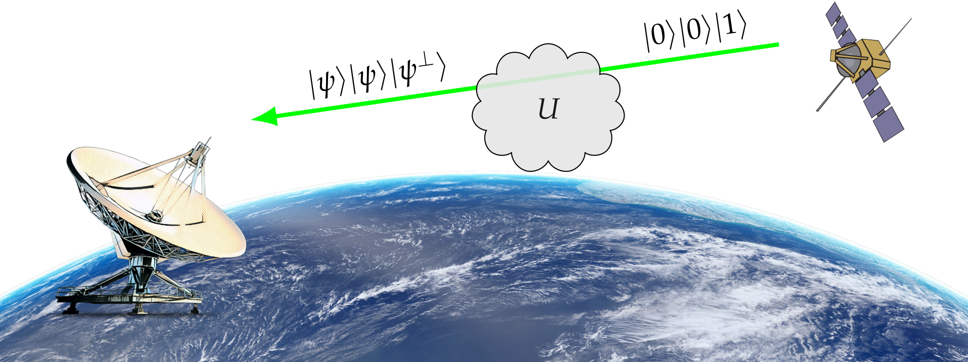

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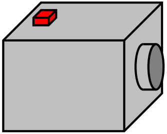

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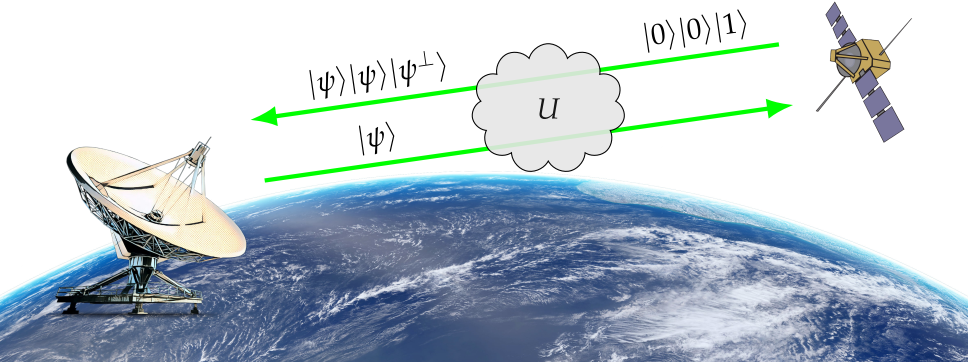

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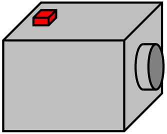

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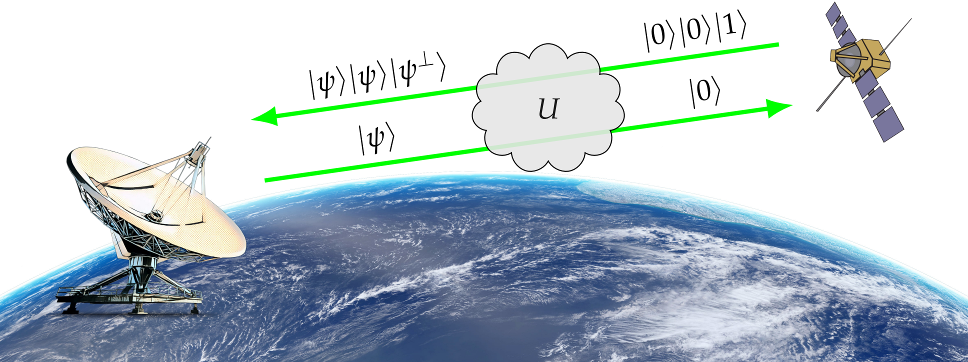

Covariance

Unitary covariance

Computing $f : \{0,1\}^n \to \{0,1\}$ in a *unitary-covariant* way:

$$|x\rangle \mapsto \quad |f(x)\rangle \qquad \forall x \in \{0,1\}^n$$

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Windows

An error has occurred. To continue:

Press Enter to return to Windows, or

Press CTRL+ALT+DEL to restart your computer. If you do this,
you will lose any unsaved information in all open applications.

Error: 0E : 016F : BFF9B3D4

Press any key to continue _

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Covariant functions

$f : \{0,1\}^n \to \{0,1\}$ is *covariant* (or *self-dual*) if

$$f(\overline{x}) = \overline{f(x)}$$

Symmetric functions

Assumption

$f(x)$ depends only on the Hamming weight of $x \in \{0, 1\}^n$

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Example: $n = 3$

$\lvert x \rvert$	0	1	2	3
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Even n is bad

$$f(0,1) = \overline{f(1,0)} = \overline{f(0,1)}$$

Problem

Assumptions

Given $f : \{0,1\}^n \to \{0,1\}$, assume

- f is covariant
- f is symmetric
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Given $U^{\otimes n}|x\rangle$, for an unknown $U \in \mathrm{U}(2)$ and $x \in \{0,1\}^n$, produce ρ that is close to $U|f(x)\rangle$.

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Worst-case fidelity

$$F_f := \max_{\Phi} \min_{x,U} \langle f(x)|U^\dagger \, \Phi\Big(U^{\otimes n}|x\rangle\langle x|U^{\dagger\otimes n}\Big) \, U|f(x)\rangle$$

Results on majority

Trivial strategy: Output any qubit at random

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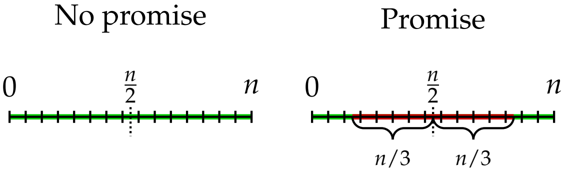

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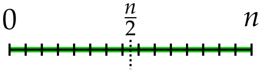
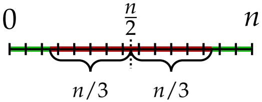
	No promise	Promise
		
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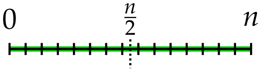
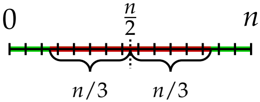
	No promise 	Promise 
Trivial	$\frac{1}{2} + \frac{1}{2n}$	$\frac{5}{6}$
Optimal	$\frac{1}{2} + \Theta\left(\frac{1}{\sqrt{n}}\right)$	$1 - \Theta\left(\frac{1}{n}\right)$

Symmetric / anti-symmetric subspace of $\mathbb{C}^2 \otimes \mathbb{C}^2$

Symmetric states:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{\otimes 2} |00\rangle = \begin{pmatrix} a^2 \\ ac \\ ca \\ c^2 \end{pmatrix} = a^2 \; |00\rangle \; + \sqrt{2}ac \; \frac{|01\rangle + |10\rangle}{\sqrt{2}} \; + c^2 \; |11\rangle$$

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Schur transform on two qubits

$$U_{\mathrm{Sch}} := \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$U_{\mathrm{Sch}}\, \mathrm{SWAP}\, U_{\mathrm{Sch}}^{\dagger} = \mathrm{diag}(-1, 1, 1, 1)$$

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Each block defines a homomorphism: $Q(MN) = Q(M)Q(N)$

Schur transform on n qubits

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Can be implemented with $O(n^4 \log n)$ gates
(Kirby & Strauch, 2017)

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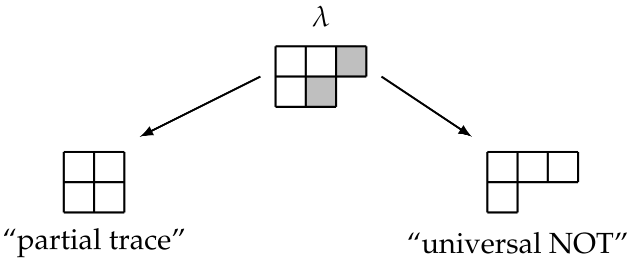
Extremal covariant channels

For each $\lambda \vdash n$, only two *extremal* covariant channels from $\mathbb{C}^{m_\lambda} \to \mathbb{C}^2$:



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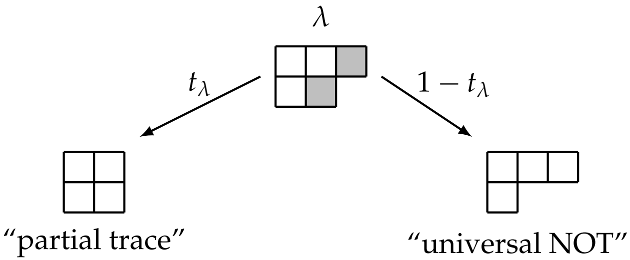

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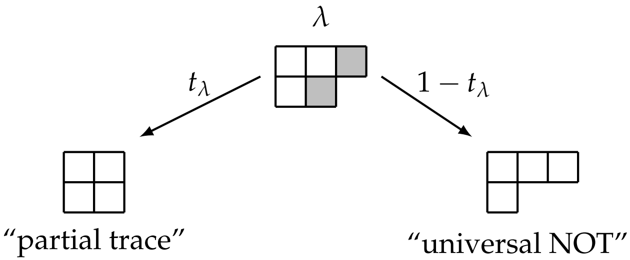

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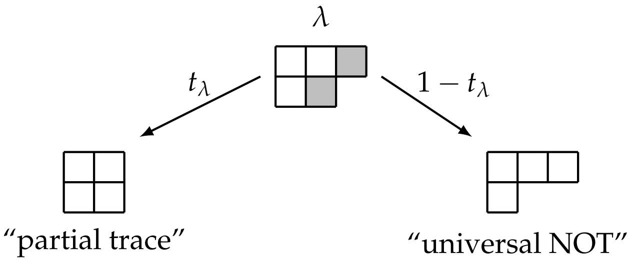

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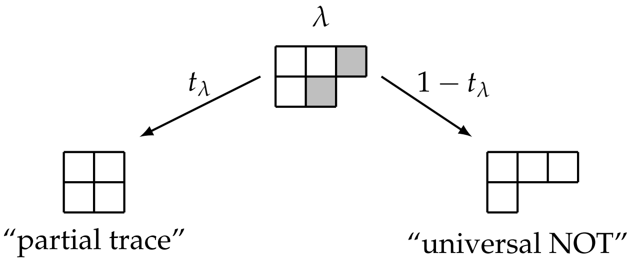

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- $O(\ell \log \ell)$ gates

Main result

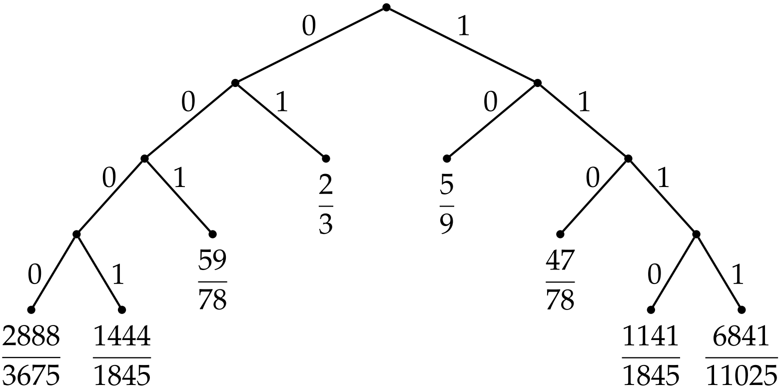
Theorem

For any symmetric and covariant n-bit boolean function

- the optimal parameters t_λ and the resulting fidelity can be determined by a linear program of size $n/2$
- optimal algorithm with $O(n^4 \log n)$ gates

Fidelities of all 7-argument functions

$\lvert x \rvert$	0	1	2	3	4	5	6	7
$f(x)$	0	1	0	0	1	1	0	1



Fidelity depends only on the gap around $n/2$ in the truth table

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- Applications!

Thank you!