Error mitigation with Clifford quantum-circuit data

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QIP 2021

LA-UR-21-20721
• Quantum advantage and error mitigation.


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Unified approach to data-driven quantum error mitigation

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Motivation

- Fault-tolerant quantum computing is not possible with near-term (NISQ) quantum computers.
- Error mitigation methods reduce noise effects without performing full error correction.
- Error mitigation is expected to be necessary for quantum advantage with NISQ devices.
- Error mitigation for quantum advantage circuits is challenging.
Clifford Data Regression (CDR)

- $X$ - an observable of interest, $|\psi\rangle$ - a quantum circuit of interest.
- $X^\text{exact}_\psi = \langle \psi | X | \psi \rangle$, $X^\text{noisy}_\psi$ - the noisy expectation value.

1. Choose near-Clifford classically simulable training circuits $S_\psi = \{|\phi_i\rangle\}$. 

**GENERATE TRAINING DATA**

- $\{X^\text{noisy}_i\}$
- $\{X^\text{exact}_i\}$
Clifford Data Regression (CDR)

- $X$ - an observable of interest, $|\psi\rangle$ - a quantum circuit of interest.
- $X^\text{exact}_\psi = \langle \psi | X | \psi \rangle$, $X^\text{noisy}_\psi$ - the noisy expectation value.

1. Choose near-Clifford classically simulable training circuits $S_\psi = \{|\phi_i\rangle\}$.
2. Construct a training set $T_\psi = \{X^\text{noisy}_{\phi_i}, X^\text{exact}_{\phi_i}\}$.
Clifford Data Regression (CDR)

- $X$ - an observable of interest, $|\psi\rangle$ - a quantum circuit of interest.
- $X^\text{exact}_\psi = \langle\psi | X | \psi\rangle$, $X^\text{noisy}_\psi$ - the noisy expectation value.

1. Choose near-Clifford classically simulable training circuits $S_\psi = \{|\phi_i\rangle\}$.
2. Construct a training set $T_\psi = \{X^\text{noisy}_\phi, X^\text{exact}_\phi\}$.
3. Learn a model for $X^\text{exact}_\phi$:
   \[
   X^\text{exact}_\phi = a_1 X^\text{noisy}_\phi + a_2,
   \]
   \[
   \arg\min_{a_1, a_2} \sum_{\phi_i \in T_\psi} (X^\text{exact}_\phi - a_1 X^\text{noisy}_\phi - a_2)^2.
   \]
Clifford Data Regression (CDR)

- $X$ - an observable of interest, $|\psi\rangle$ - a quantum circuit of interest.
- $X_{\psi}^{\text{exact}} = \langle \psi | X | \psi \rangle$, $X_{\psi}^{\text{noisy}}$ - the noisy expectation value.

1. Choose near-Clifford classically simulable training circuits $S_{\psi} = \{|\phi_i\rangle\}$.
2. Construct a training set $T_{\psi} = \{X_{\phi_i}^{\text{noisy}}, X_{\phi_i}^{\text{exact}}\}$.
3. Learn a model for $X^{\text{exact}}$:
   
   $$X^{\text{exact}} = a_1 X^{\text{noisy}} + a_2,$$
   
   $$\arg\min_{a_1, a_2} \sum_{\phi_i \in T_{\psi}} (X_{\phi_i}^{\text{exact}} - a_1 X_{\phi_i}^{\text{noisy}} - a_2)^2.$$
4. Use the model to correct $X_{\psi}^{\text{noisy}}$. 

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Error mitigation with Clifford quantum-circuit data
Near-Clifford circuits with up to $N \approx 50$ non-Clifford gates can be simulated classically.

Replace non-Clifford gates by Clifford gates.

An algorithm for IBM:

\[ R_Z(\alpha) \rightarrow S^n, \quad S = R_Z(\pi/2), \quad n = 0, 1, 2, 3. \]
QAOA for the quantum Ising model

- The problem: Ground state simulation of a transverse-field 1D quantum Ising model \( (g = 2) \).

\[
H = -g \sum_j \sigma_j^x - \sum_{\langle j, j' \rangle} \sigma_j^z \sigma_{j'}^z
\]

- The QAOA (Quantum Alternating Operator Ansatz) ansatz:

\[
H = H_1 + H_2, \quad H_2 = -g \sum_j \sigma_j^x, \quad H_1 = - \sum_{\langle j, j' \rangle} \sigma_j^z \sigma_{j'}^z,
\]

\[
|\psi(\beta_1, \gamma_1, \ldots, \beta_p, \gamma_p)\rangle = \prod_{j=p, p-1 \ldots, 1} e^{i\beta_j H_2} e^{i\gamma_j H_1} |+\rangle^\otimes Q, \quad |+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle).
\]

Local minima of the optimization are simulated with IBM’s Almaden quantum computer.

A factor of 15 improvement obtained.
At least a factor of 10 improvement obtained.
\[ G^\alpha = \alpha G^{\text{noisy}} + (1 - \alpha) G^{\text{exact}} \quad \alpha = \frac{4}{p} \]

\[ Q = 8, N = 28 \]

At least a factor of 10 improvement up to \( p = 24 \).
Choose training circuits as in the case of CDR.

Multiply noise level $j = 1, \ldots, n$ as in the case of Zero Noise Extrapolation (ZNE).

Construct a training set $\mathcal{T}_\psi = \{X_{\phi_i}^{\text{noisy},j}, X_{\phi_i}^{\text{exact}}\}$.

Learn a model for $X^{\text{exact}}$:

$$X^{\text{exact}} = \sum_j a_j X_{\psi}^{\text{noisy},j}.$$ 

Use the model to correct $X_{\psi}^{\text{noisy},j}$. 

Error mitigation with Clifford quantum-circuit data
A factor of 33 improvement.
Random quantum circuits mitigation

\[ U(\theta, \phi, \lambda) = R_Z(\phi + \pi)PR_Z(\theta + \pi)PR_Z(\lambda) \]

Systematic improvement over ZNE.

\[ p = 4, N = 20 \]

\[ Q = 8, N = 20 \]
Conclusions

- Error mitigation methods based on learning the noise effects from classically simulable near-Clifford circuits.
- vnCDR unifies CDR and ZNE.
- Good scaling with number of qubits.
- A factor of 10 improvement for the 16-qubit hardware QAOA implementation.
- A factor of 10 improvement for the 64-qubit QAOA implementation with realistic noise.
- Systematic improvement over ZNE.