The ghost in the radiation: Robust encodings of the black hole interior

Isaac Kim, **Eugene Tang**, John Preskill

Quantum Information Processing 2021 - Munich

February 4, 2021

arXiv: 2003.05451

The Entanglement Structure of Hawking Radiation

Consider a simple model for black hole evaporation based on the pair creation picture for Hawking radiation:

1. We begin with a pure state $|\psi\rangle_M$ representing the state of the black hole on an initial time slice.
2. Hawking radiation is then induced from pair creation.

   The schematic evolution of our state is given by

$$|\psi\rangle_M \mapsto |\psi'\rangle_M \otimes \frac{1}{\sqrt{2}} \left(|00\rangle_{\tilde{B}_1 B_1} + |11\rangle_{\tilde{B}_1 B_1} \right). \tag{1}$$

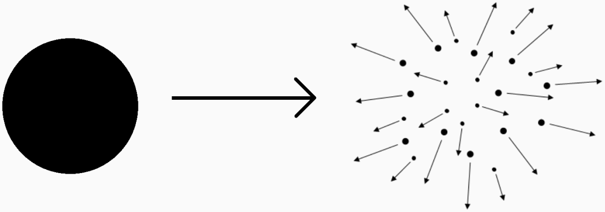
3. Continual time evolution repeatedly produce new correlated pairs, with the state at step N being

$$|\Psi_N\rangle = |\psi''\rangle_M \otimes \frac{1}{\sqrt{2^N}} \bigotimes_{k=1}^{N} \left(|00\rangle_{\tilde{B}_k B_k} + |11\rangle_{\tilde{B}_k B_k} \right). \tag{2}$$

The Black Hole Firewall Problem

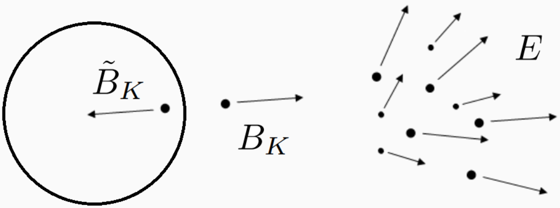
If we trace away the interior modes $\tilde{B}_k$, then the exterior modes B_k are left in a maximally mixed state: the Hawking radiation is *thermal*.

If evaporation continues until the black hole has completely radiated away, then we are left with a mixed thermal state, violating the unitarity of quantum mechanics.



We can sharpen the above argument using entanglement entropy.

The Black Hole Firewall Problem



Consider some late outgoing mode B_K, and denote the collection of early modes $\{B_k \mid k < K\}$ as E.

From the toy-model above, B_K must be (maximally) entangled with its partner mode $\tilde{B}_K$ in the interior:

$$S(B_K \tilde{B}_K) = 0. \tag{3}$$

If the final state of evaporation is to remain unitary, then B_K must purify the early radiation:

$$S(B_K E) < S(E). \tag{4}$$

The Black Hole Firewall Problem

We have the conditions:

$$S(ABC) + S(B) \leq S(AB) + S(BC), \tag{5}$$

$$S(B_K \tilde{B}_K) = 0, \tag{6}$$

$$S(B_K E) < S(E). \tag{7}$$

Collectively, these conditions are contradictory:

$$\begin{aligned} S(B_K) + S(E) &= S(B_K) \; + \; S(EB_K \tilde{B}_K) & (9) \\ &\leq S(B_K \tilde{B}_K) + S(B_K E) & (10) \\ &= 0 + S(B_K E) & (11) \\ &< S(E) & (12) \end{aligned}$$

This formulation of the black hole information problem is called the *firewall paradox*.

The Black Hole Firewall Problem

There are several proposed solutions of the firewall paradox, each with its own merits and flaws. We will focus on the $\tilde{B} \subseteq E$ proposal.

Problem: An outgoing mode B must be entangled with both an interior mode $\tilde{B}$ *and* the early radiation E. Violation of monogamy.

Solution: Identify the two problematic subsystems. Embed the interior partner mode $\tilde{B}$ *within* the exterior radiation E.

Appears to cause just as many problems as it solves:

- If the interior is actually embedded within the exterior, how can such an embedding respect the causal structure of the black hole?
- How do we protect the interior from outside observers?
- How can such an embedding be realized in practice?

Black Holes as Quantum Error-Correcting Codes

1. We postulate that black holes are efficient “scramblers” in a very precise way, namely that the exterior Hawking radiation emitted by a black hole is a *computationally pseudorandom* state.

2. Through the pseudorandomness hypothesis, we show that black holes define a natural encoding of each interior mode into the exterior Hawking radiation.

3. Such an encoding forms an error-correcting code which protects against all operations with sufficiently small complexity.

A sufficiently powerful observer can detect violations of causality and locality, but only provided that they are able to perform operations which are of exponential complexity in the entropy of the remaining black hole.

Pseudorandomness

A quantum state ρ is said to be *computationally pseudorandom* if for all polynomial time algorithms $\mathcal{A}$, we have

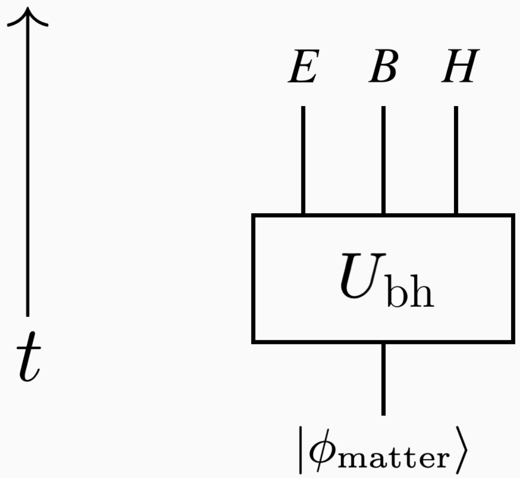
$$\left| \Pr[\mathcal{A}(\rho) = 1] - \Pr[\mathcal{A}(\sigma) = 1] \right| = \text{error}. \tag{13}$$

Not clear if pseudorandom states even exists. Assuming standard cryptographic primitives, it can be shown that there exists efficiently computable pseudorandom quantum states [1711.00385].

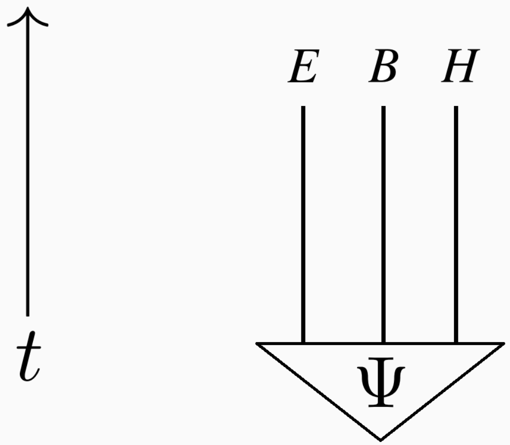
Harlow and Hayden [1301.4504] argued that any experiment witnessing firewalls must be exponentially complex.

Exponentially complex operations are unphysical. A physically theory should come with restrictions not just to low energies, *but also low complexities*.

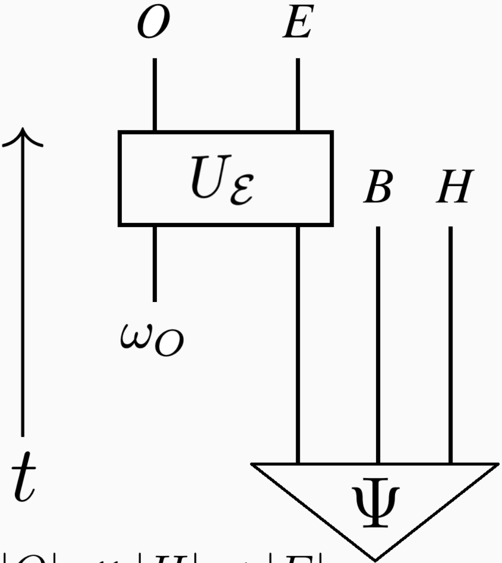
Hawking radiation appears thermal *to a computationally bounded observer*.


E B H
U_bh
|φ_matter⟩
t

Setup



Setup



$$|O| \ll |H| < |E|$$

Let $|\Psi\rangle_{EBH}$ denote the state of the black hole and the exterior radiation. Let $\sigma_{EB} = I_{EB}/d_{EB}$ be the maximally mixed state of EB, and let $\rho_{EB} = \mathrm{Tr}_H(|\Psi\rangle\langle\Psi|)$.

Definition: We say that the state $|\Psi\rangle_{EBH}$ is *pseudorandom* on the radiation EB if there exists some constant $\alpha > 0$ such that

$$\left| \Pr[\mathcal{M}(\rho_{EB}) = 1] - \Pr[\mathcal{M}(\sigma_{EB}) = 1] \right| \leq 2^{-\alpha|H|}, \tag{14}$$

for any two-outcome measurement $\mathcal{M}$ with quantum complexity polynomial in $|H|$, the entropy of the remaining black hole.

The pseudorandomness hypothesis can be seen as an axiomization of the thermality of Hawking radiation as restricted to low-complexity observers.

Implications of Pseudorandomness

We show the following:

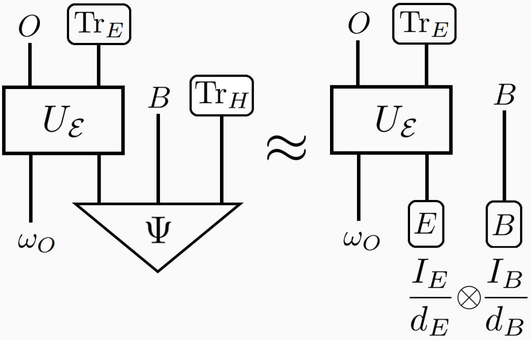
1. There exists an encoding $V : \mathcal{H}_{\tilde{B}} \to \mathcal{H}_{EH}$ of each interior mode $\tilde{B}$ into the Hilbert space of the exterior radiation.
2. The encoding V defines a quantum error-correcting code, which we call the black hole code.
3. The black hole code protects against all operations performed by a computationally bounded observer. Specifically, the code corrects against all channels of sufficiently small complexity and Kraus rank.
4. Moreover, there exists a complete set of logical operators, the *ghost operators*, which commutes with all correctable errors of the code. The ghost operators serve as witness to the smoothness of the horizon and the preservation of causality.

The Decoupling Bound

The pseudorandomness hypothesis immediately allows us to prove a key decoupling bound.

Theorem (Decoupling Bound): Let ρ_{OEBH} denote any state obtained from $\omega_O \otimes |\Psi\rangle_{EBH}$ by acting with some polynomial-size circuit $U_{\mathcal{E}}$. Then

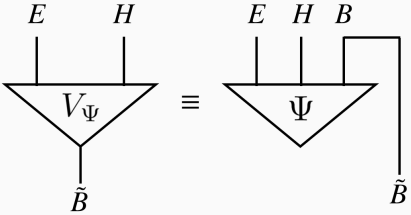
$$\|\rho_{OB} - \rho_O \otimes \rho_B\|_1 \leq 6 \cdot 2^{-(\alpha|H|-|O|)}. \qquad (19)$$



The Black Hole Encoding

The decoupling bound allows us to define a natural code subspace associated with the black hole. Letting $|\Psi\rangle_{EBH}$ denote the state of the black hole and radiation, we define an operator $V_\Psi : \mathcal{H}_{\tilde{B}} \to \mathcal{H}_{EH}$ by

$$V_\Psi |i\rangle_{\tilde{B}} = 2 \left(I_{EH} \otimes \langle \omega|_{B\tilde{B}} \right) \left(|\Psi\rangle_{EHB} \otimes |i\rangle_{\tilde{B}} \right). \tag{20}$$



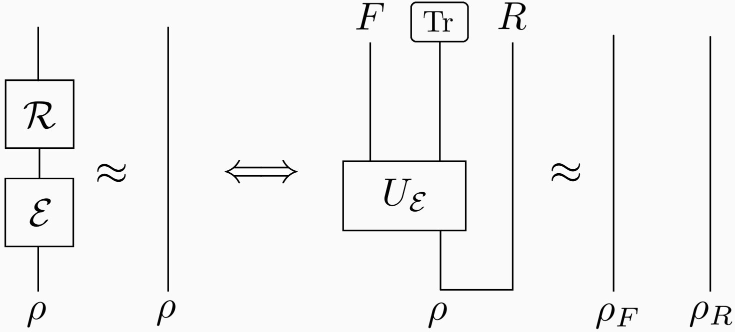
One can use the decoupling bound to show that V_Ψ is ϵ-close to an isometric embedding, where $\epsilon = 2 \cdot 2^{-\alpha|H|}$.

The code subspace defined by V_Ψ will be called the *black hole encoding*, which defines the desired embedding of the partnered interior modes $\tilde{B}$ into the exterior radiation (more precisely, into EH).

The Information-Disturbance Tradeoff

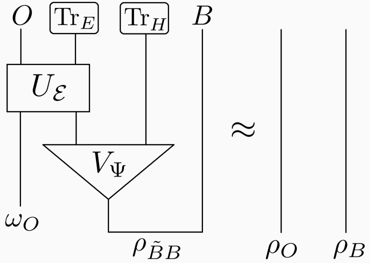
Having identified a suitable codespace, we now characterize its error-correcting capabilities.

The *information-disturbance relation* says that a code can protect quantum information from noise if and only if the environment of the noise channel $\mathcal{E}$ learns nothing about the logical information.



The Black Hole Error-Correcting Code

Compare the information-disturbance tradeoff to the decoupling condition:



This shows that the black hole code corrects against all channels of polynomial complexity in $|H|$ which have sufficiently low-rank.

Theorem: Let $\mathcal{E}$ be an error channel on E with purification $U_{\mathcal{E}}$. Suppose that the decoupling bound holds. Then $\mathcal{E}$ is ϵ-correctable for V_Ψ, where

$$\epsilon = \sqrt{\frac{3}{2}} \cdot 2^{-(\alpha|H|-|O|)/2}. \tag{22}$$

Ghost Logical Operators

We can recast the error-correcting conditions of the black hole code into an algebraic form.

Let $T_{\tilde{B}}$ be an operator which acts on the interior mode $\tilde{B}$. By microcausality, any operator $\mathcal{O}$ that an exterior agent can apply onto the early radiation should commute with $T_{\tilde{B}}$, i.e.,

$$[\mathcal{O}, T_{\tilde{B}}] = 0. \tag{23}$$

The corresponding logical operator T_{EH} on the code subspace should satisfy the same relation when restricted to the code subspace:

$$[\mathcal{O}, T_{EH}]|\Psi\rangle_{EBH} = 0. \tag{24}$$

We will call operators which satisfy such a relation *ghost logical operators*.

Ghost Logical Operators

Theorem: Let T_B be any operator acting on an outgoing mode B. Then there exists a corresponding logical operator T_{EH} such that

$$T_B|\Psi\rangle_{EBH} \approx T_{EH}|\Psi\rangle_{EBH}, \tag{25}$$

$$[E_a, T_{EH}]|\Psi\rangle_{EBH} \approx 0, \tag{26}$$

where $\{E_a\}$ is some set of operators that a computationally bounded external observer can apply on the radiation (such that the decoupling bound holds).

The first relation serves as a witness to the entanglement between the outgoing mode and the encoded interior mode, as required for the existence of a smooth horizon.

The second relation serves as a witness to microcausality between the encoded interior and an external observer, as required to maintain the correct causal structure.

Conclusion

- We find that the $\tilde{B} \subseteq E$ proposal for the firewall problem can be given reasonable meaning within the context of quantum information theory and quantum complexity.
- Under a reasonable pseudorandomness hypothesis, black holes form natural error-correcting codes protecting against low-complexity operations.
- Error-correcting properties of black holes lead to the existence of ghost operators, which certify the preservation of geometry and causality.
- Operations with large complexity or high rank can see violations of causality and create firewalls. Effective field theory should be restricted to low complexity, not simply low energy.

Epilogue

Many questions remain:

- Is there a more robust relationship between our ghost operators and the entanglement islands proposal? To the Python's Lunch?
- More precise relation to Papadodimas and Raju's mirror operators?
- Can we make use of holography in our construction?
- Is there anything more we can say about the complexity of the Ads/CFT dictionary, or the quantum Church-Turing thesis?
- What exactly happens at the end stages of black hole evaporation, when the black hole is no-longer macroscopic?
- What is the role of state dependence?
- Do the ghost logical operators we construct have practical uses for quantum information theory in general?