Interactive quantum advantage with noisy, shallow Clifford circuits

Nathan Ju (University of Illinois)

joint work with

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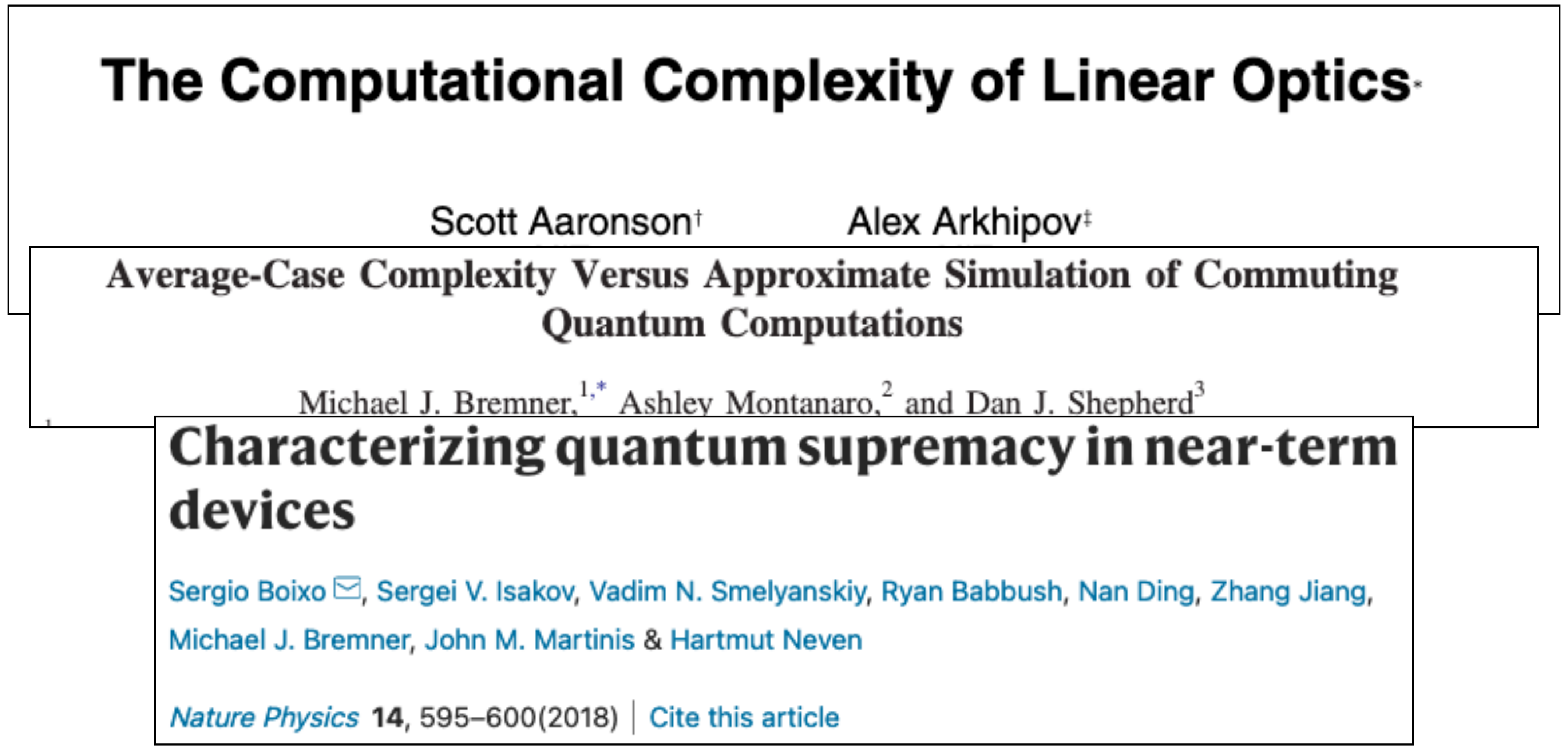
IQC Institute for Quantum Computing

Motivation: Quantum advantage?

Near-term, noisy quantum computers solve sampling tasks that are classically intractable, **assuming some conjectures**


The Computational Complexity of Linear Optics

Scott Aaronson† Alex Arkhipov‡

Average-Case Complexity Versus Approximate Simulation of Commuting Quantum Computations

Michael J. Bremner,[1,*] Ashley Montanaro,[2] and Dan J. Shepherd[3]

Characterizing quantum supremacy in near-term devices

Sergio Boixo, Sergei V. Isakov, Vadim N. Smelyanskiy, Ryan Babbush, Nan Ding, Zhang Jiang, Michael J. Bremner, John M. Martinis & Hartmut Neven

Nature Physics **14**, 595–600(2018) | Cite this article

Compare noisy and shallow quantum computers against <u>shallow/weak</u> classical computers instead, **with fewer or no conjectures**?

Noisy quantum advantage against weak circuits

Improving on this breakthrough result from 2018…

Quantum advantage with shallow circuits

Sergey Bravyi[1], David Gosset[1,*], Robert König[2,†]

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Science 19 Oct 2018:
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Noisy quantum advantage against weak circuits

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A classical probabilistic circuit with bounded fan-in gates and constant-depth (NC^0) solves the task with probability at most $9/10$ over a uniform input.

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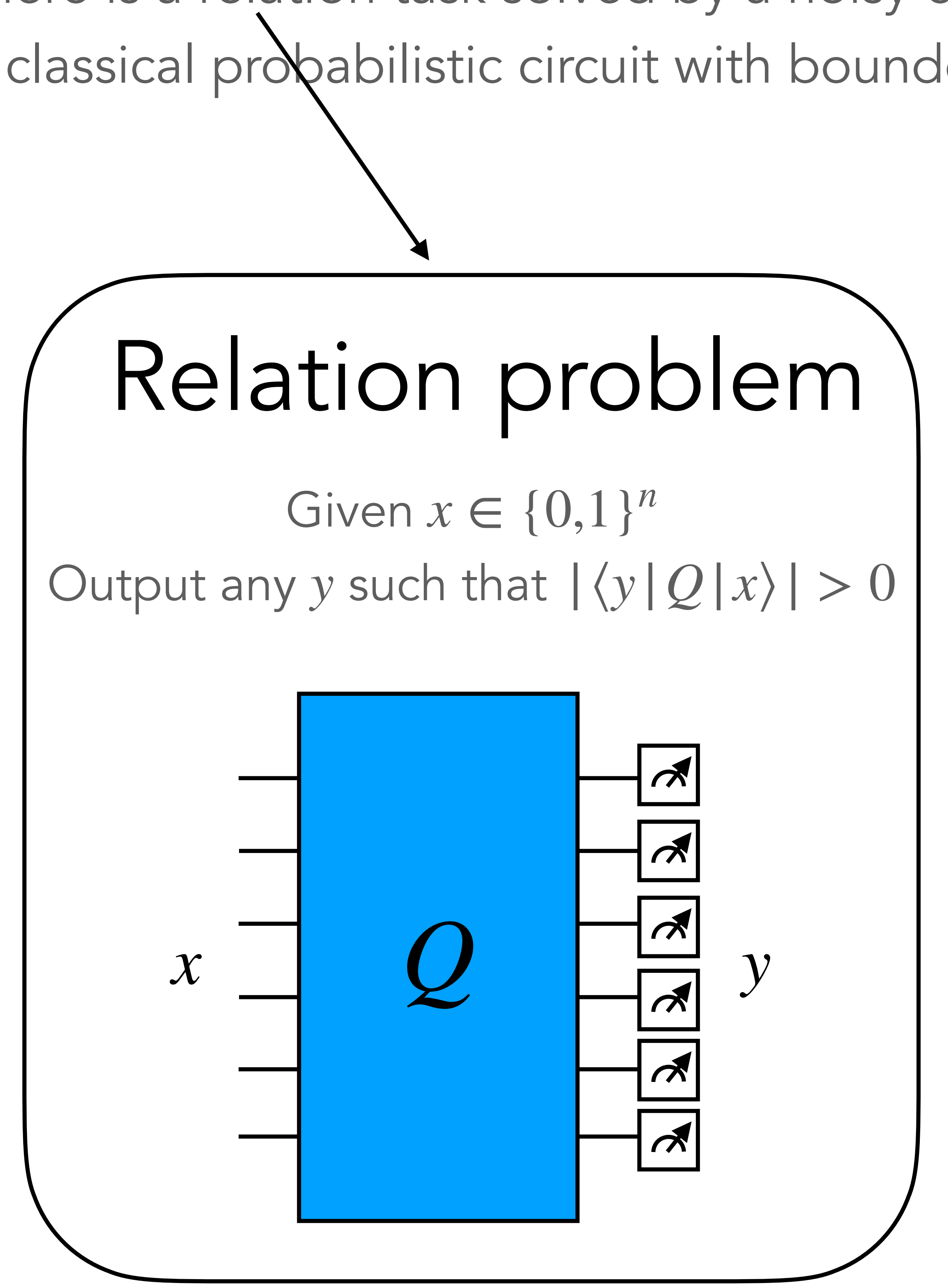

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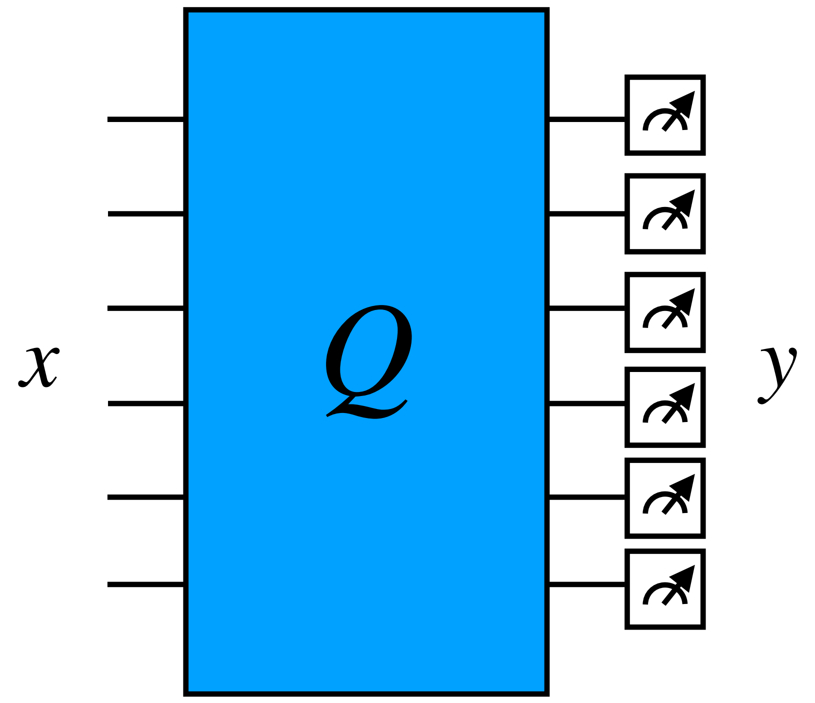
Relation problem

Given $x \in \{0,1\}^n$

Output any y such that $|\langle y|Q|x\rangle| > 0$

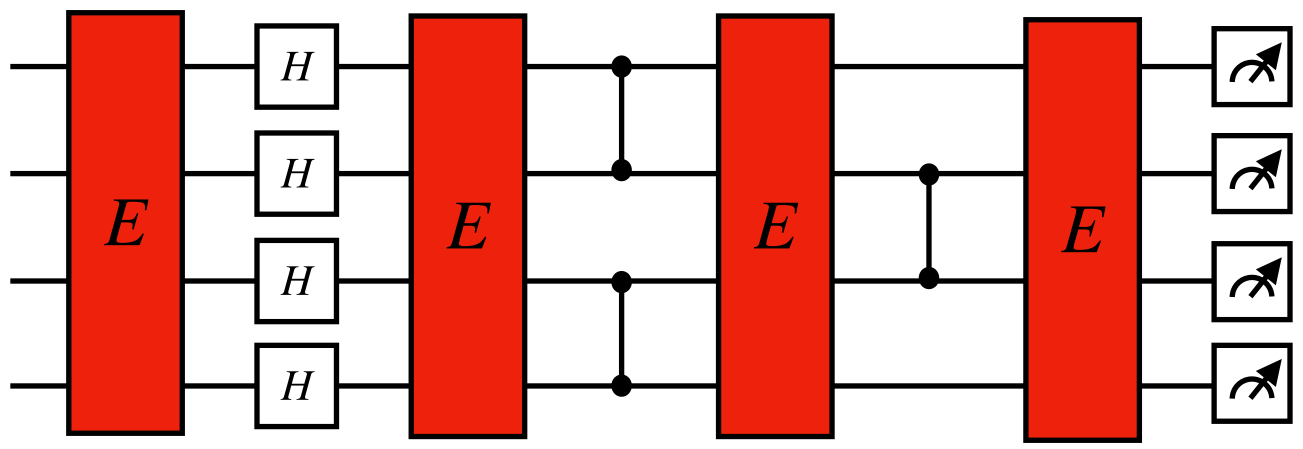


Local stochastic noise model

[Aliferis, Gottesman, Preskill 2007]

Random n-qubit Pauli E is *local stochastic with noise rate* p

if it acts non-trivially on qubits $F \subseteq [n]$ with probability $\leq p^{|F|}$



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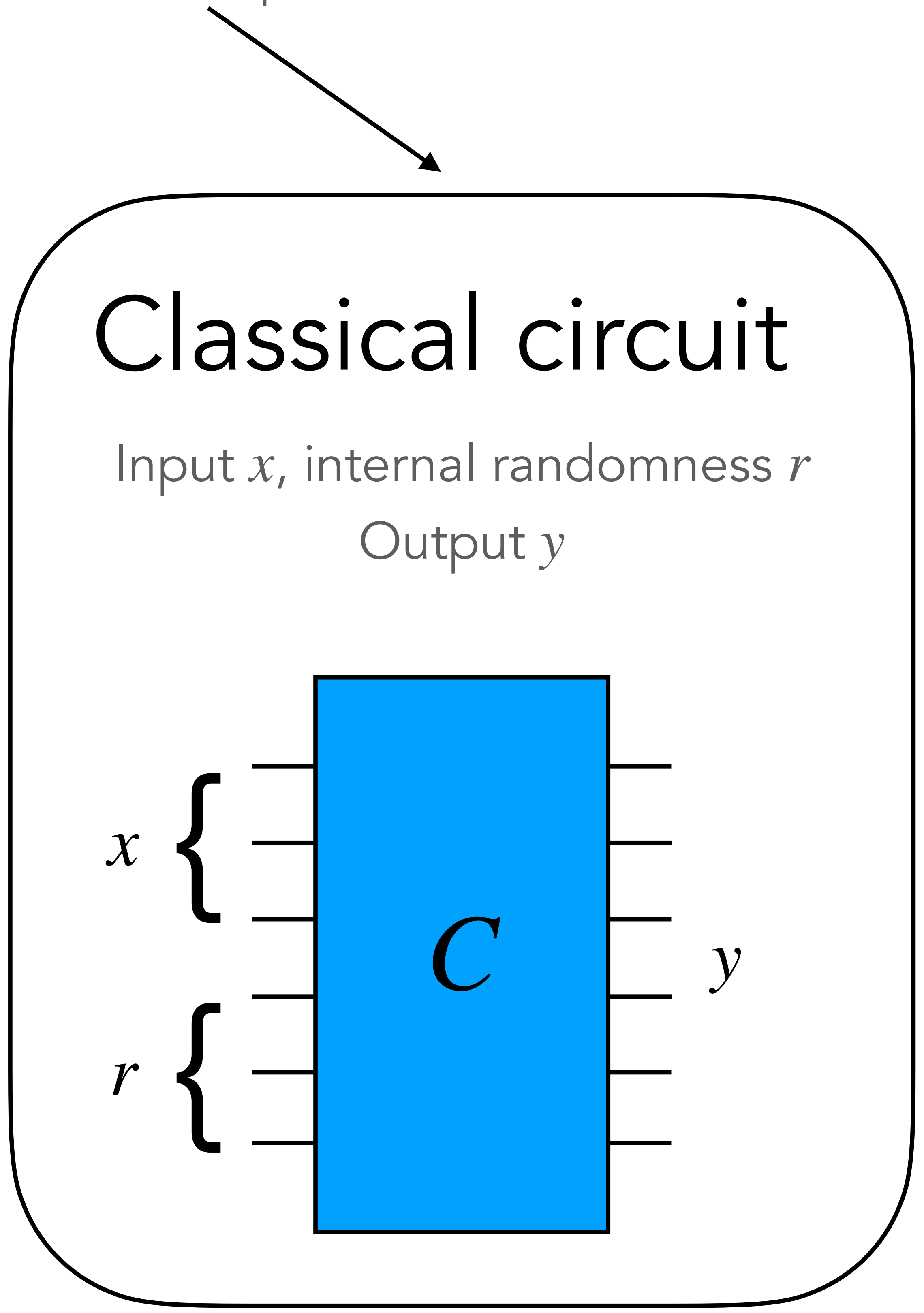

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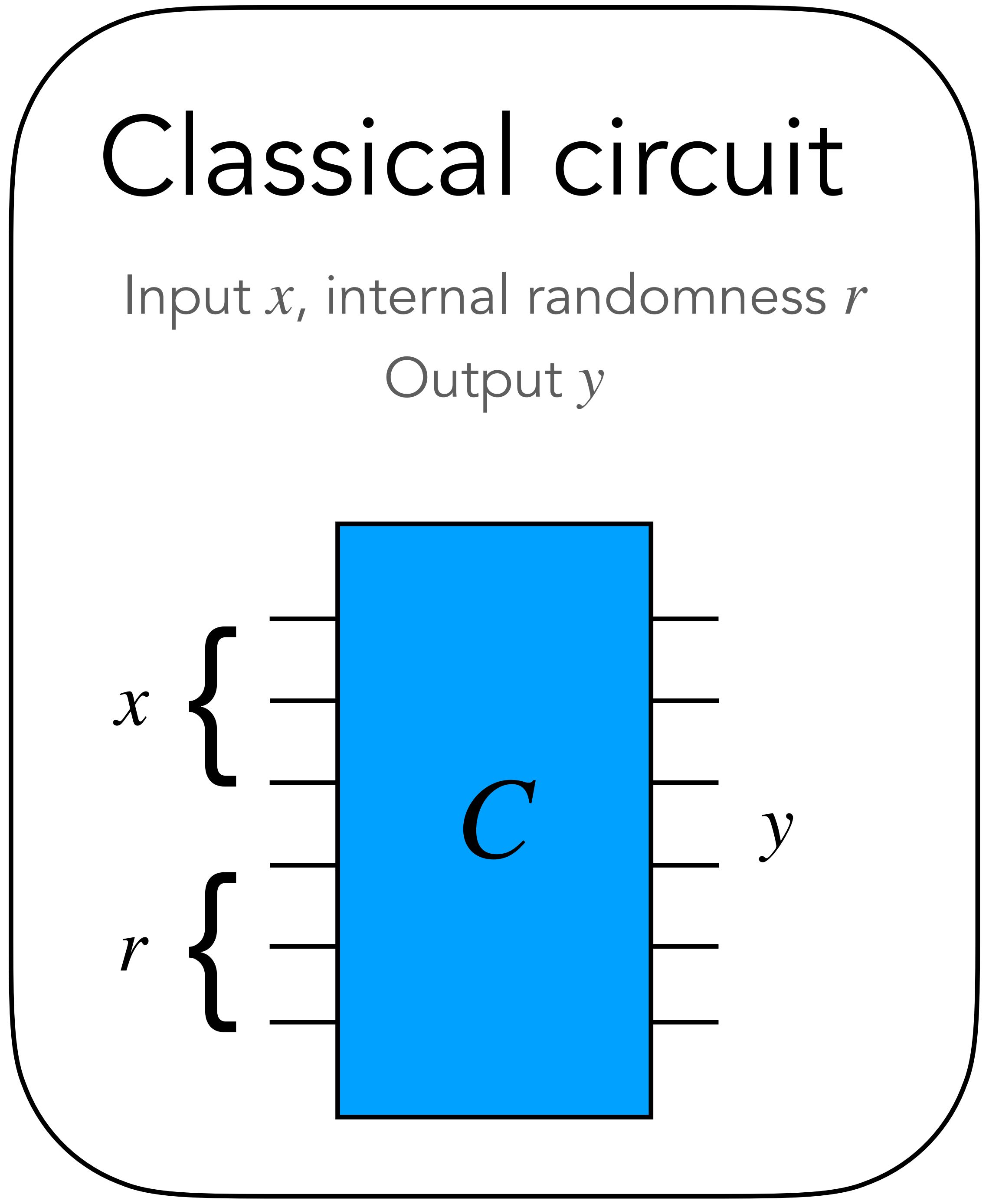

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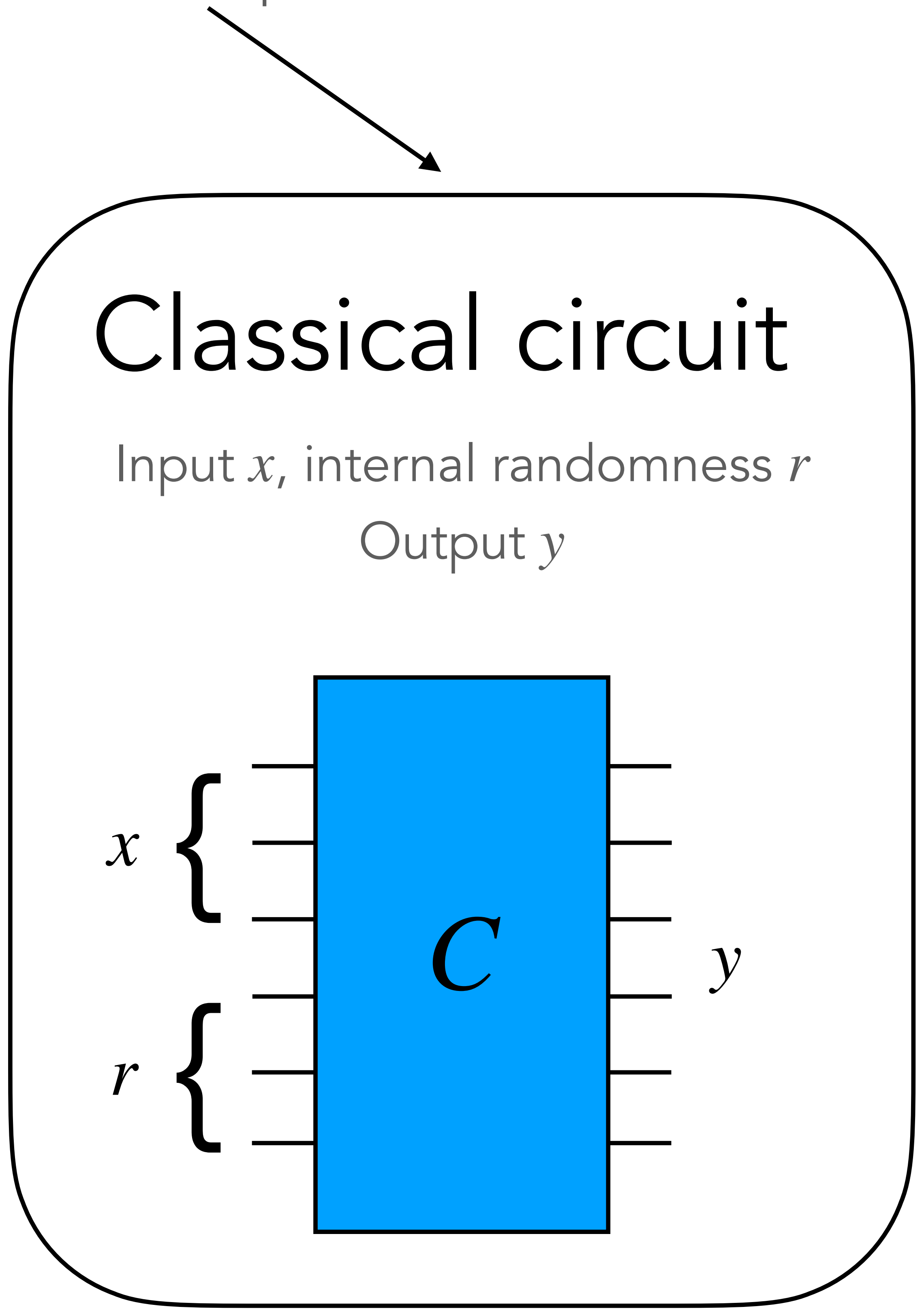

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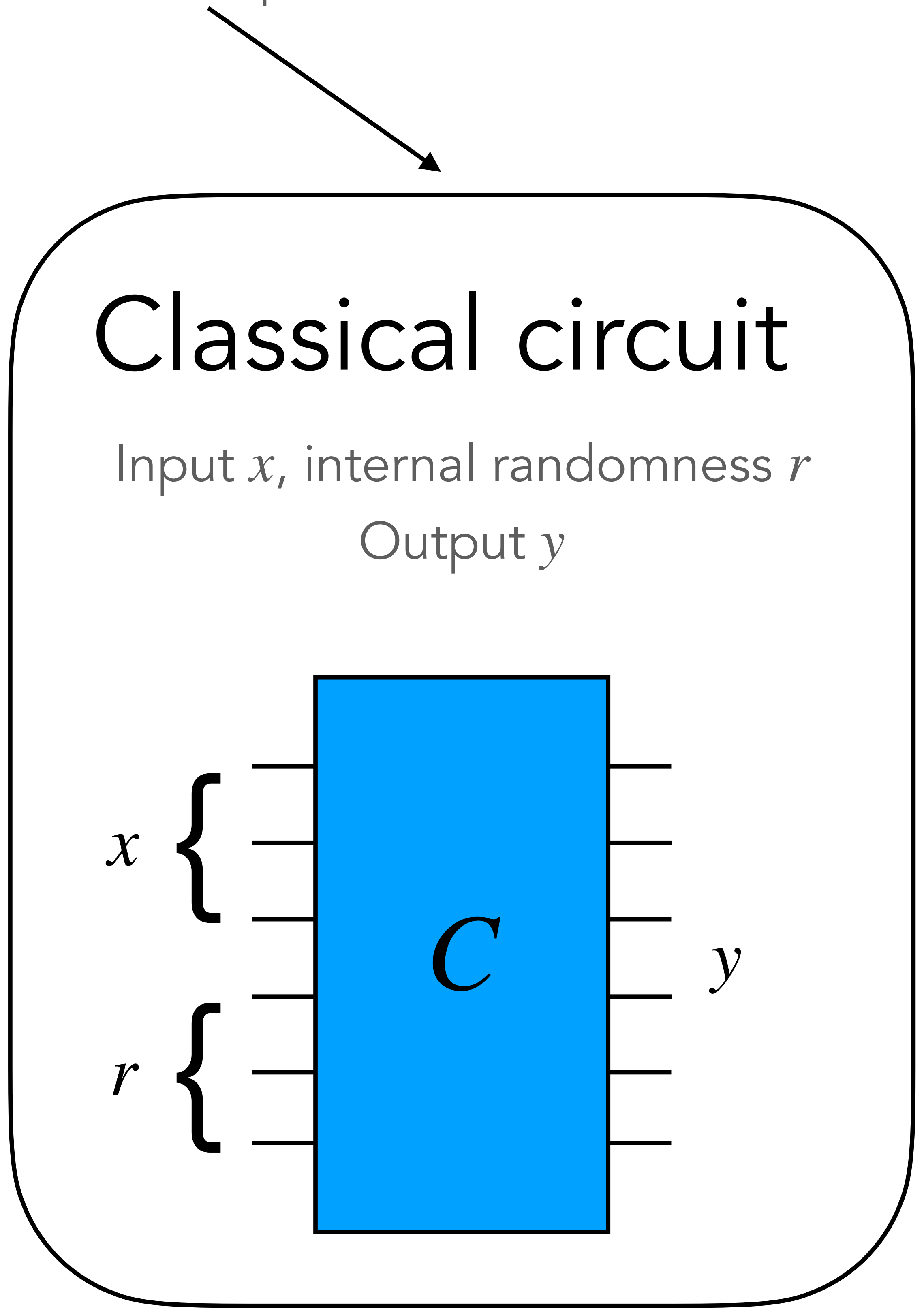

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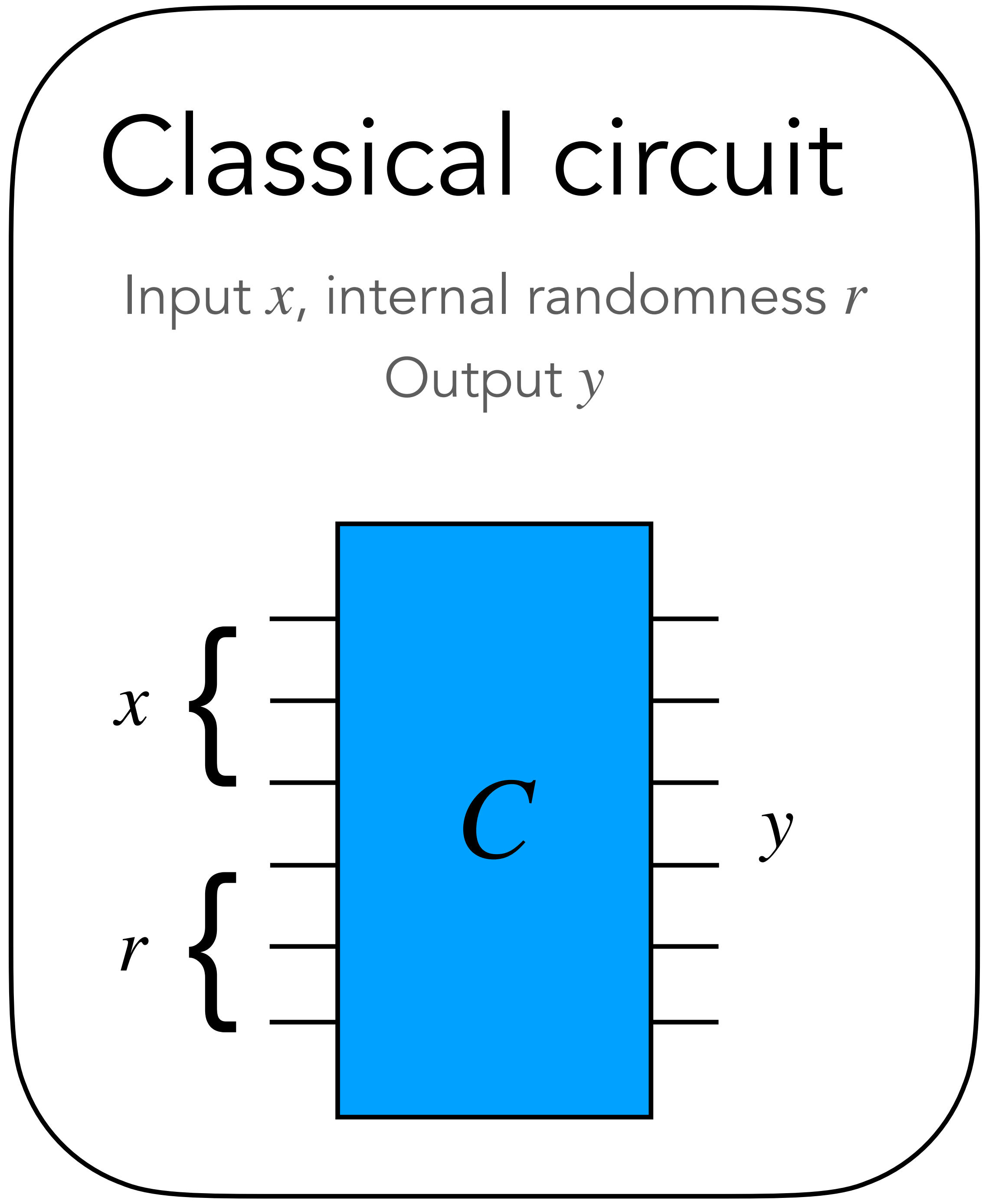

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Separation between:

	Noiseless quantum circuit	Noisy quantum circuit
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Stronger classical circuit (Above NC^0)		

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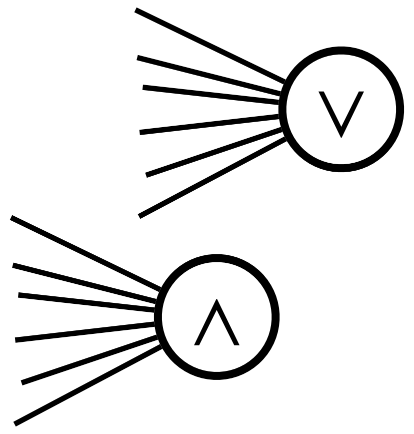

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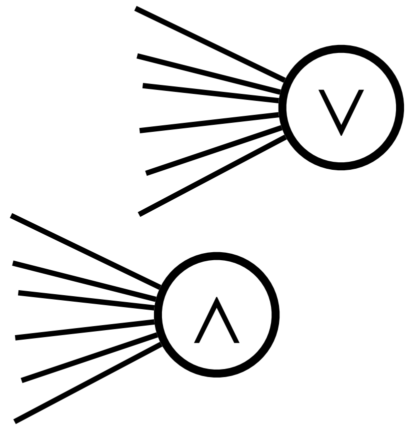

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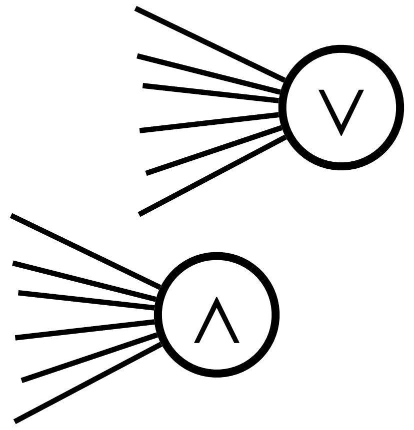

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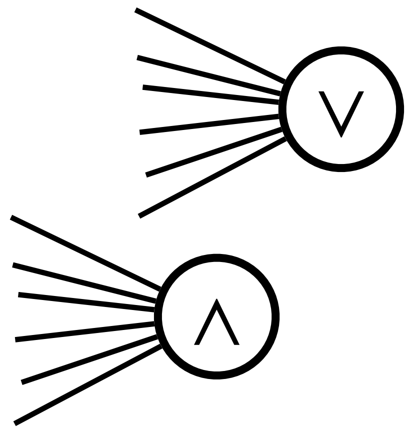

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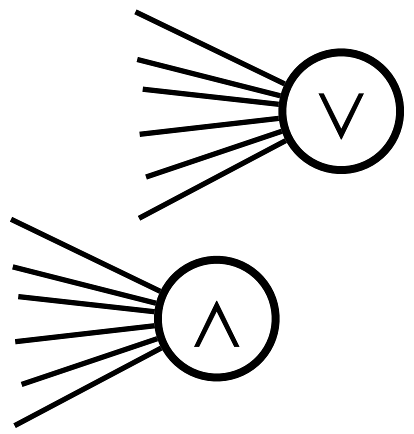

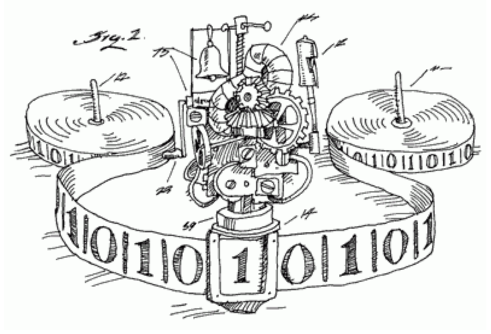

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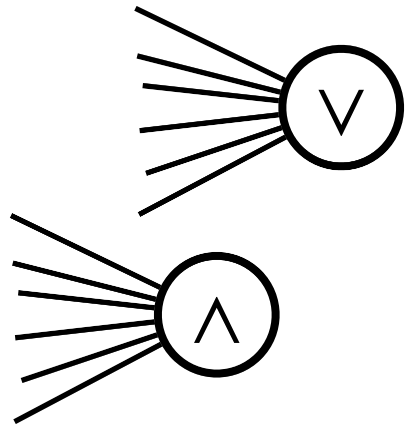

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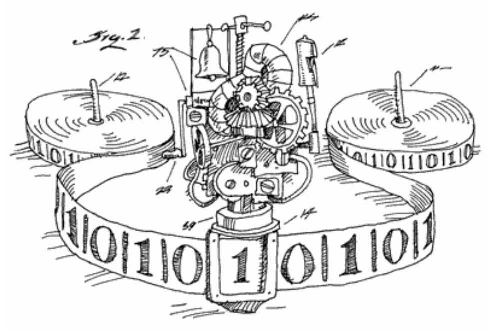

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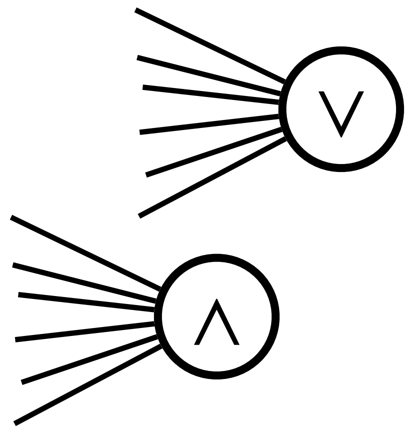

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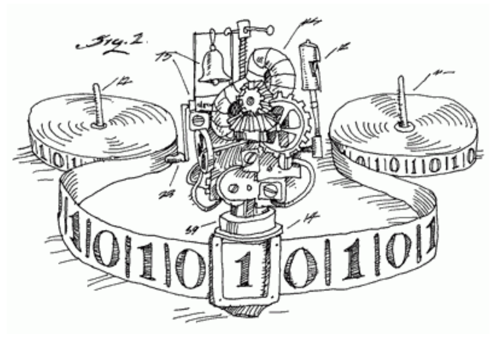

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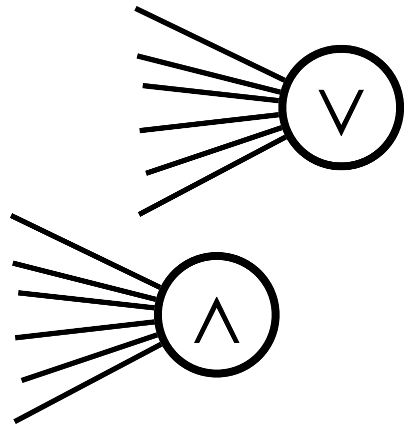

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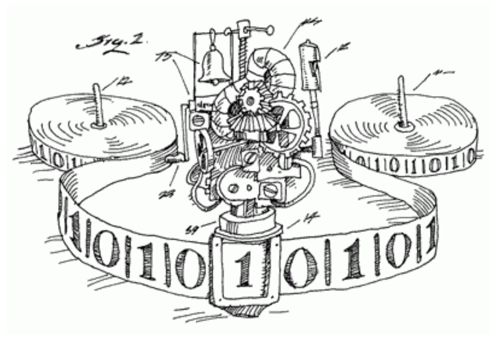

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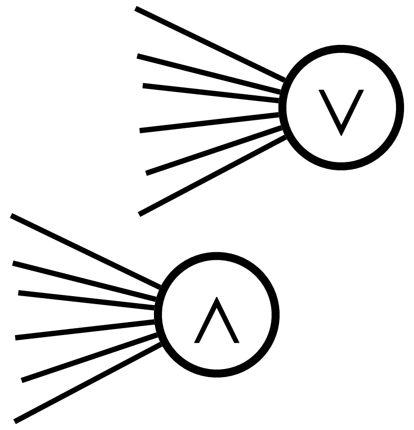

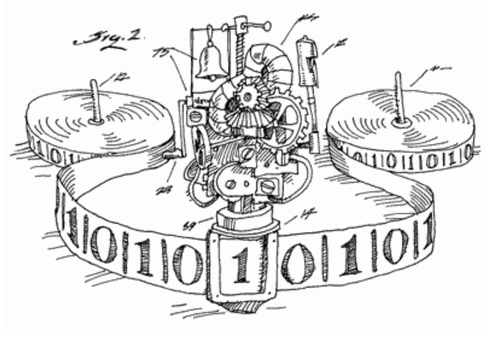

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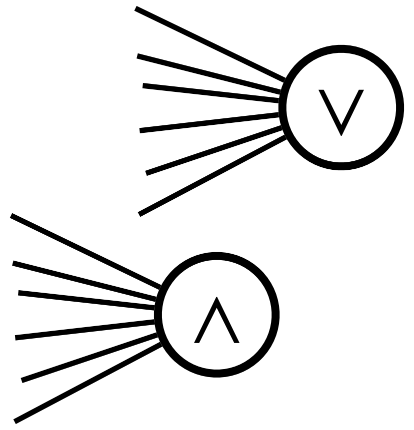

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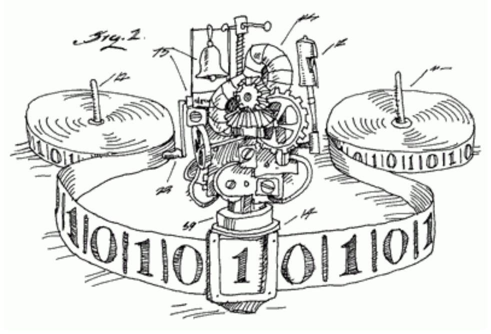

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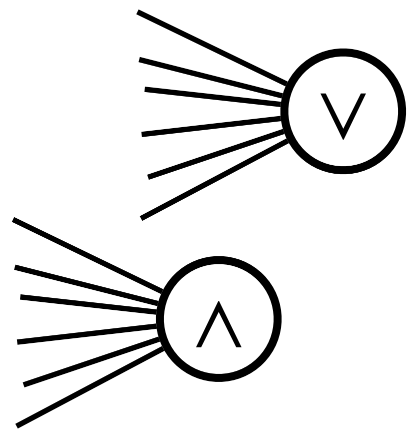

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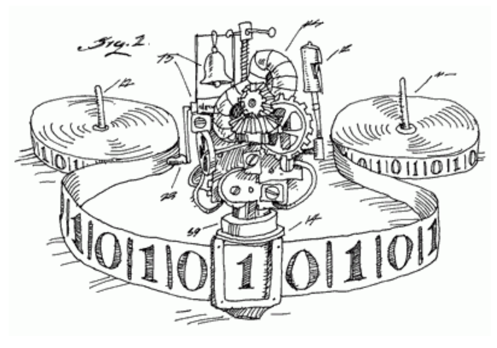

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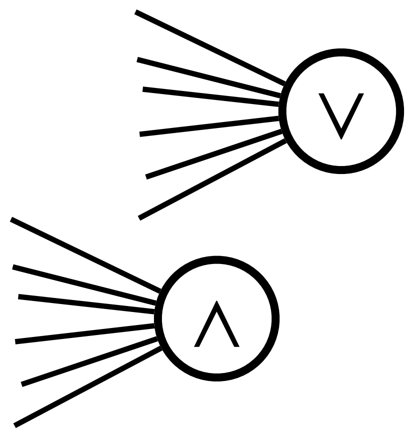

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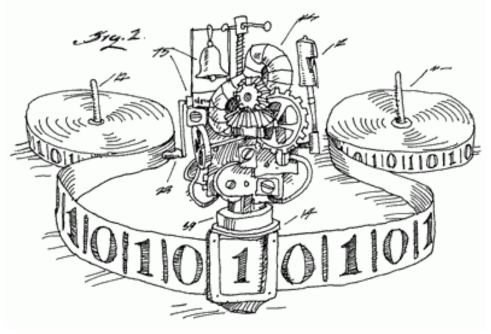

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Outline: Three steps to prove a noisy separation

Noiseless average-case separation:
Let $\mathscr{I}$ be a task solved by a **noiseless** QNC^0 circuit on all inputs with certainty.
Prove that a classical probabilistic machine solves $\mathscr{I}$ with probability at most $1 - \delta$ on a uniformly random input.

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Extend the definition of the problem to account for noise [BGKT19]:
Let $\mathcal{I}$ be the task above. Suppose y is a valid output on input x.
For the "extended" task $\mathcal{I}'$, all $\mathcal{Y}$ such that $Dec(\mathcal{Y}) = y$ are valid outputs on input x.

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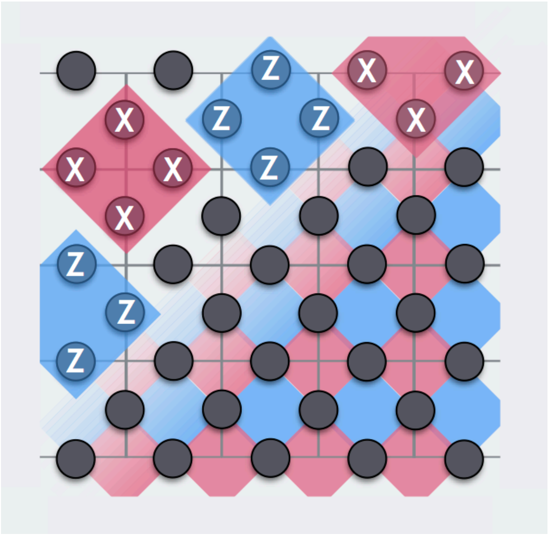
$Dec($  $) = y$

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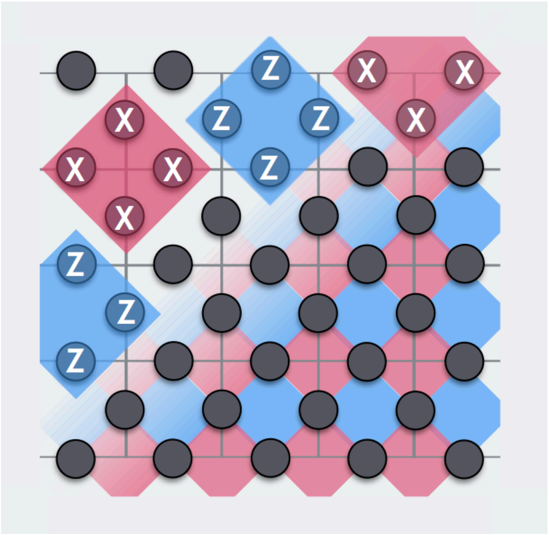
$Dec($  $) = y$

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Noisy separation:
The task $\mathscr{I}'$ is solved by a **noisy** QNC^0 circuit on all inputs w/p $1 - o(1)$.
A classical probabilistic machine solves $\mathscr{I}'$ with probability at most $1 - \delta$ on a uniformly random input.

Main result

Noiseless average-case separation (This work):
There is an interactive task solved by a **noiseless** QNC^0 circuit on all inputs with certainty.
Let $\mathscr{R}$ be a classical probabilistic machine that solves the same task with probability $420/421$ over uniform input. Then $\oplus\mathsf{L} \subseteq (\mathsf{AC}^0)^{\mathscr{R}}$.

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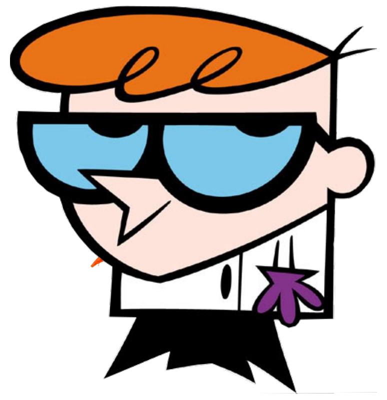
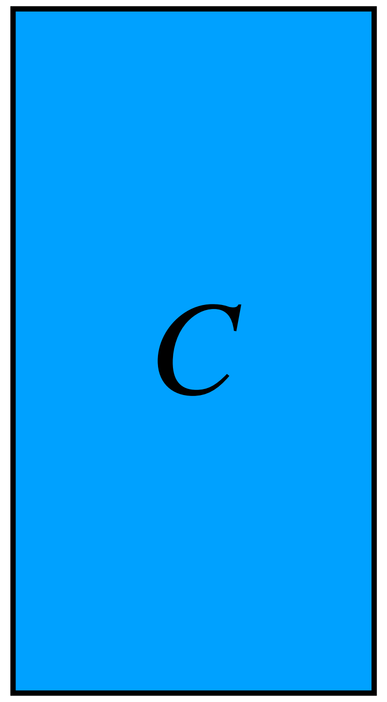
Experimenter





Main result

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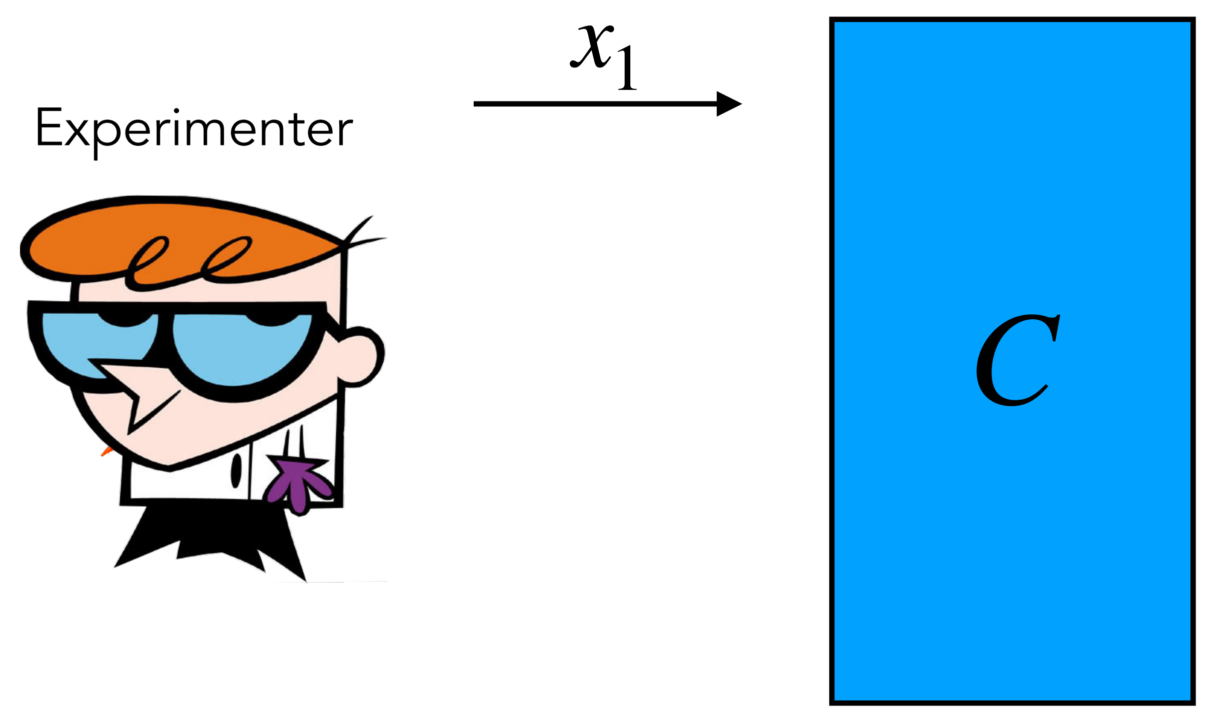

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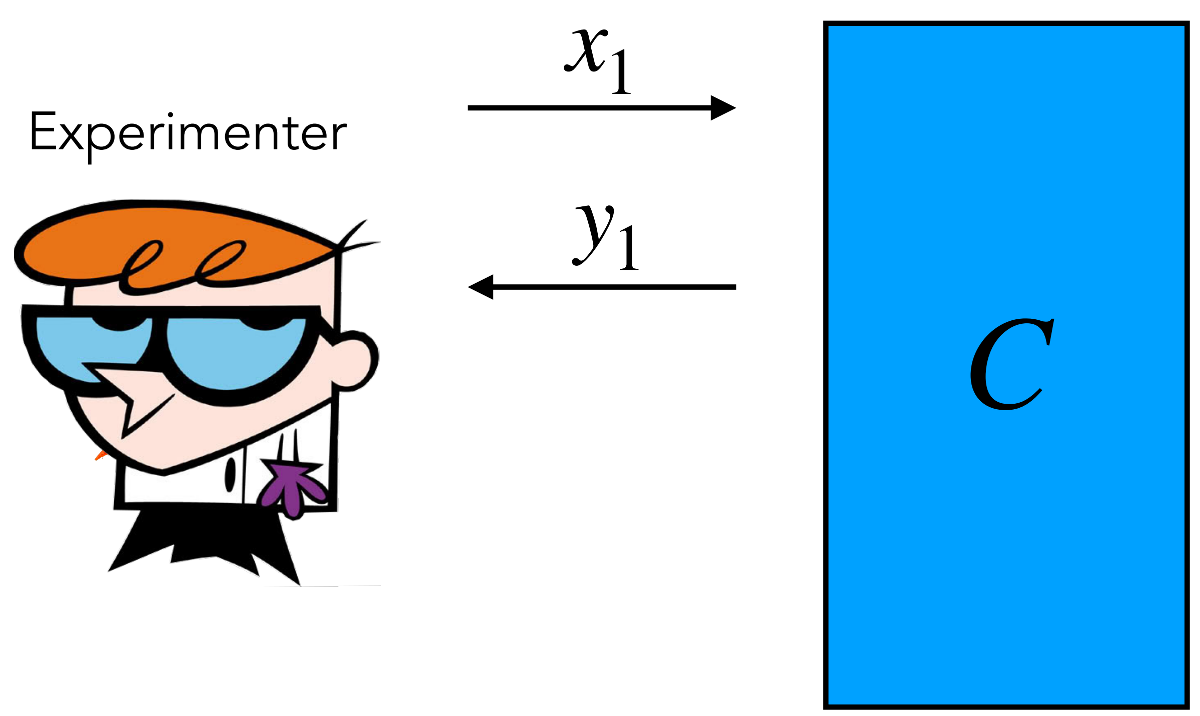

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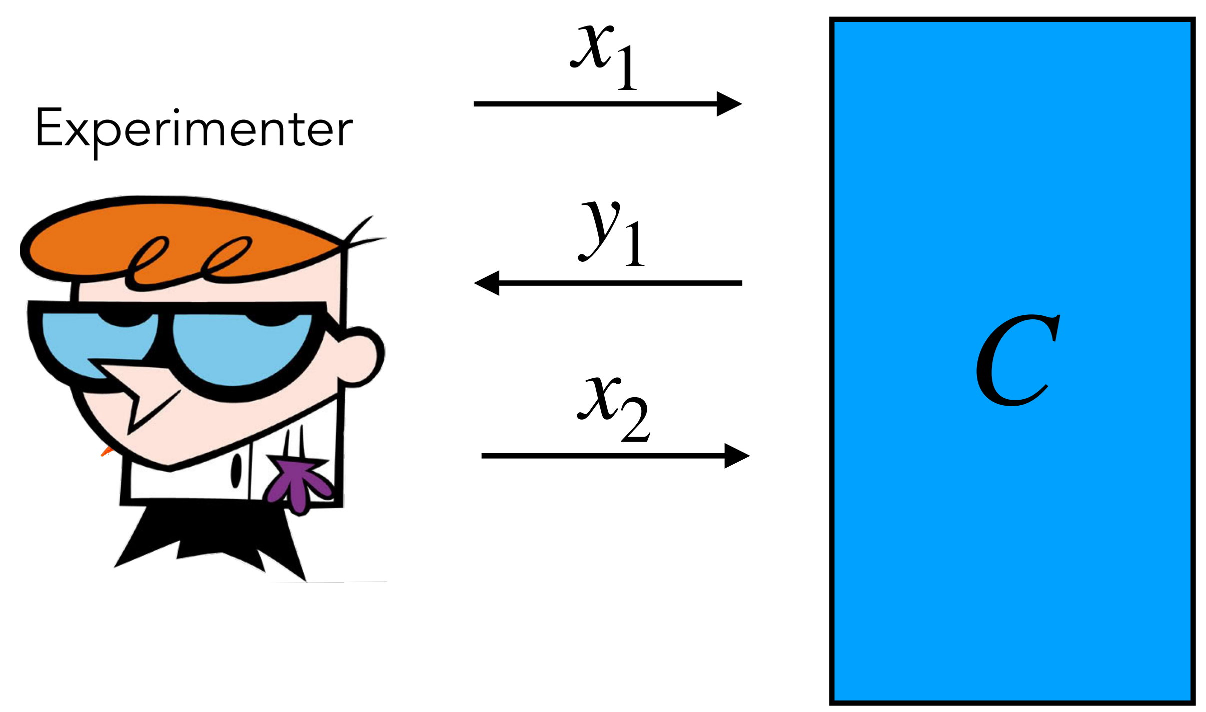

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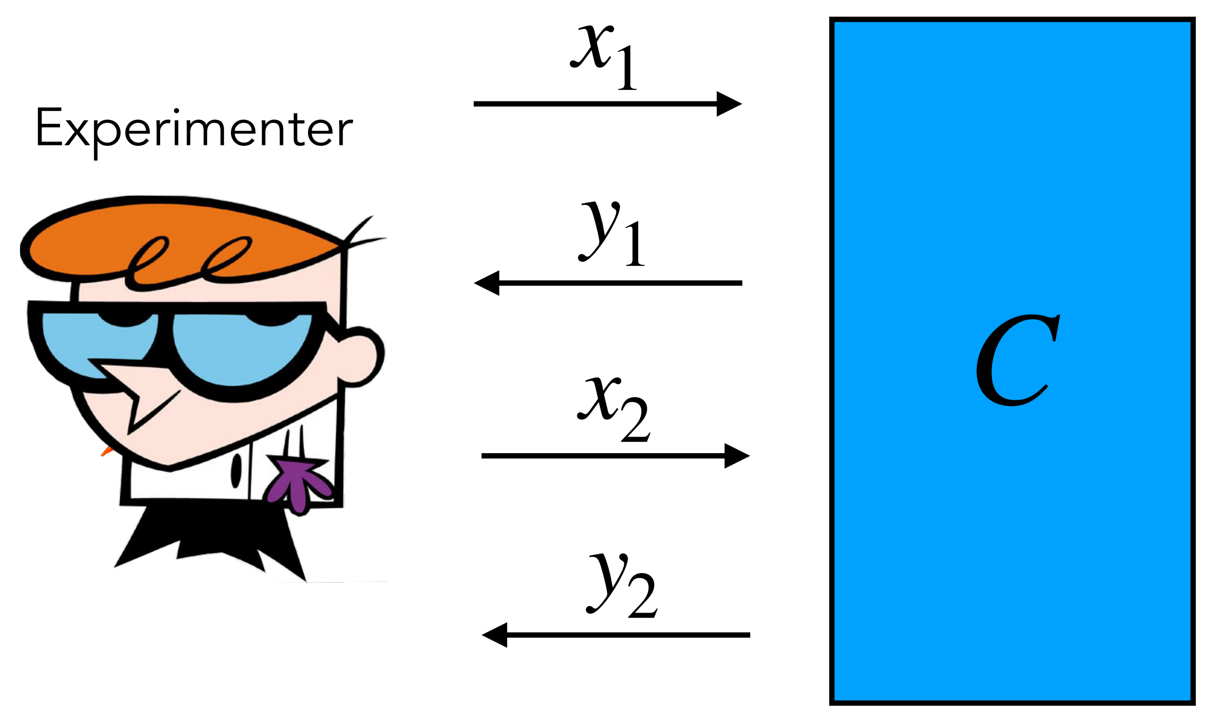

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Extend the definition of the interactive problem to account for noise [BGKT19]:
Let $\mathscr{I}$ be the interactive problem above.
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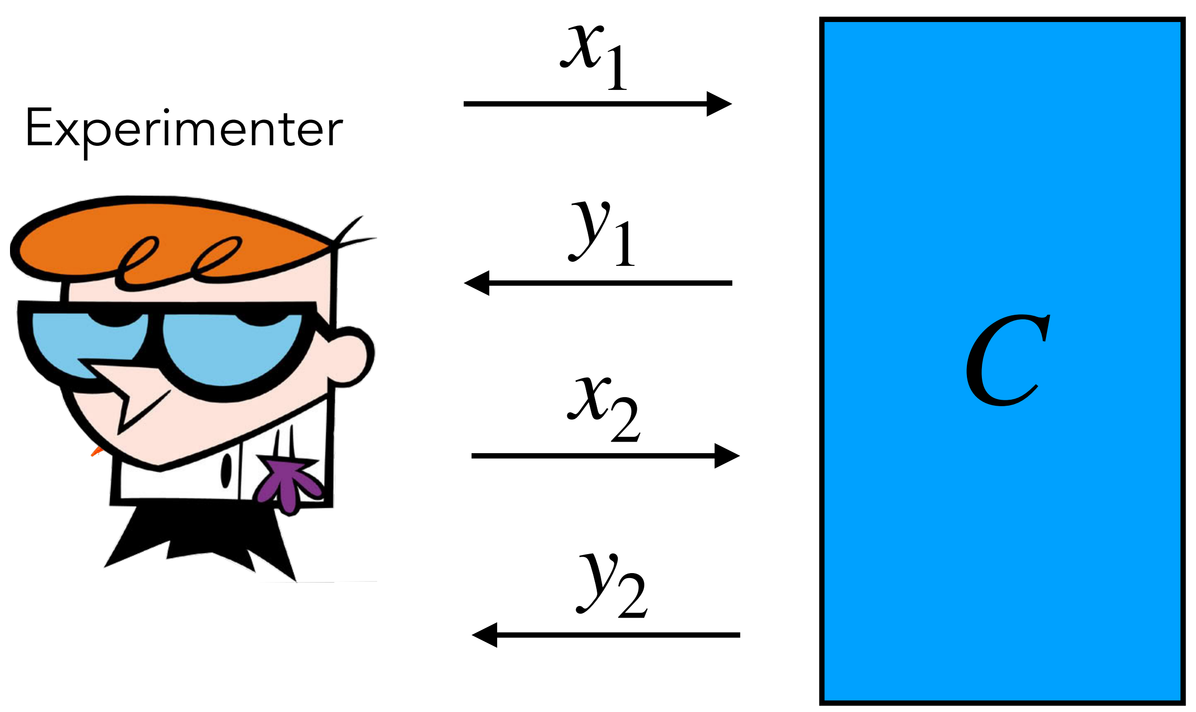

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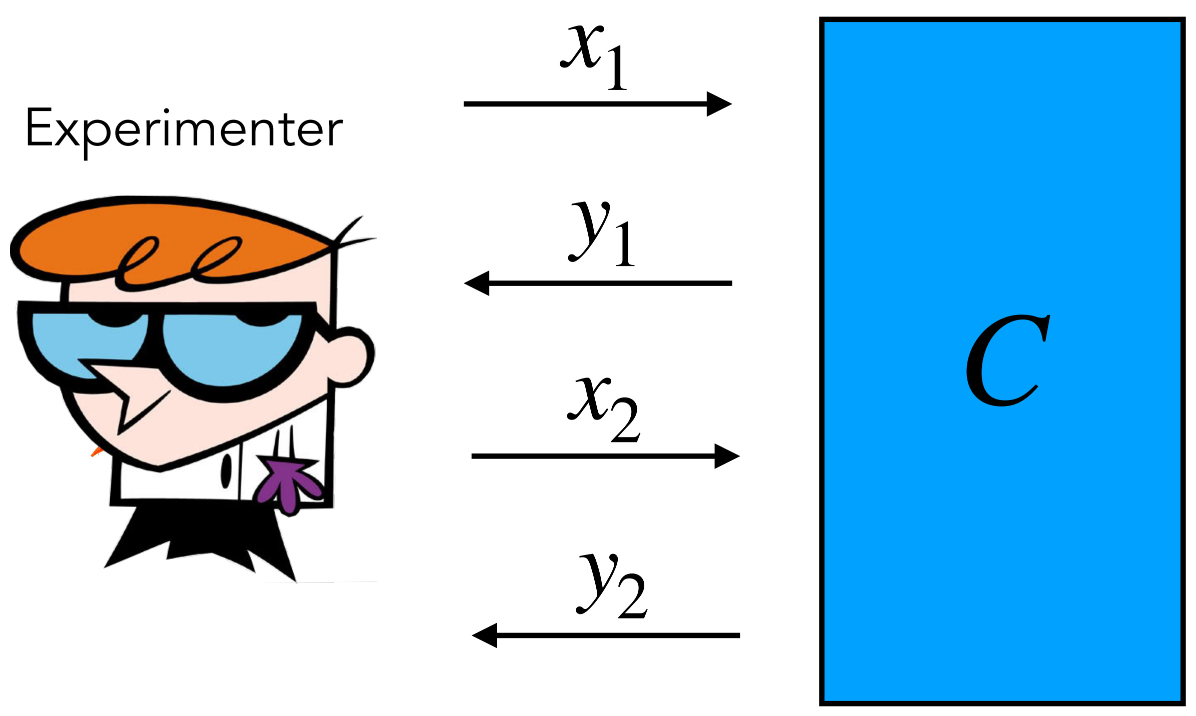

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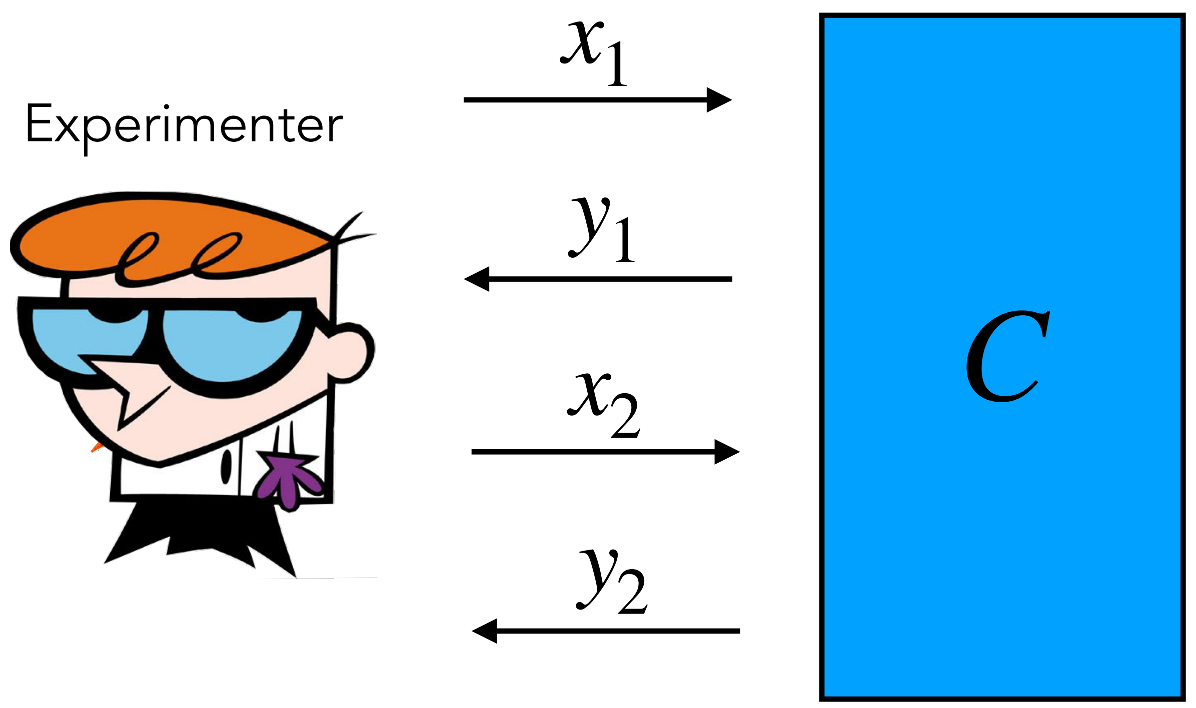

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$\Longrightarrow$

Unconditional: Noisy QNC^0 vs. $\mathsf{AC}^0[p]$ separation

Conditional: If $\oplus\mathsf{L} \not\subseteq (\mathsf{qAC}^0)^{\mathsf{L}}$, then noisy QNC^0 vs. L separation

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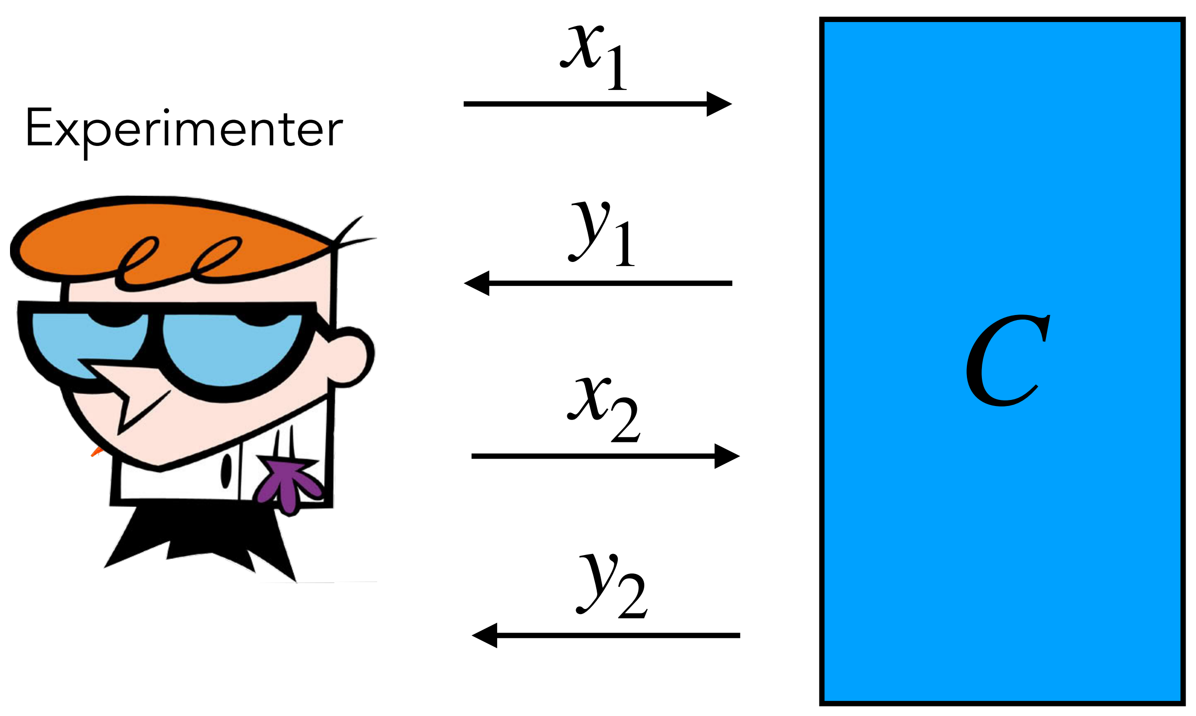

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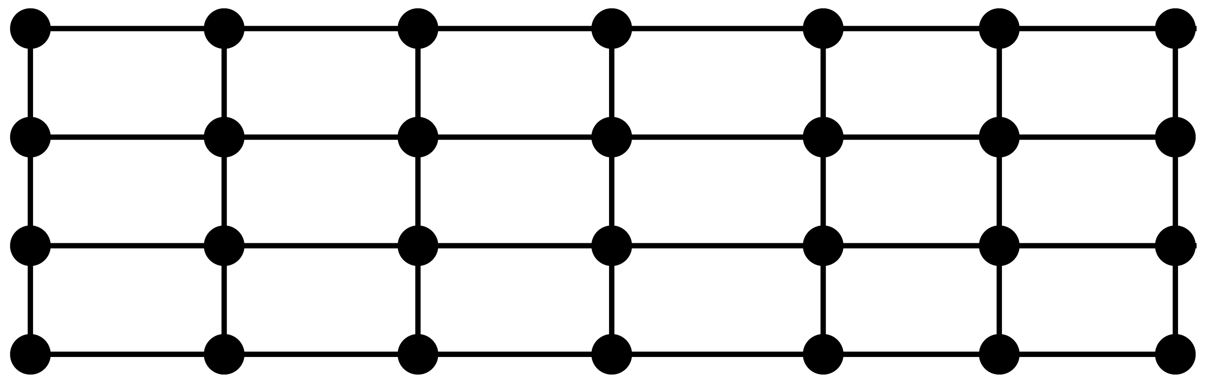

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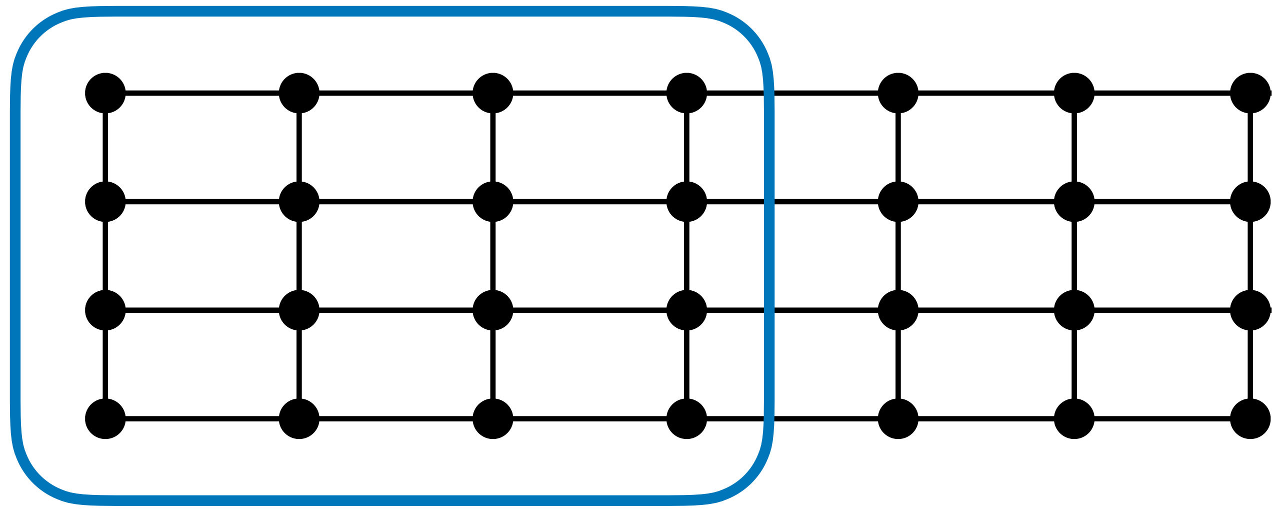
Round 1:



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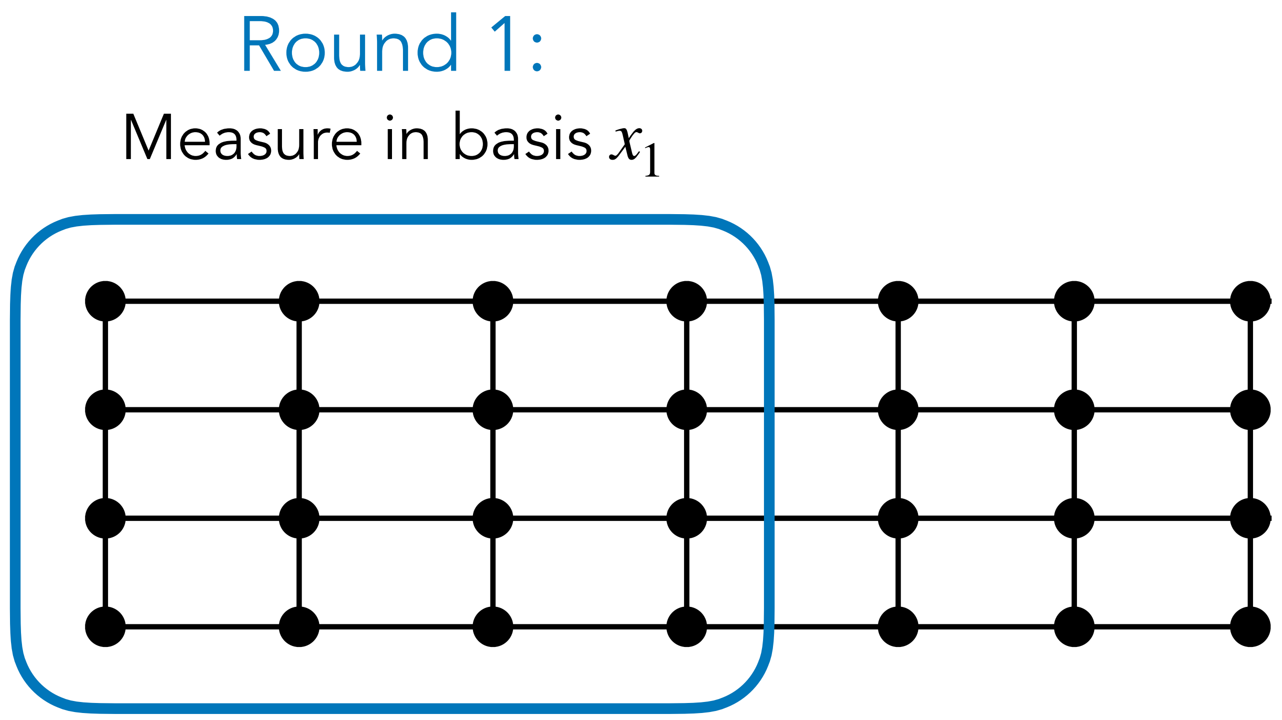

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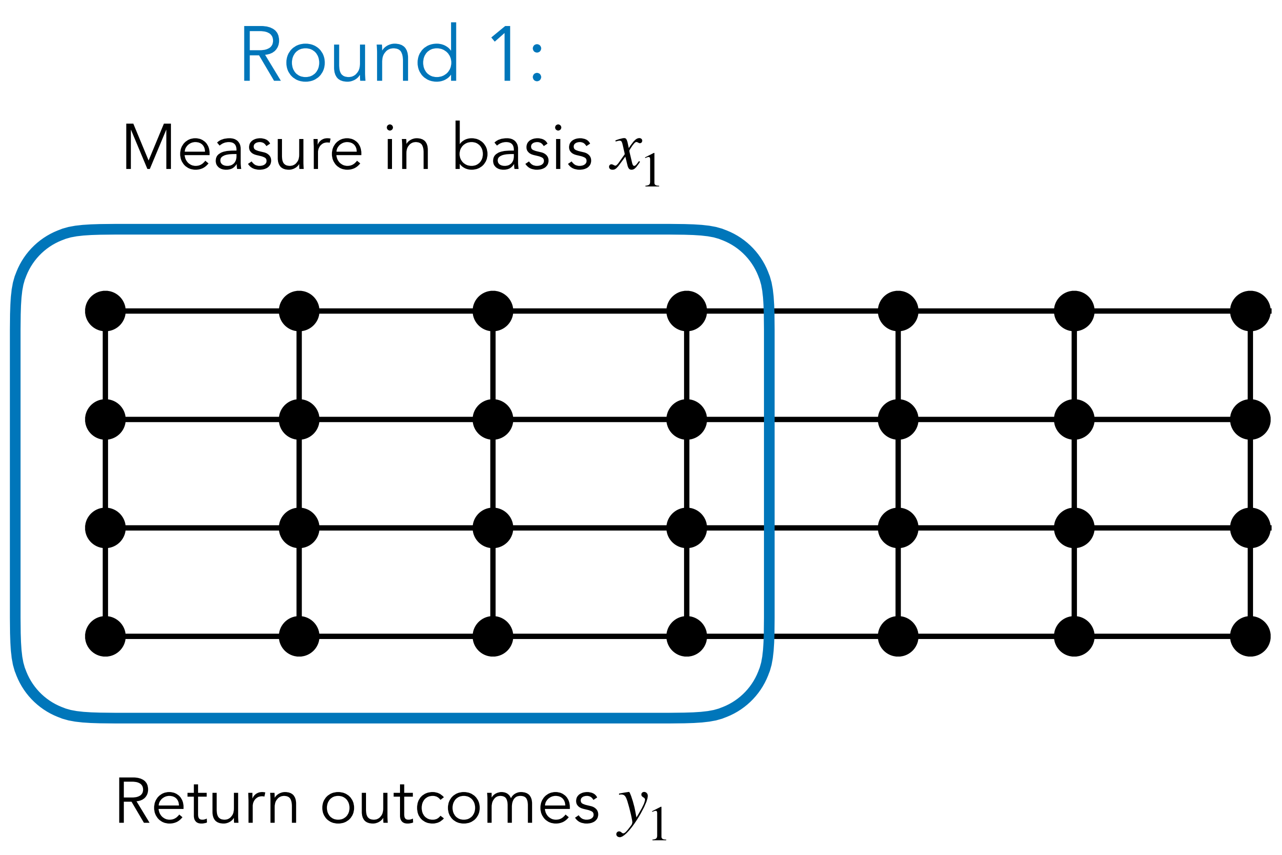

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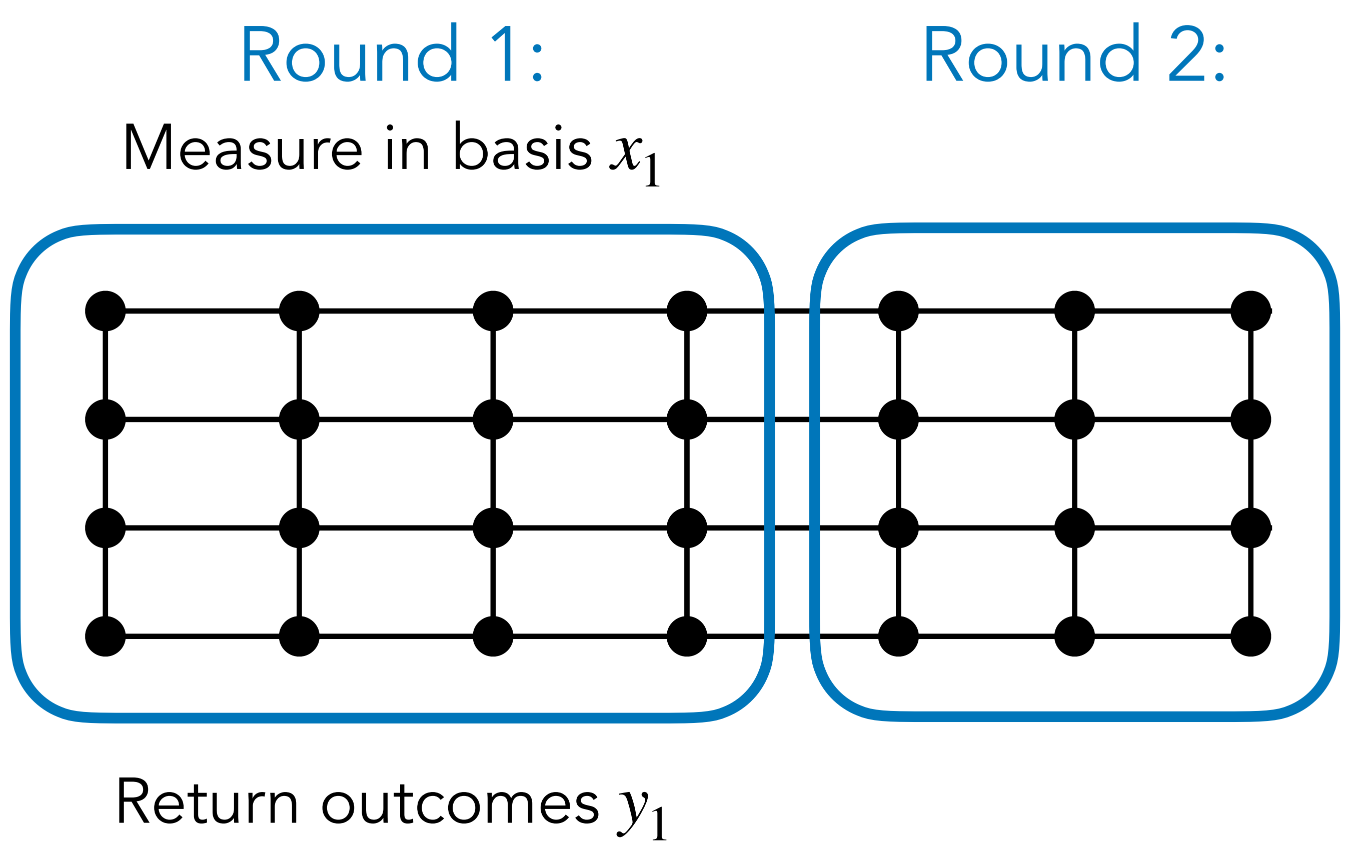

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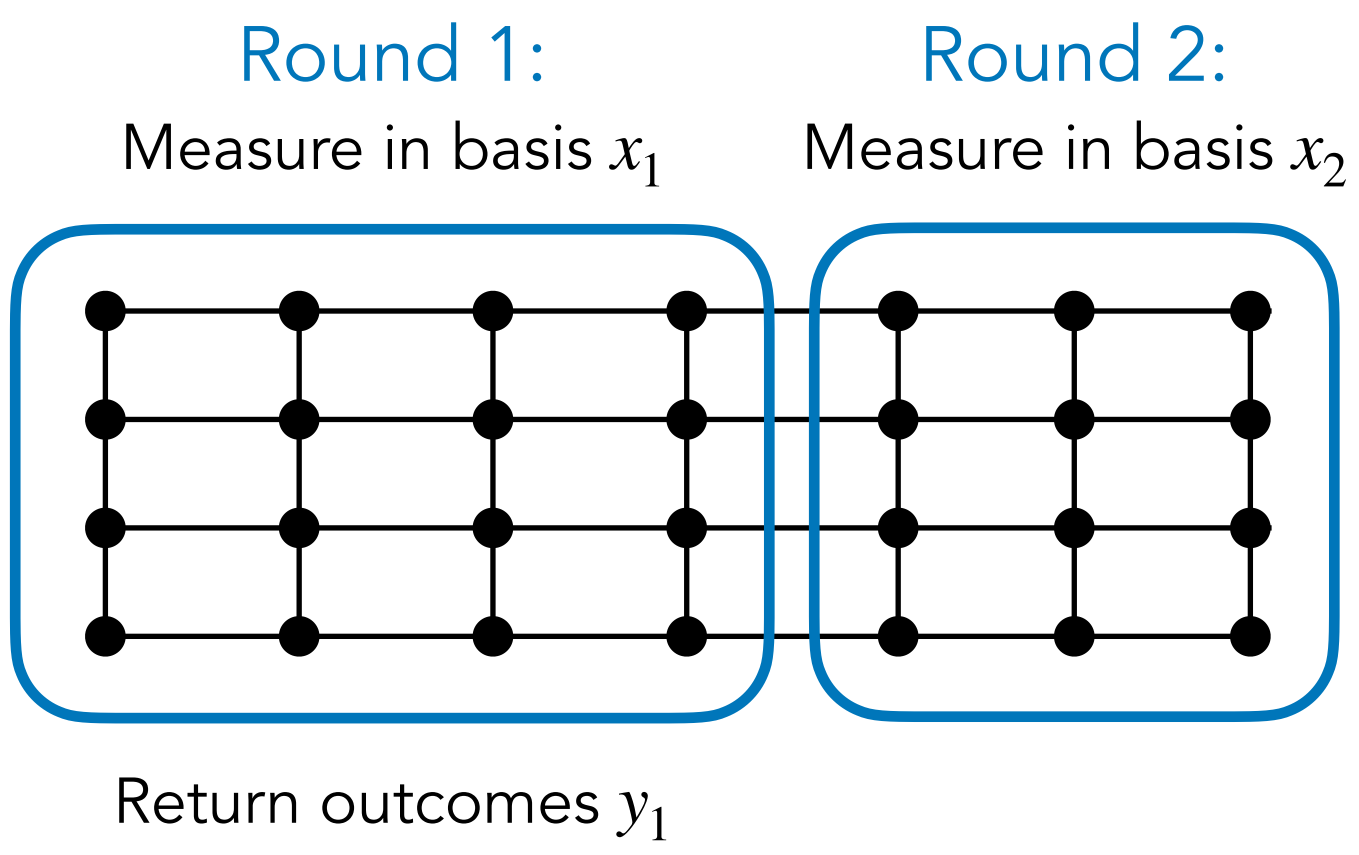

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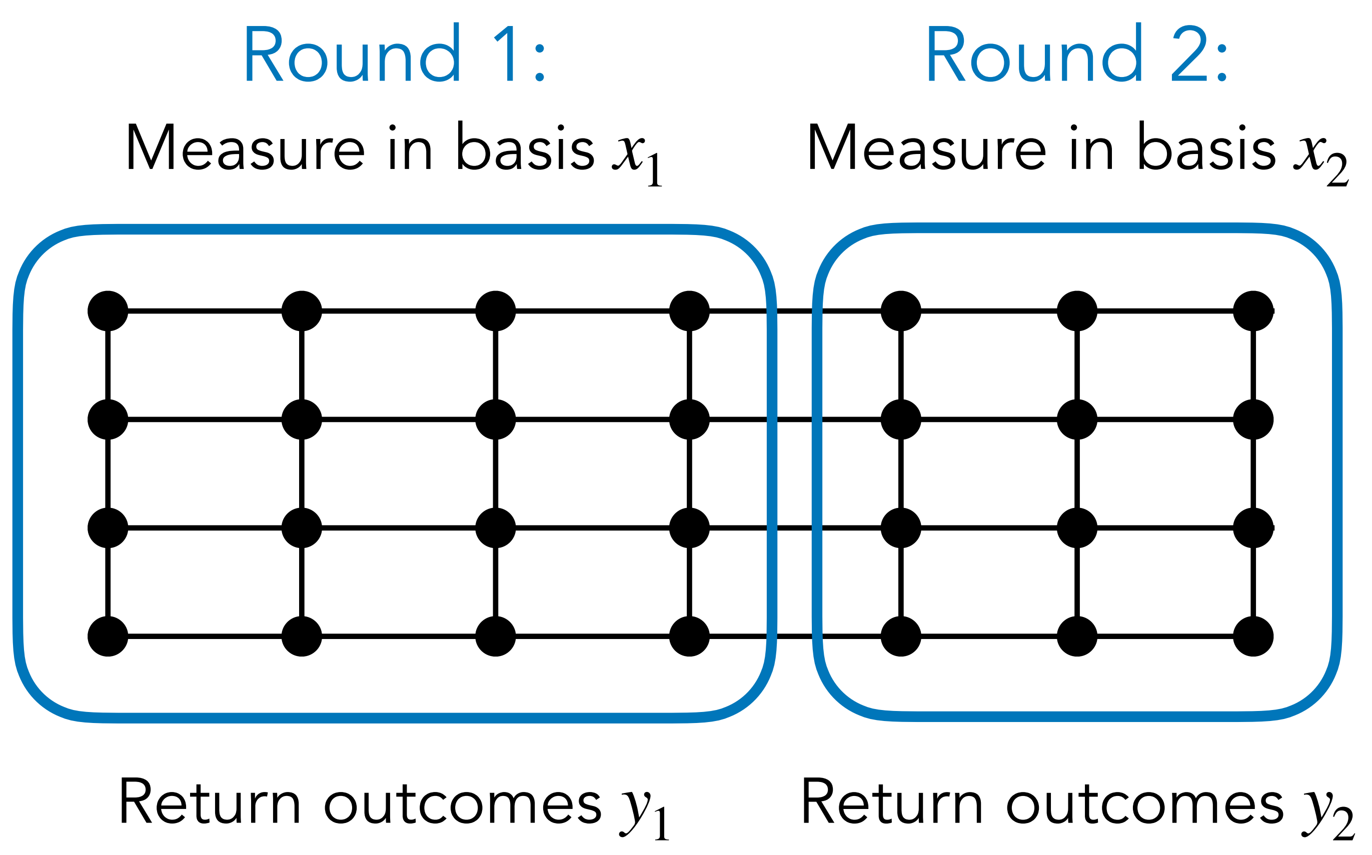

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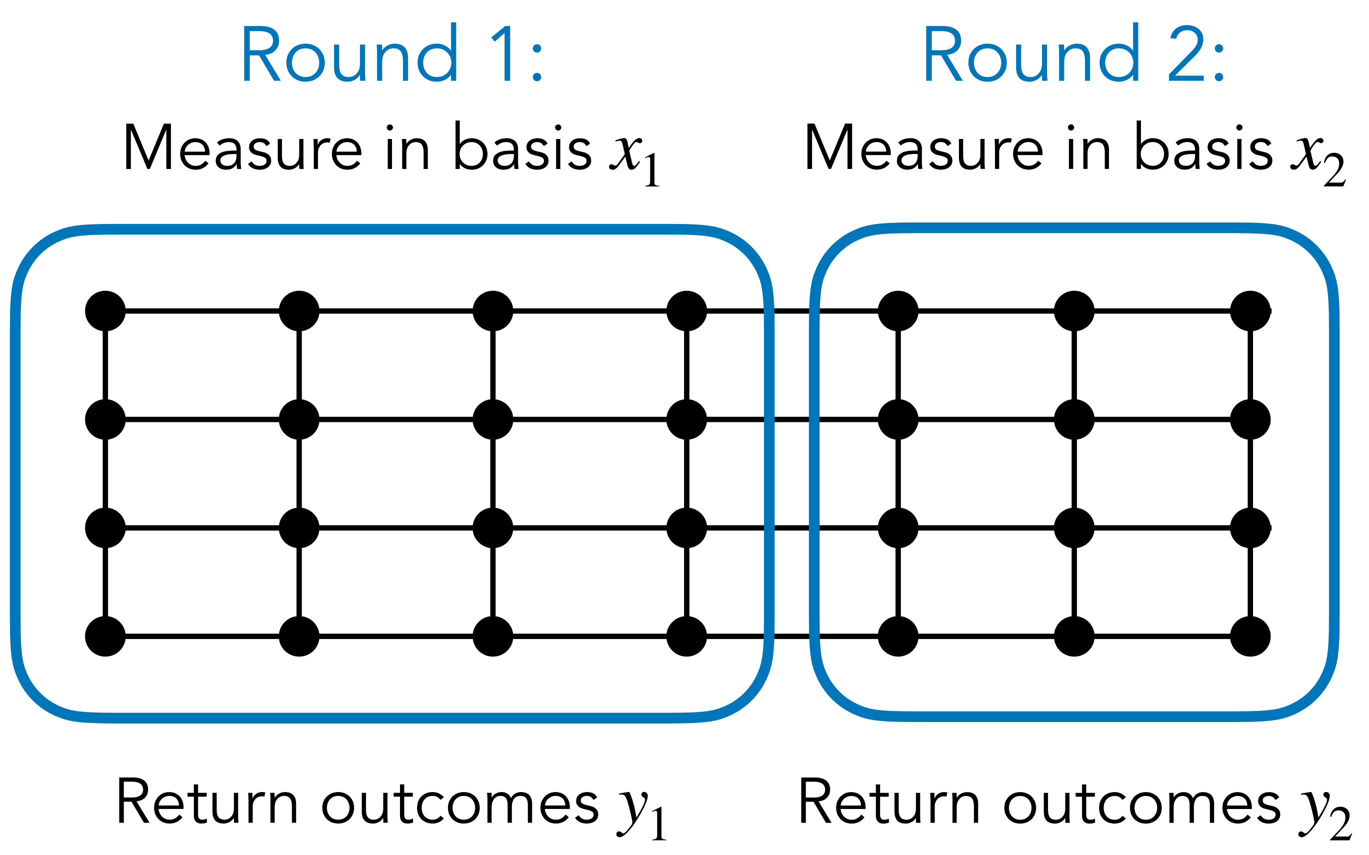
promise



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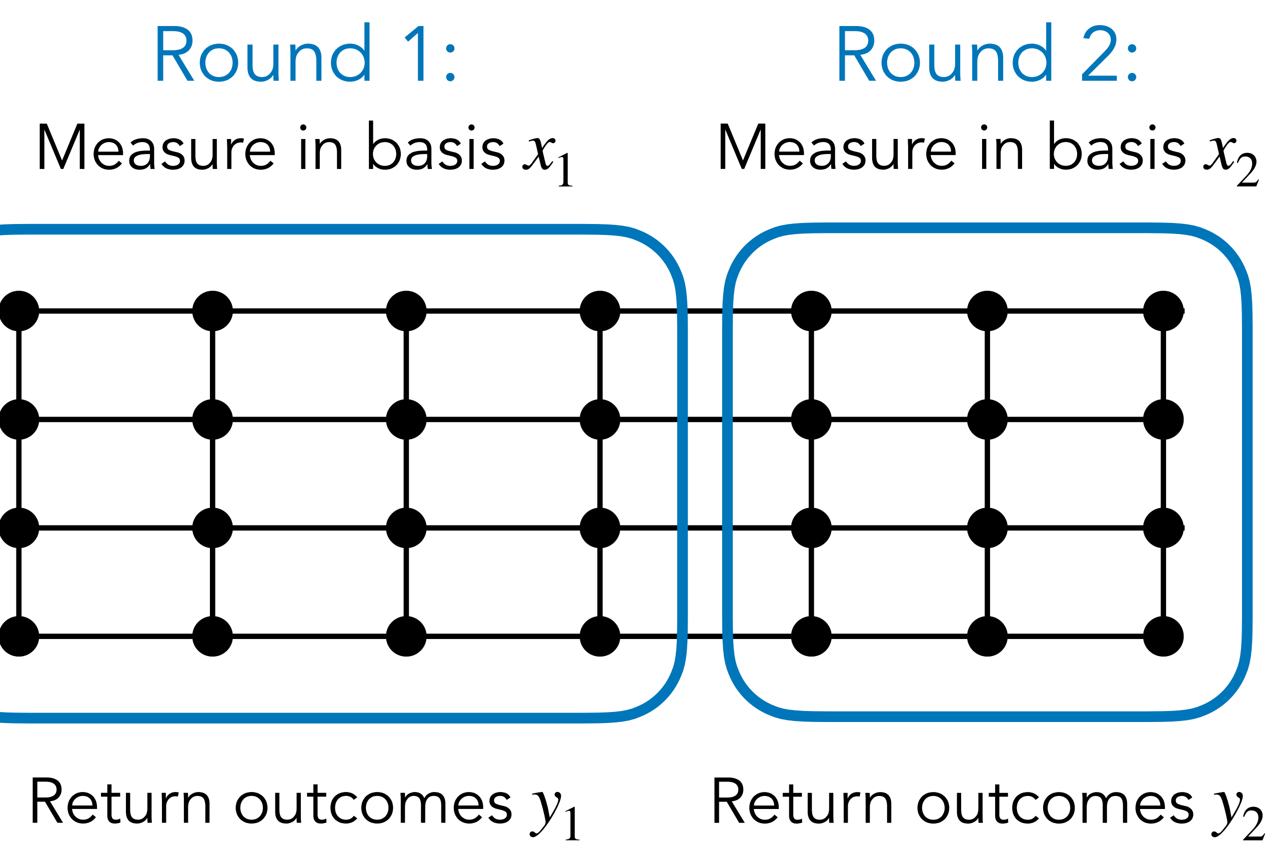

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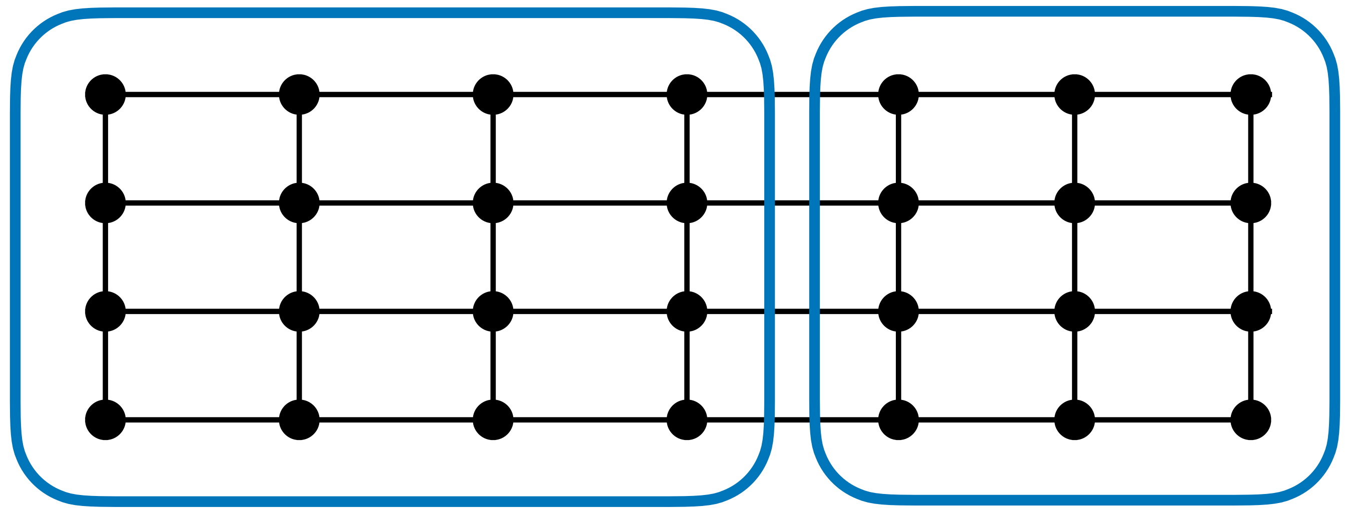
Round 1:
Measure in basis x_1

Round 2:
Measure in basis x_2



Return outcomes y_1

Return outcomes y_2

It is **average-case** $\oplus$L-hard to return valid Pauli measurement outcomes of a grid state*.

* = For an interactive promise problem

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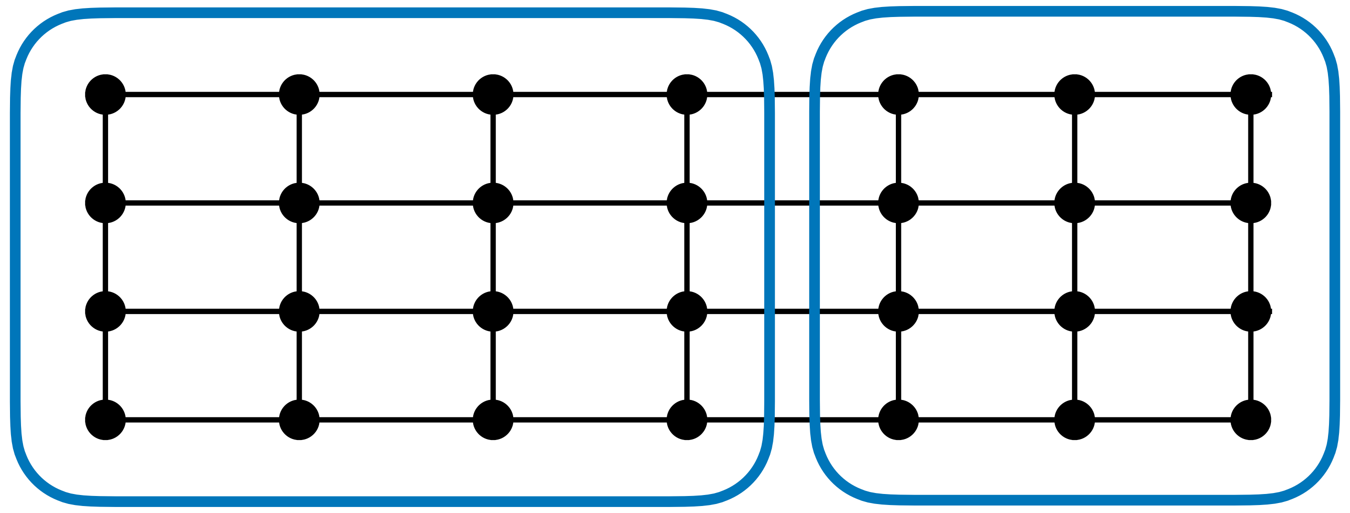

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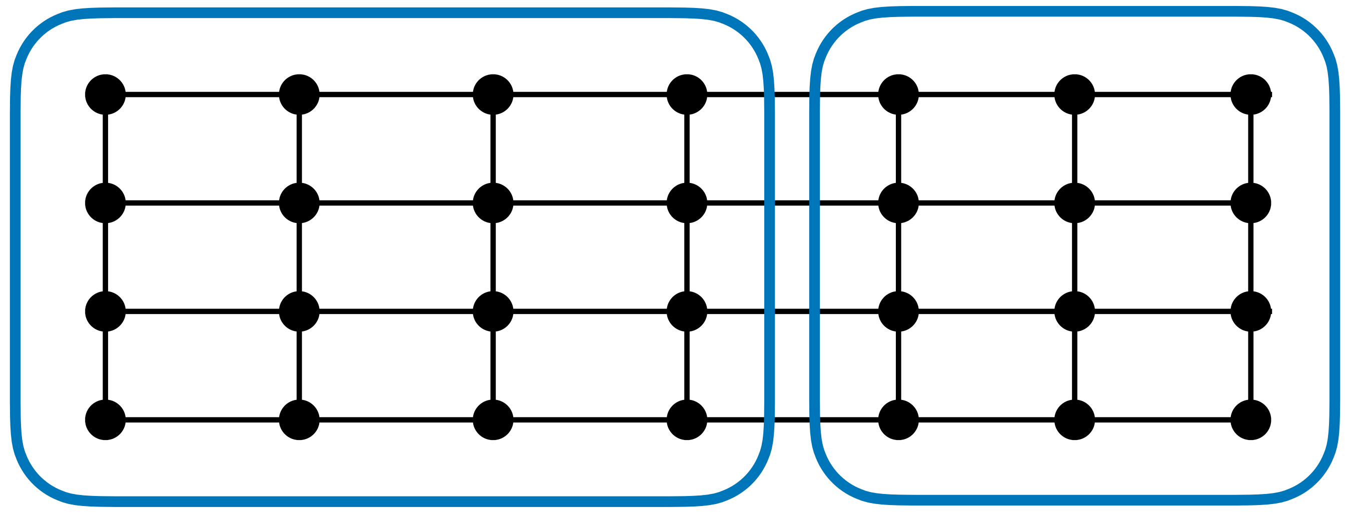

It is **average-case** $\oplus$L-hard to return valid Pauli measurement outcomes of a grid state*.

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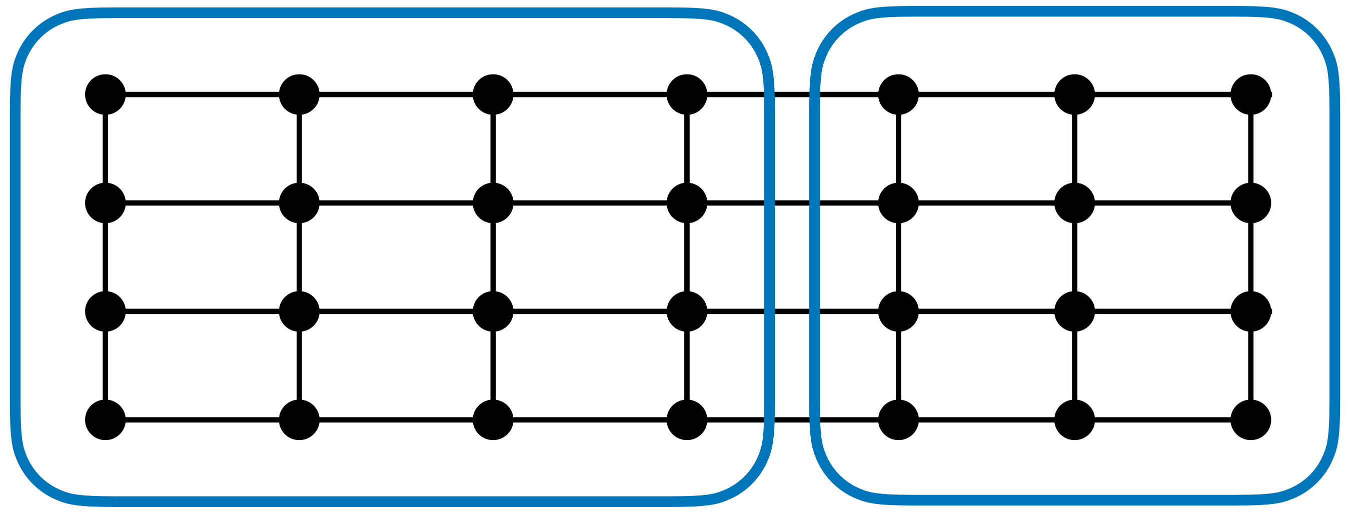
How to prove average-case $\oplus$L-hardness for the interactive problem?
Construct a worst-to-average-case reduction from a $\oplus$L-hard problem.

Noiseless average-case separation



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It is **average-case** ⊕L-hard to return valid Pauli measurement outcomes of a grid state*.

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How to prove average-case ⊕L-hardness for the interactive problem?

Construct a worst-to-average-case reduction from a ⊕L-hard problem.

→ Use the ability to classically solve the interactive problem on **random** (average-case) instances to solve a **worst-case** ⊕L-hard problem instance

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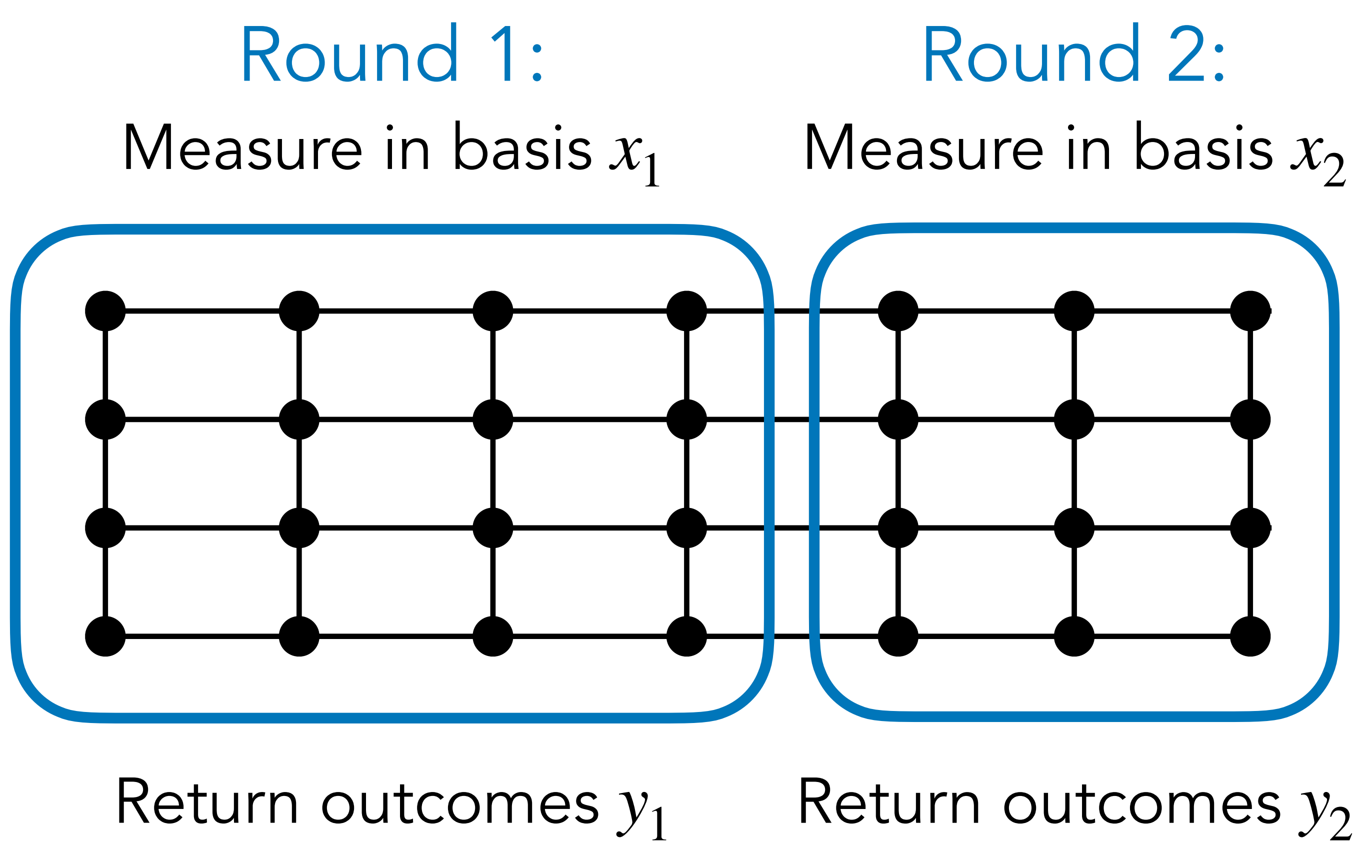

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Construct a worst-to-average-case reduction from a $\oplus$L-hard problem.

(different from [BGKT], who prove average-case hardness using nonlocal games instead)

→ Use the ability to classically solve the interactive problem on **random** (average-case) instances to solve a **worst-case** $\oplus$L-hard problem instance

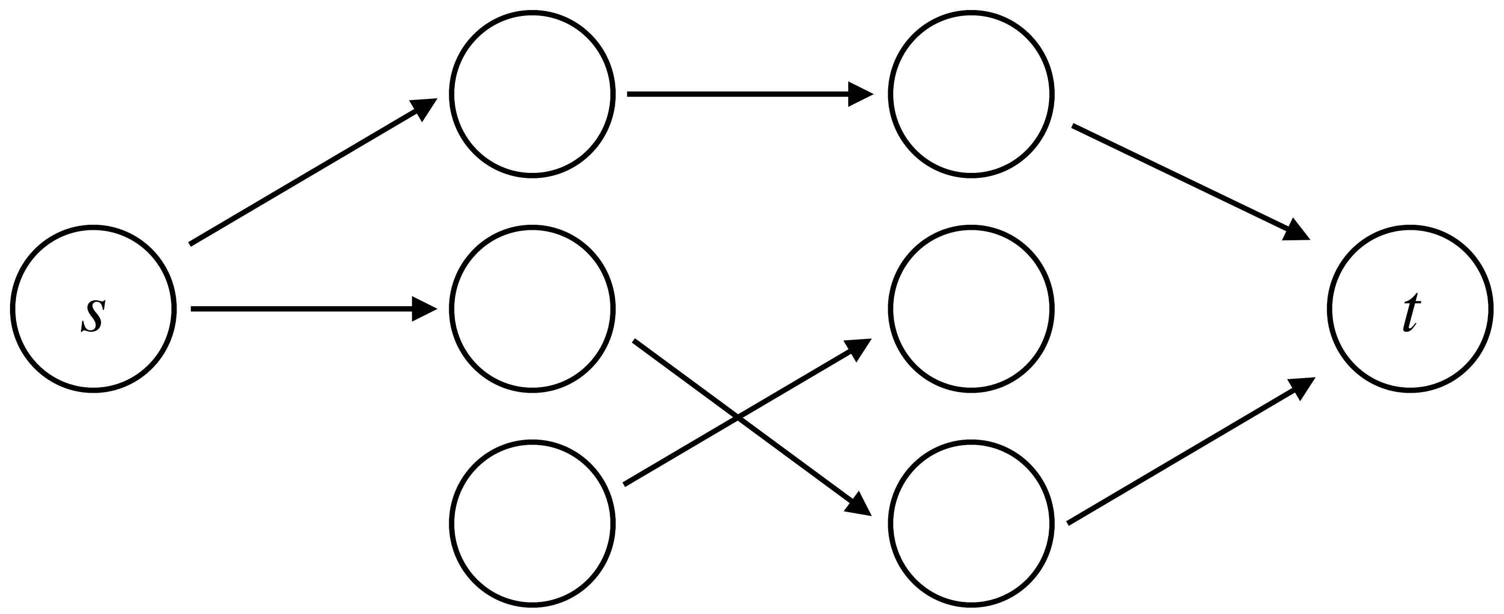
Worst-case $\oplus$L-hard problems

It is $\oplus$L-hard to determine the parity of the number of $s \to t$ paths in a (layered) DAG.

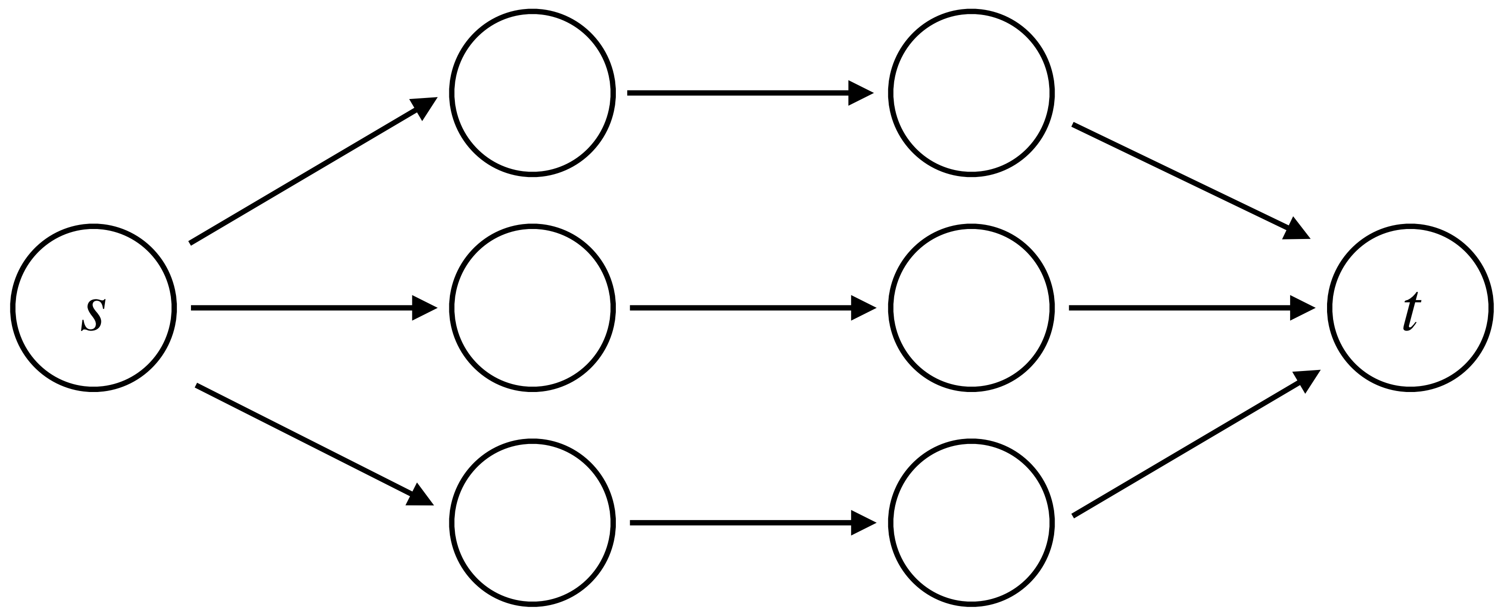
Worst-case ⊕L-hard problems

It is ⊕L-hard to determine the parity of the number of $s \to t$ paths in a (layered) DAG.

Even parity



Odd parity



Worst-case $\oplus$L-hard problems

It is $\oplus$L-hard to determine the parity of the number of $s \to t$ paths in a (layered) DAG.

[GS20]: It is also $\oplus$L-hard to determine whether CNOT gates $g_1, \ldots, g_n$ multiply to 3-cycle or identity.

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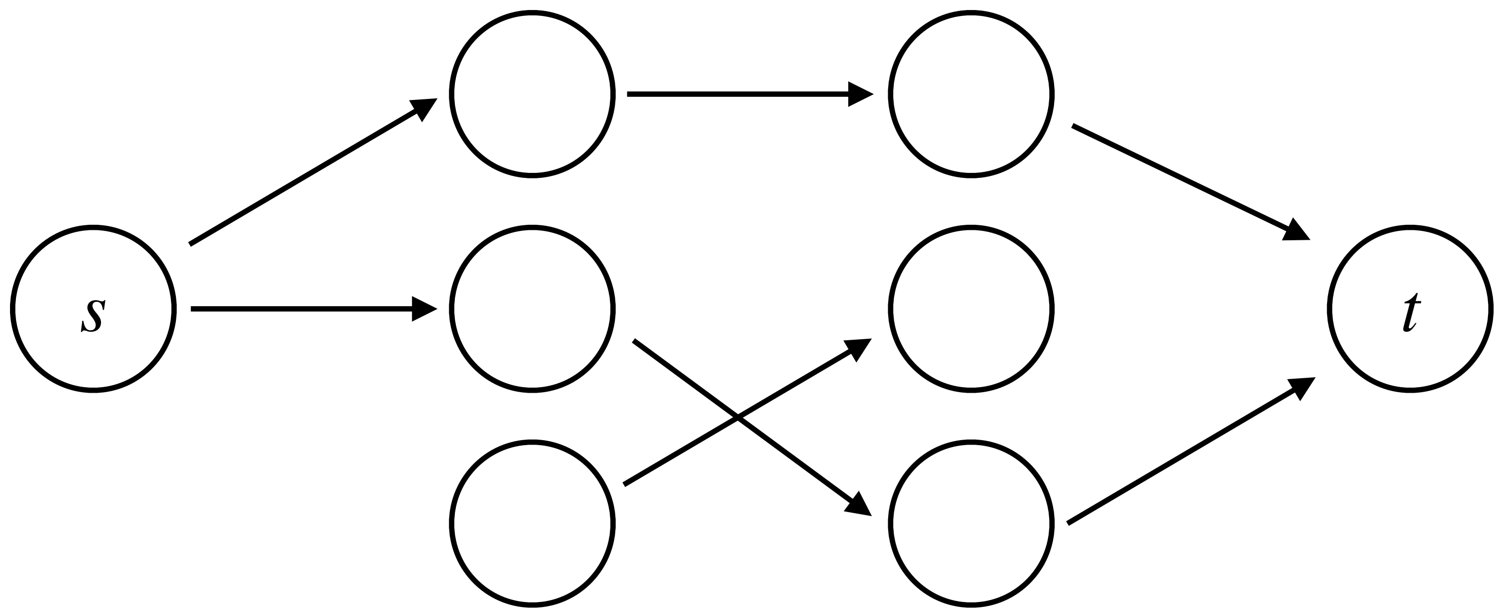

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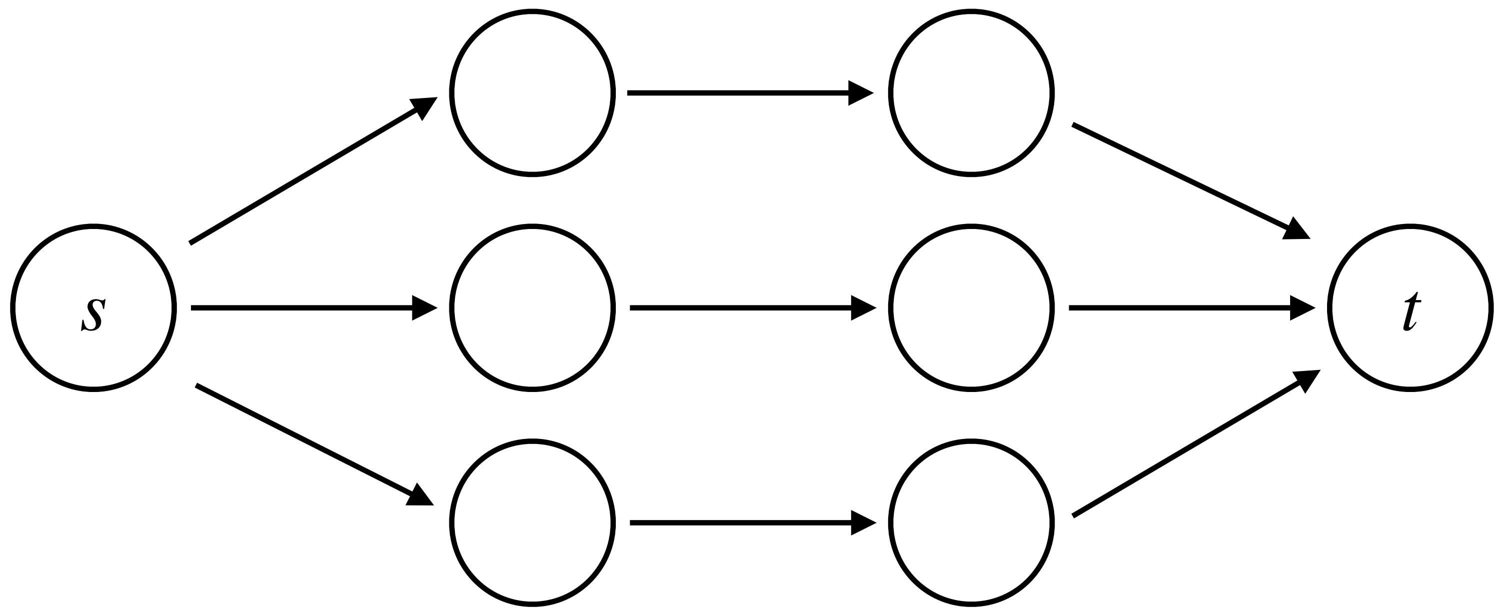

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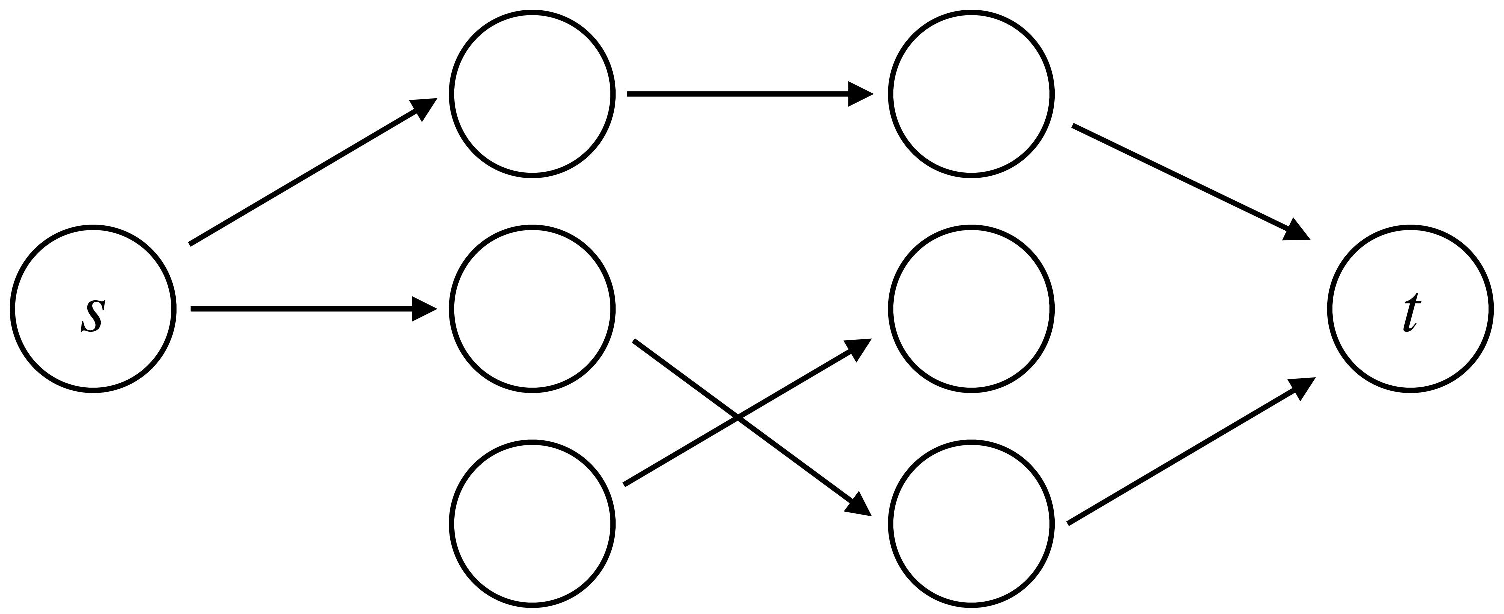
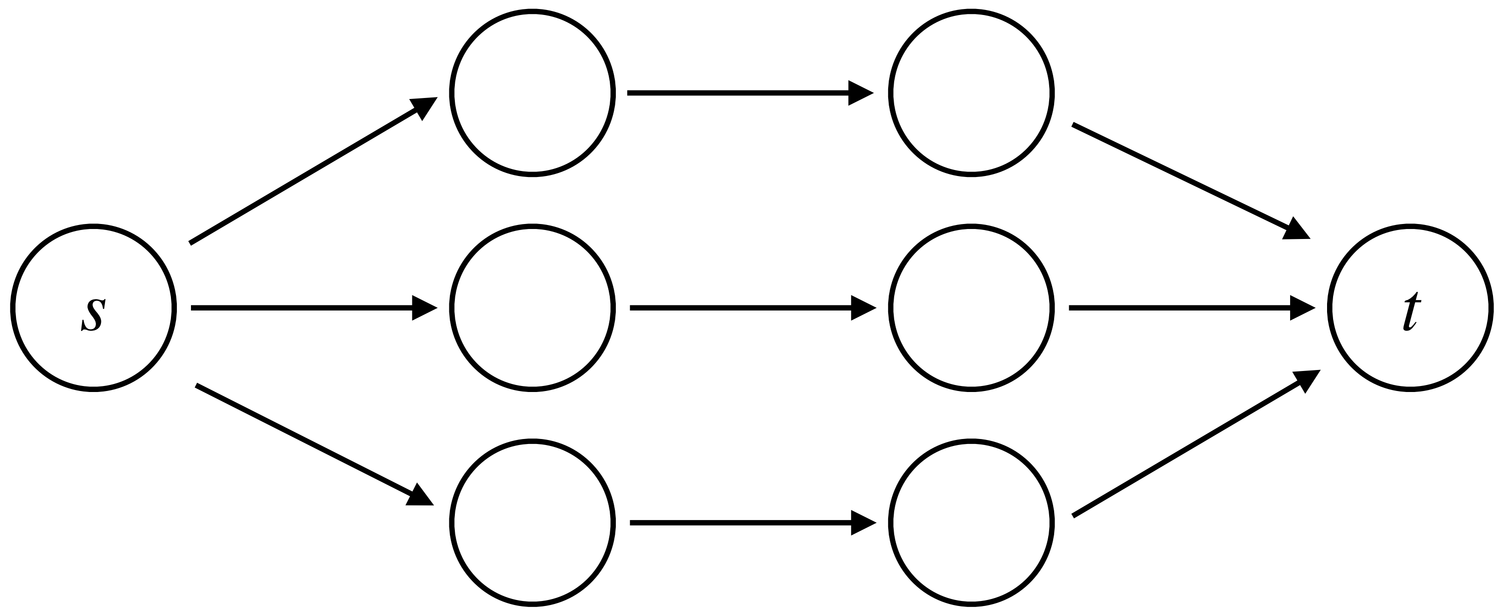


$$\Longrightarrow \quad g_1 g_2 \ldots g_n = I$$

Odd parity



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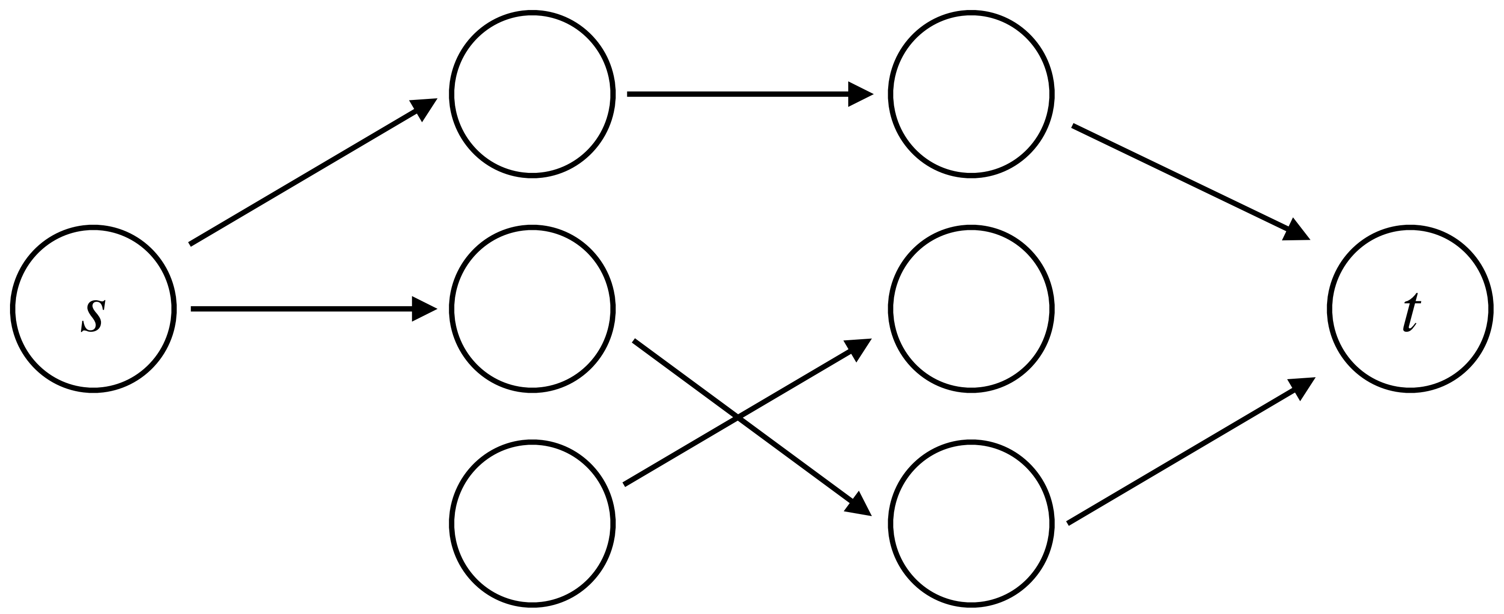

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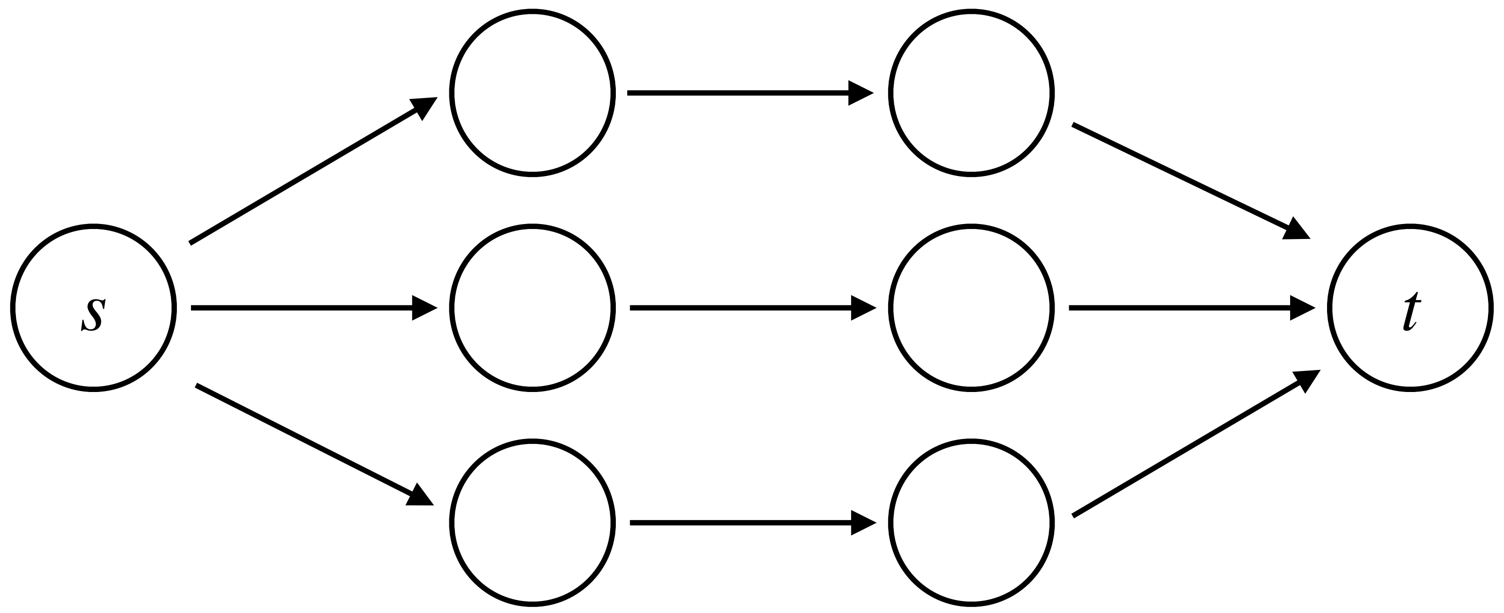

$$\implies \quad g_1 g_2 \cdots g_n = C_3$$

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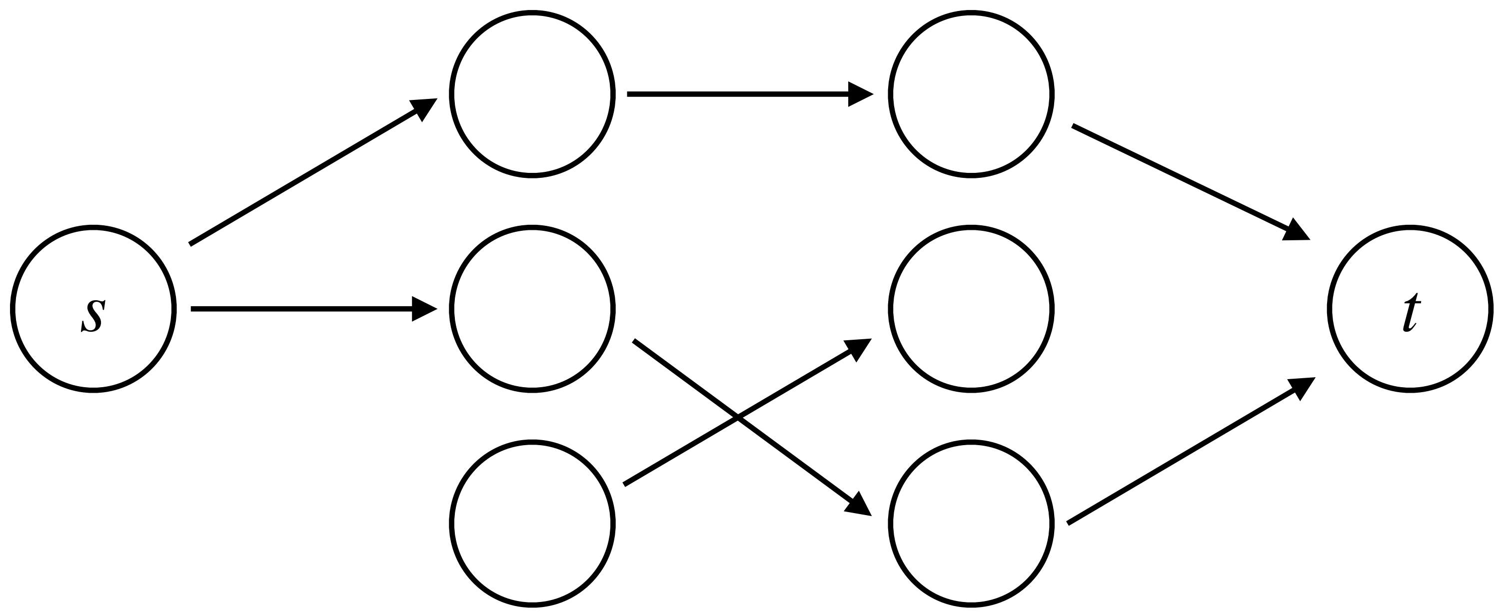

$\implies$ $g_1 g_2 \ldots g_n = I$

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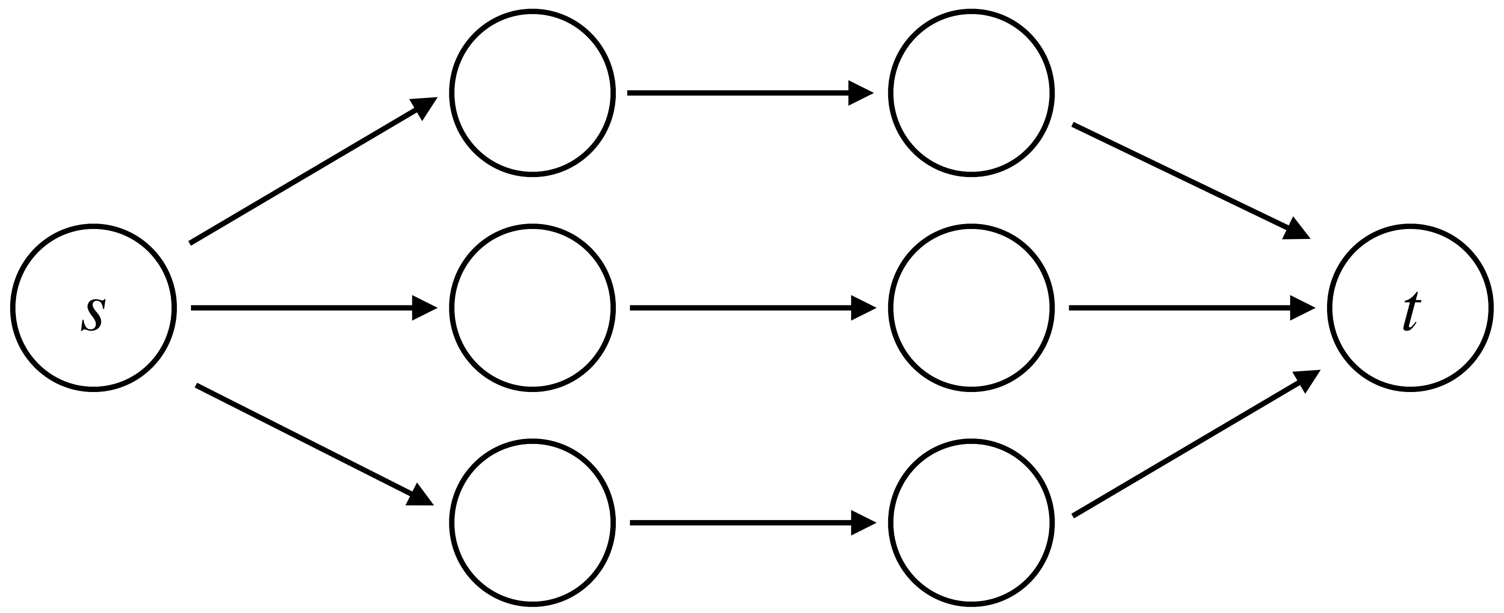

$\implies$ $g_1 g_2 \ldots g_n = C_3$

[GS20]: Measurement results from grid state determine whether $g_1 \ldots g_n = I$ or C_3.

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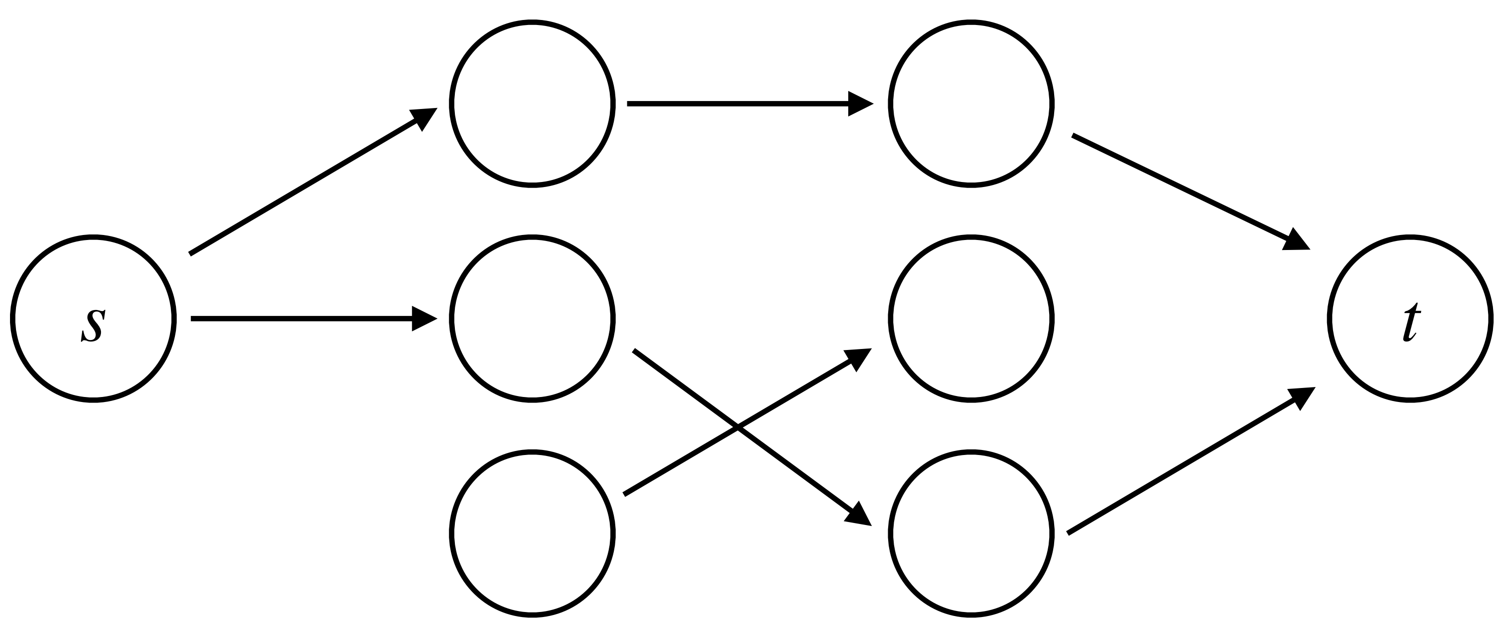

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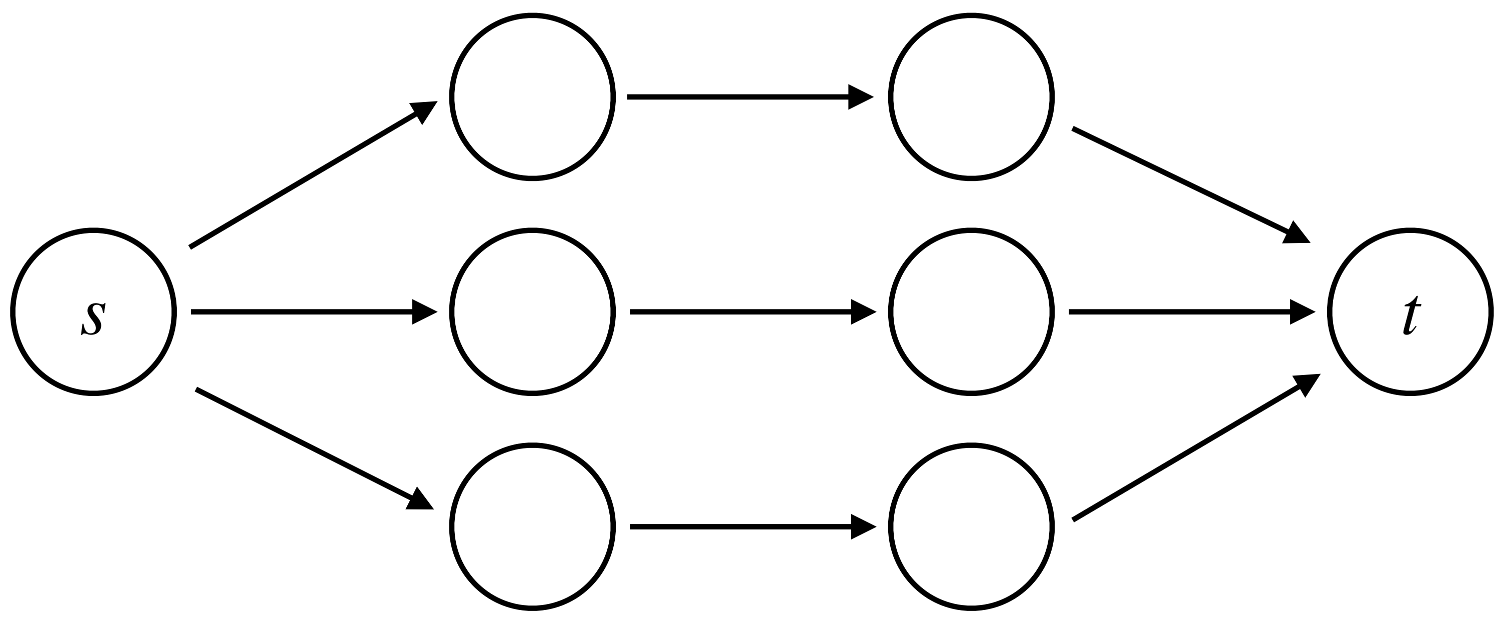

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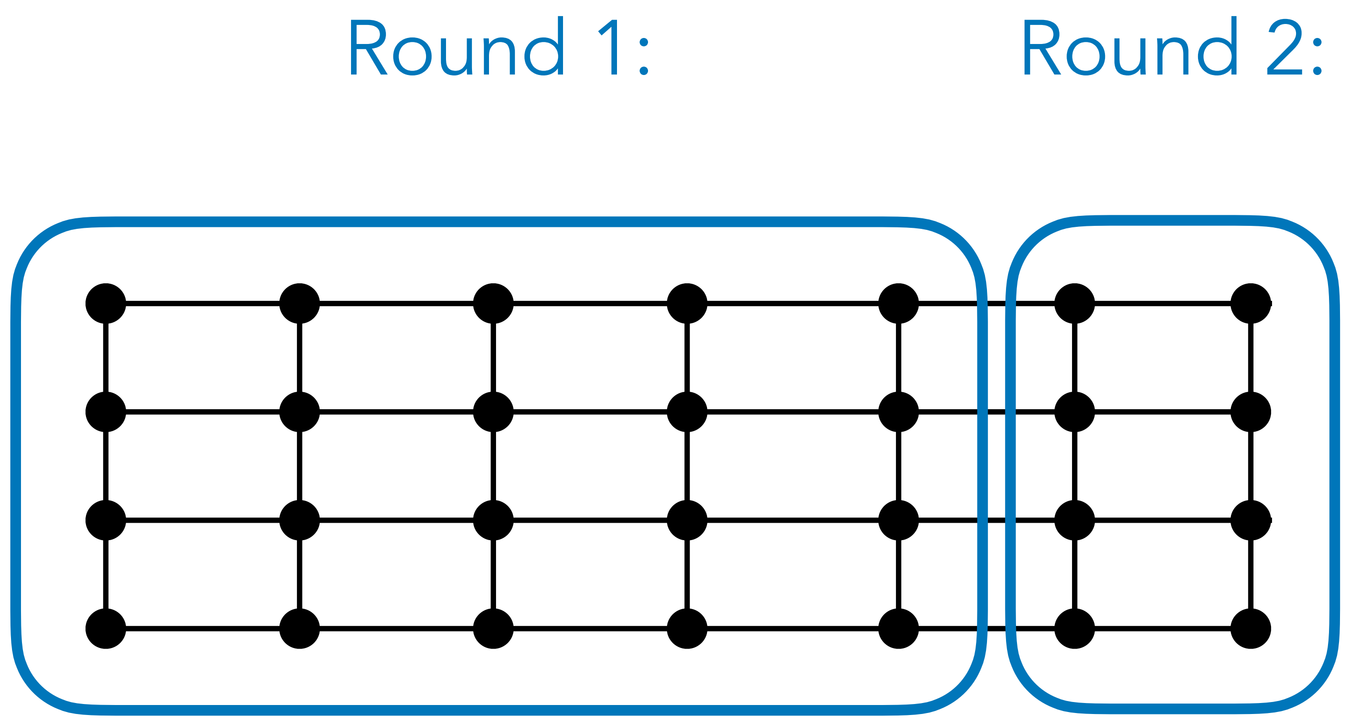
[GS20]: Measurement results from grid state determine whether $g_1 \ldots g_n = I$ or C_3.

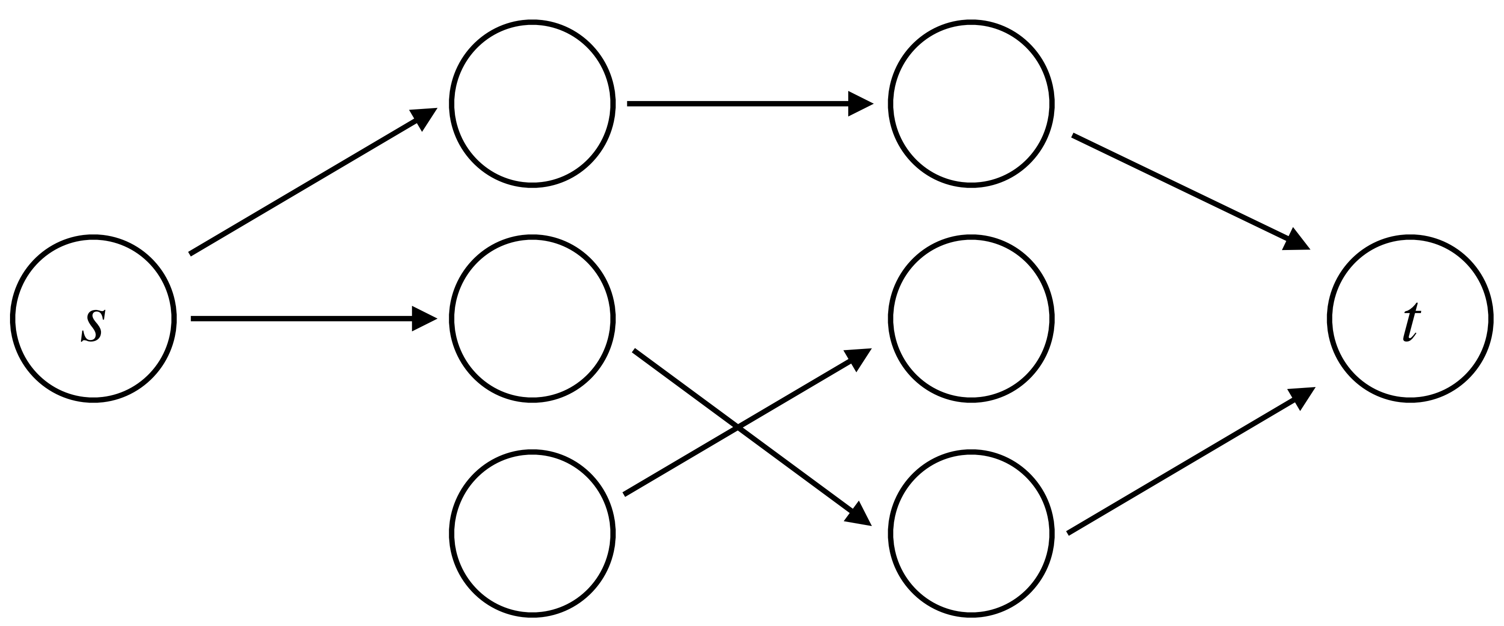


Worst-case ⊕L-hard problems

It is ⊕L-hard to determine the parity of the number of $s \to t$ paths in a (layered) DAG.

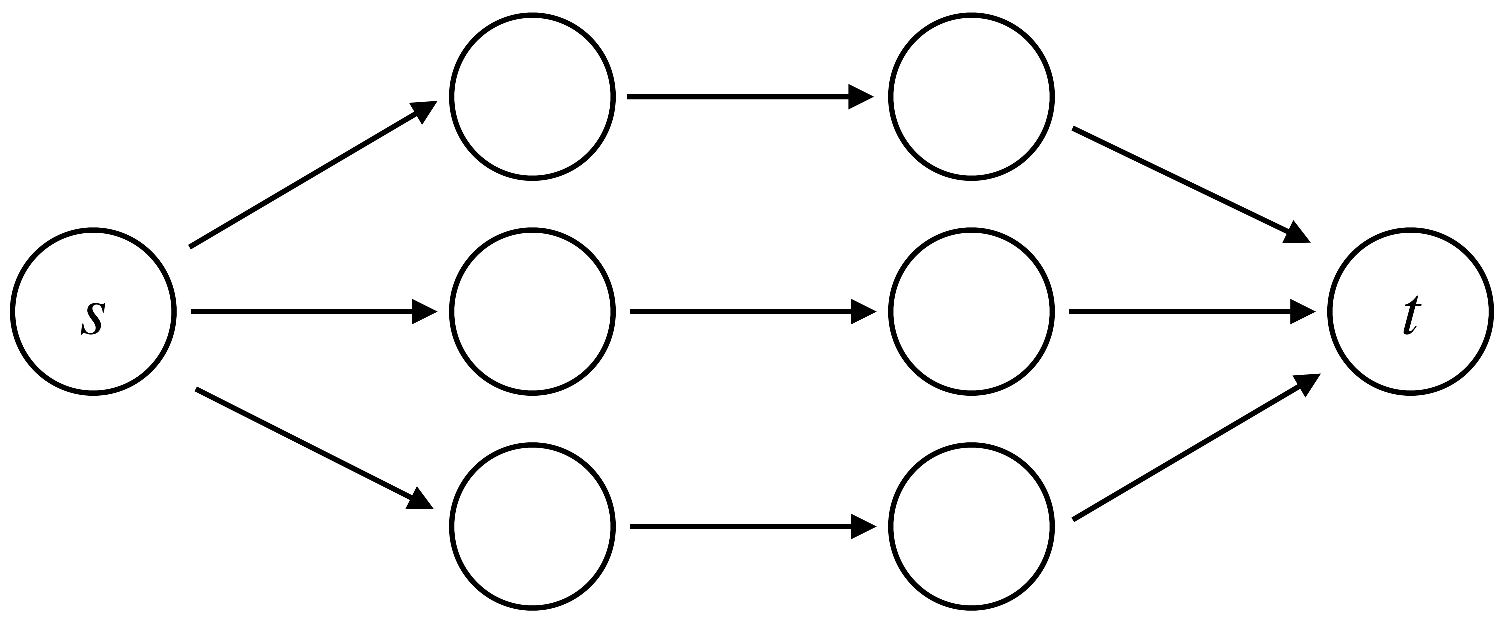
[GS20]: It is also ⊕L-hard to determine whether CNOT gates $g_1, \ldots, g_n$ multiply to 3-cycle or identity.

Even parity



$\Longrightarrow \quad g_1 g_2 \cdots g_n = I$

Odd parity



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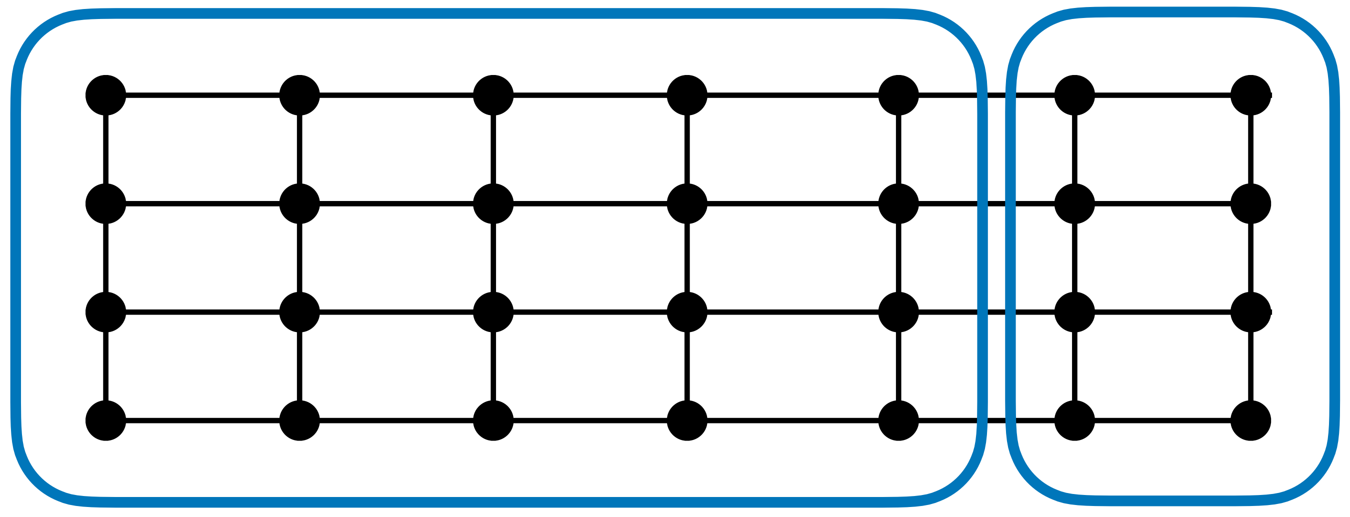

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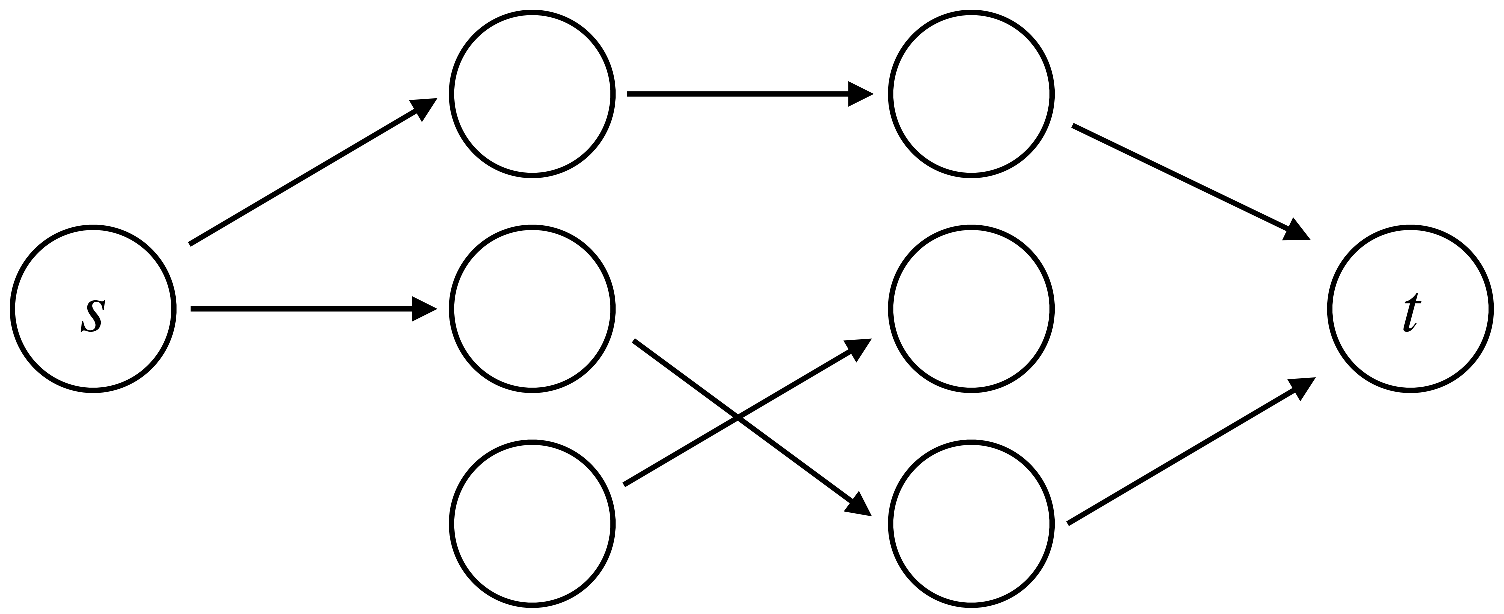

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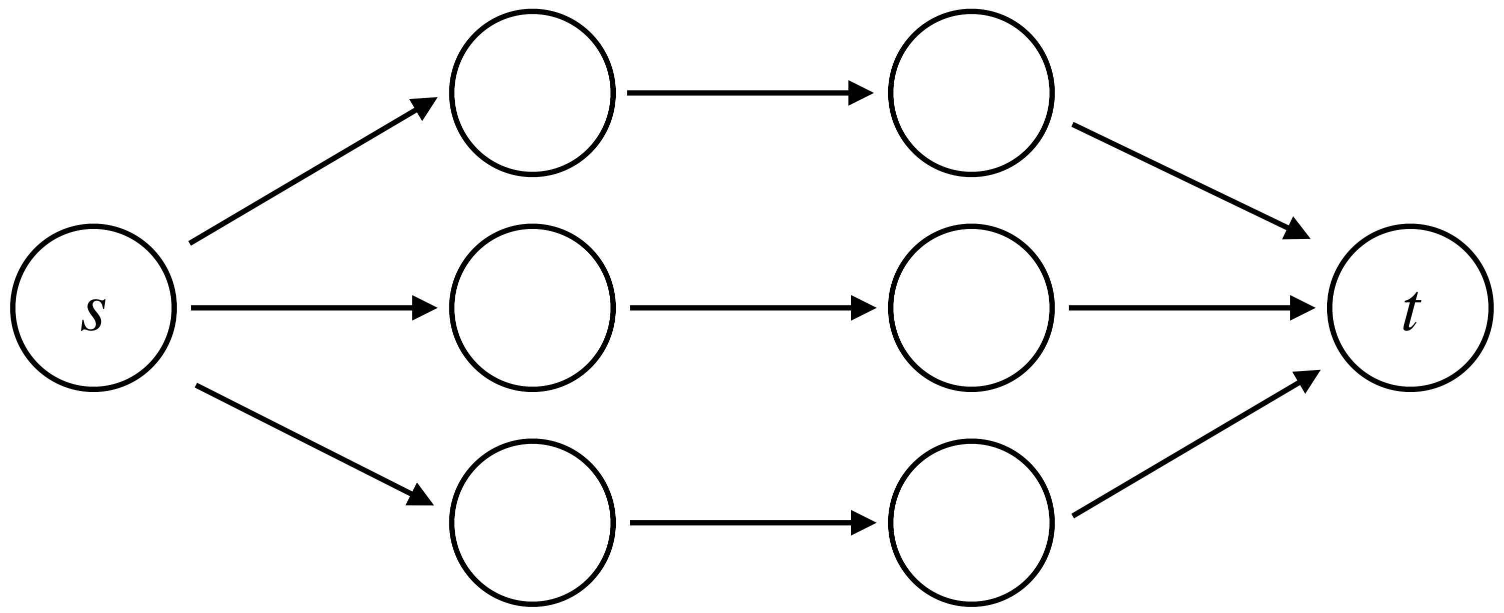

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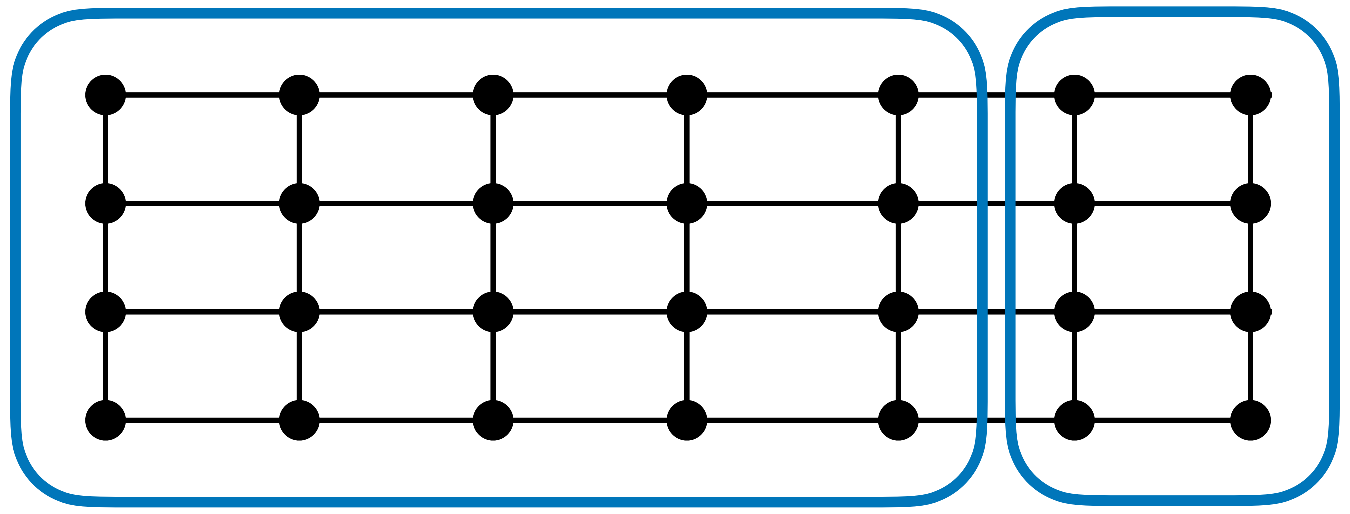

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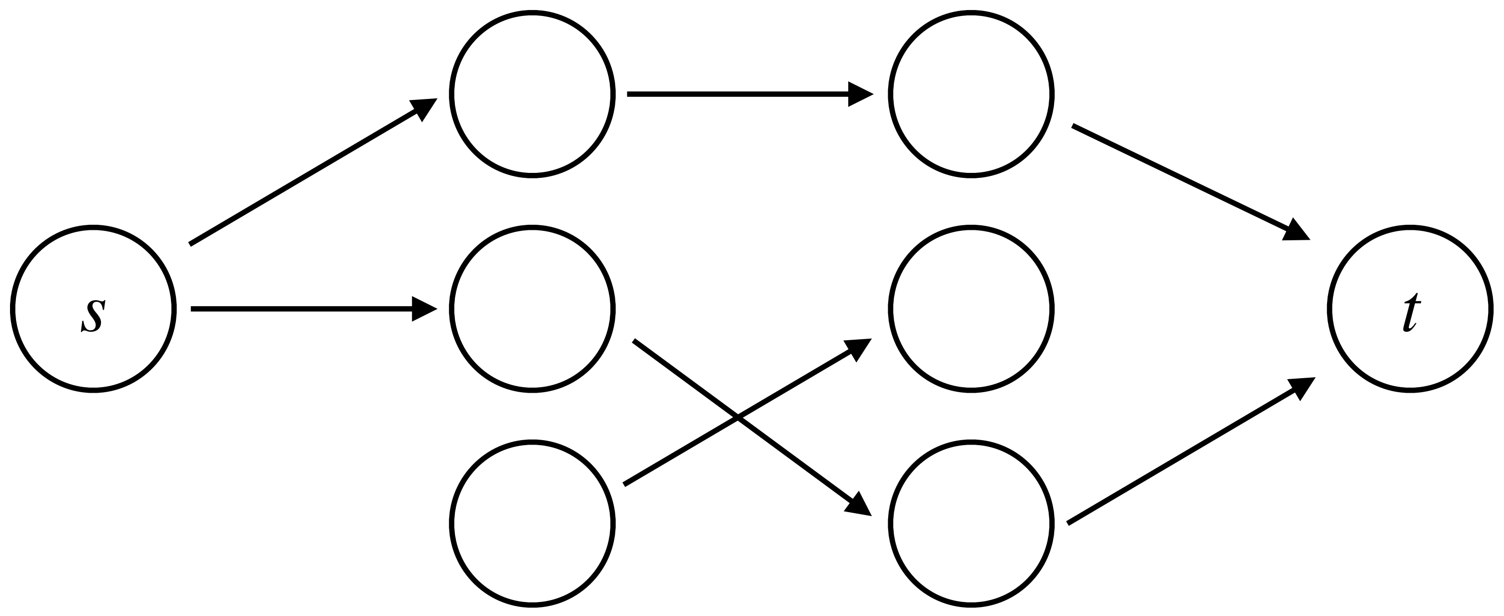

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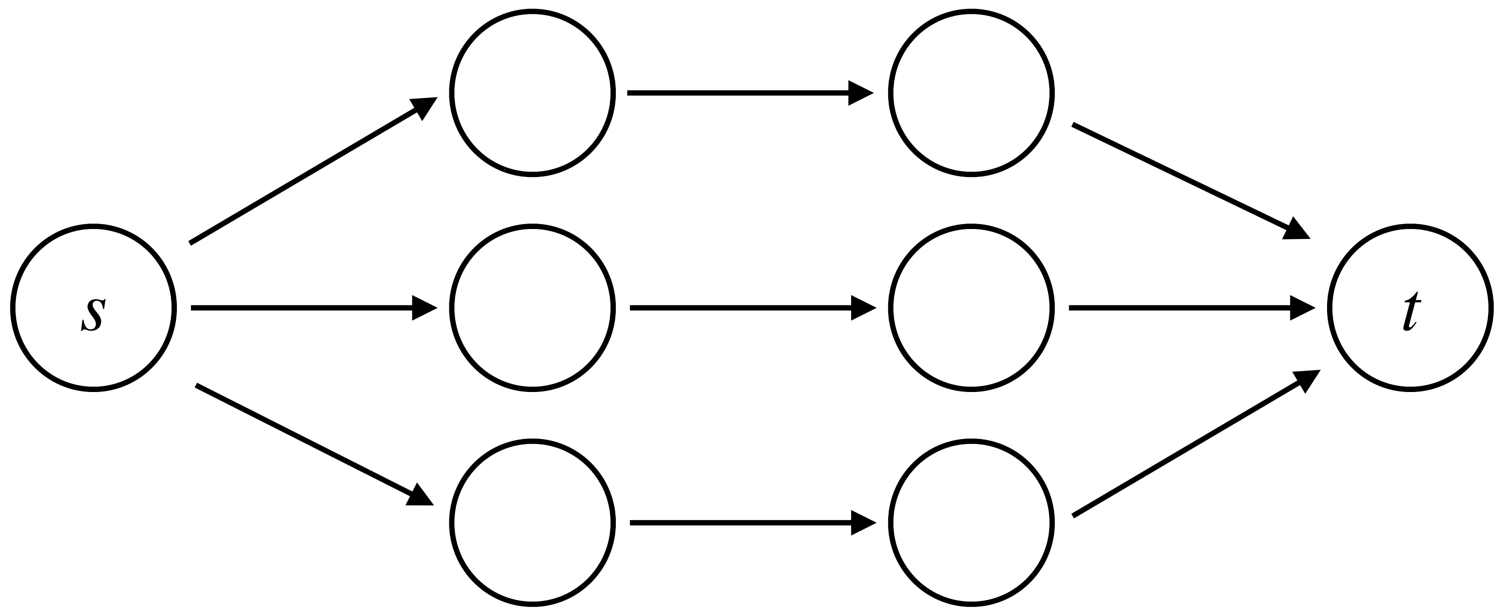

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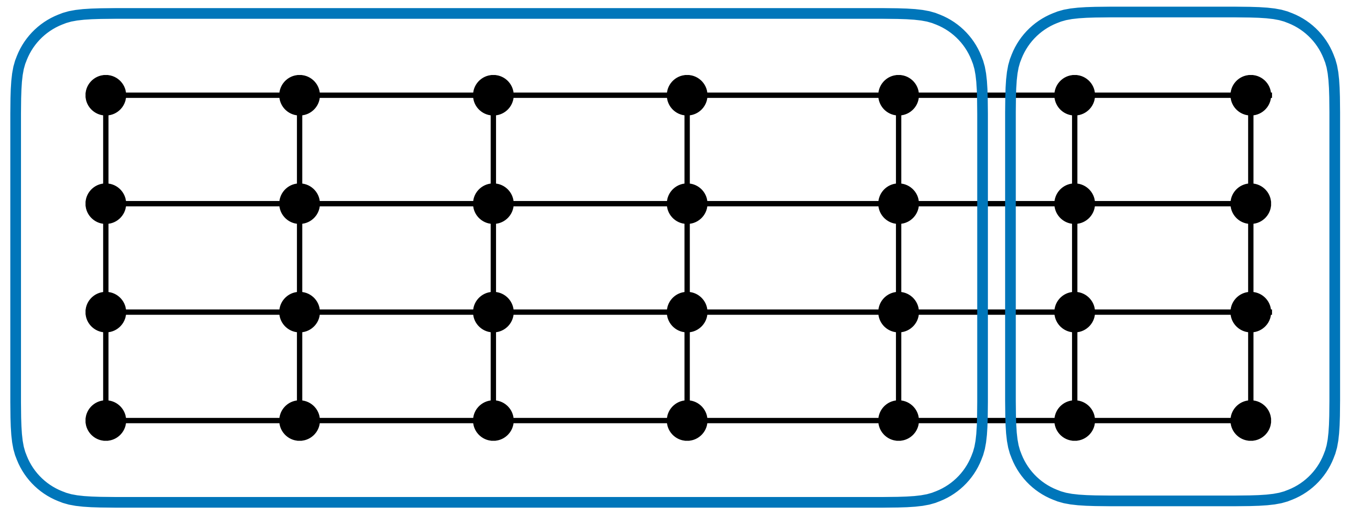

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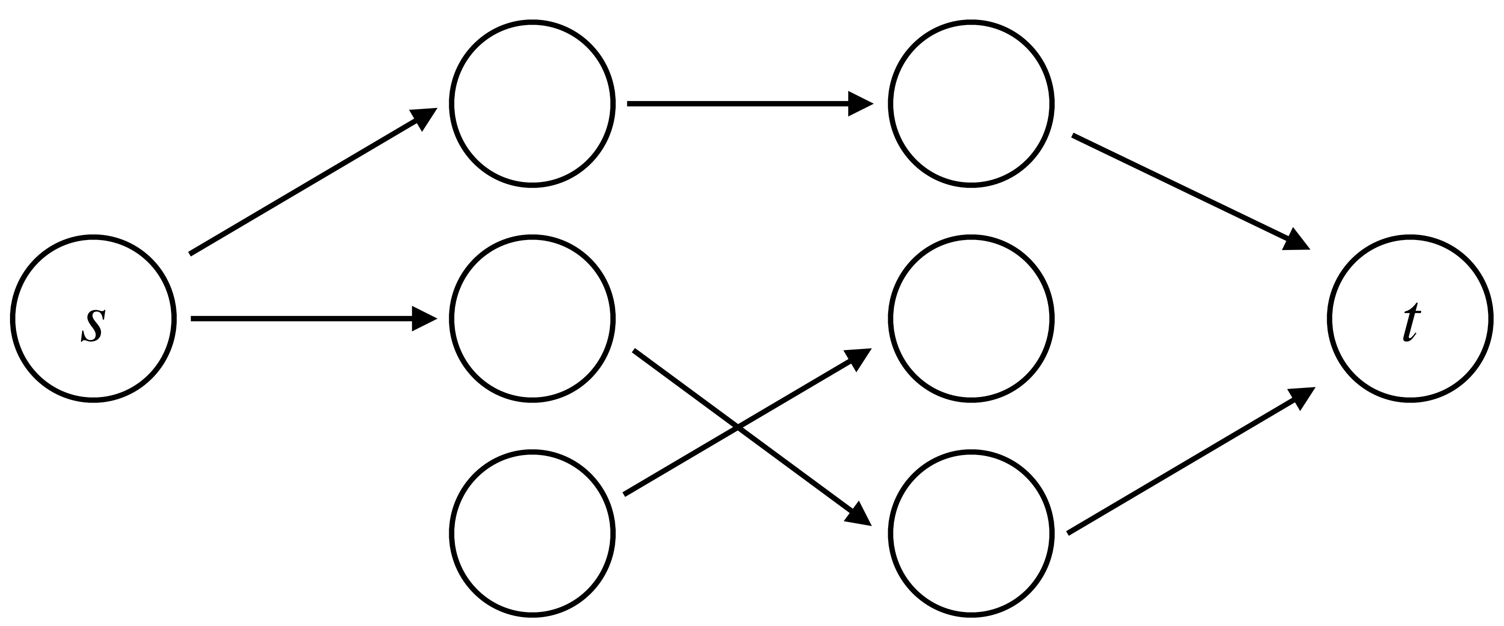

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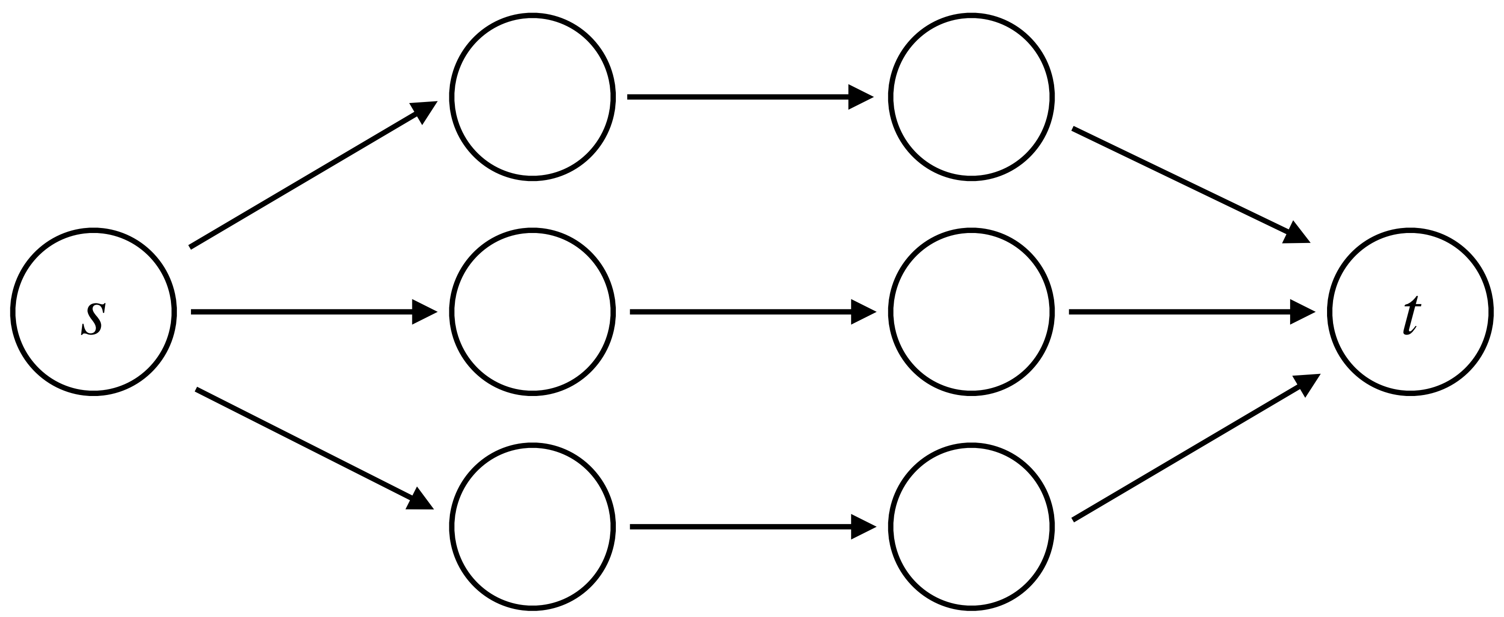

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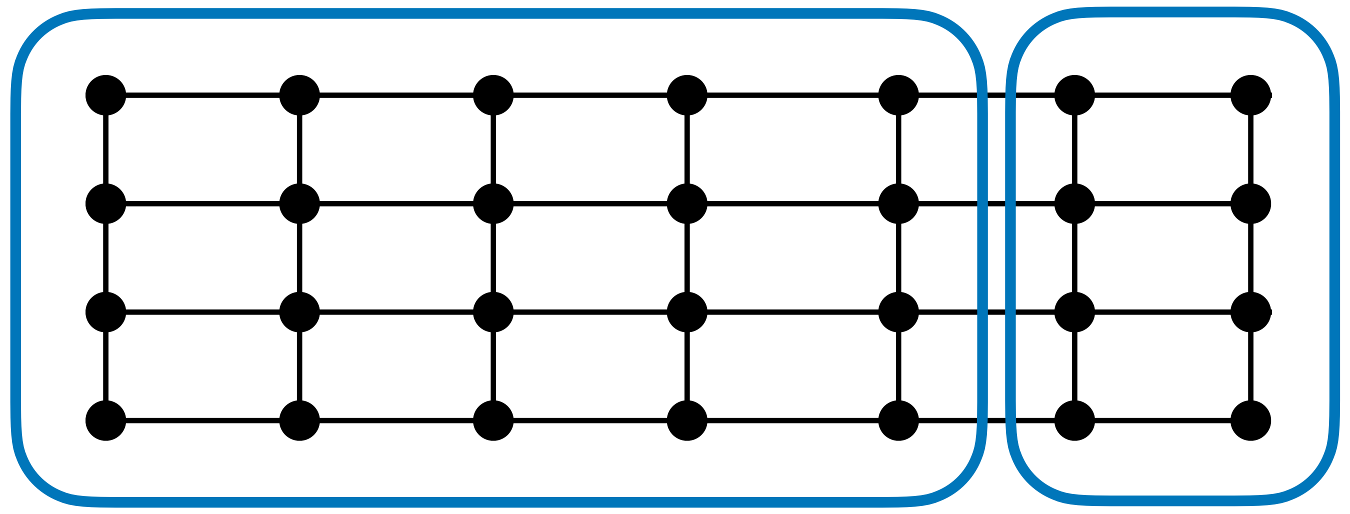

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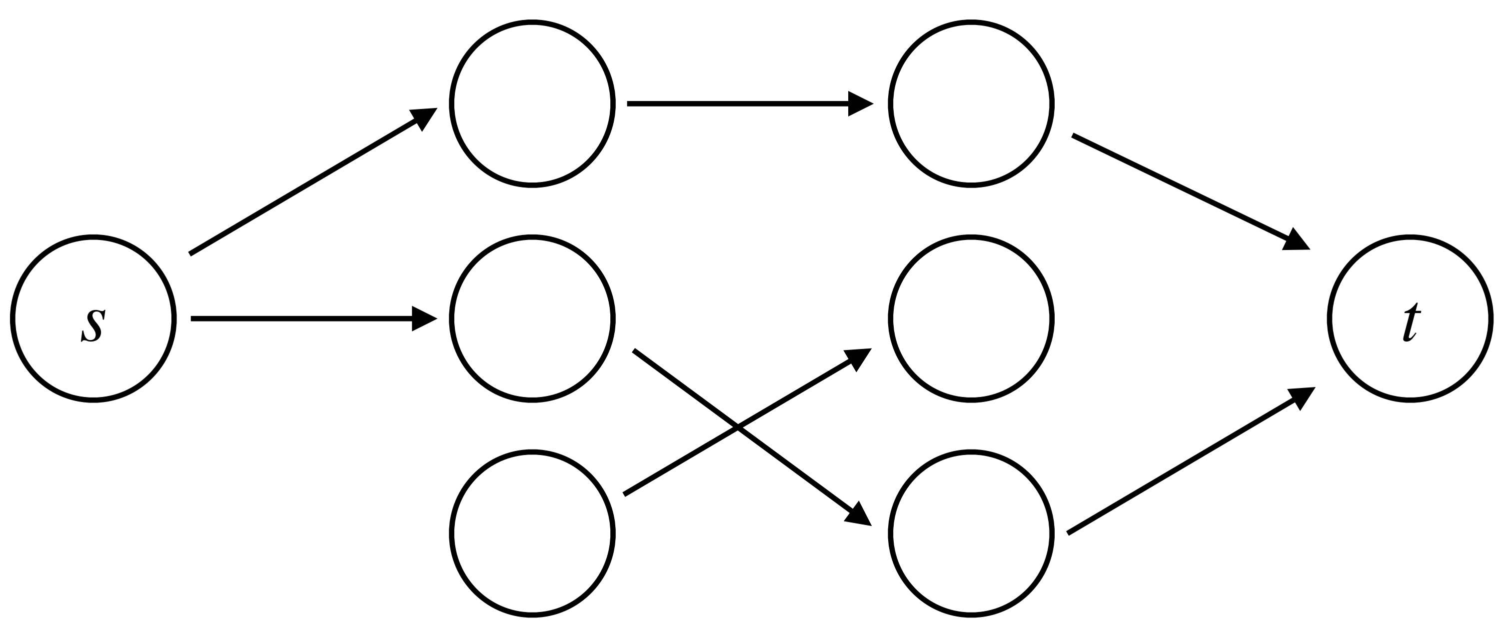

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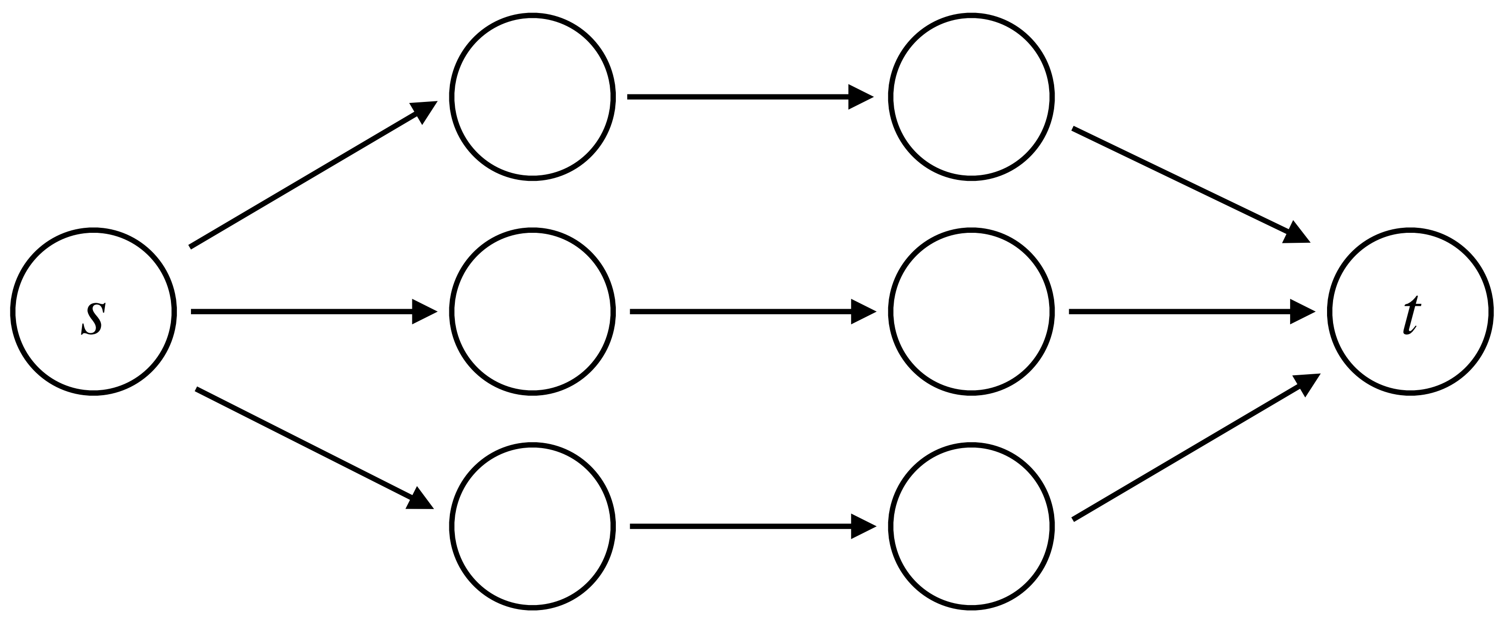

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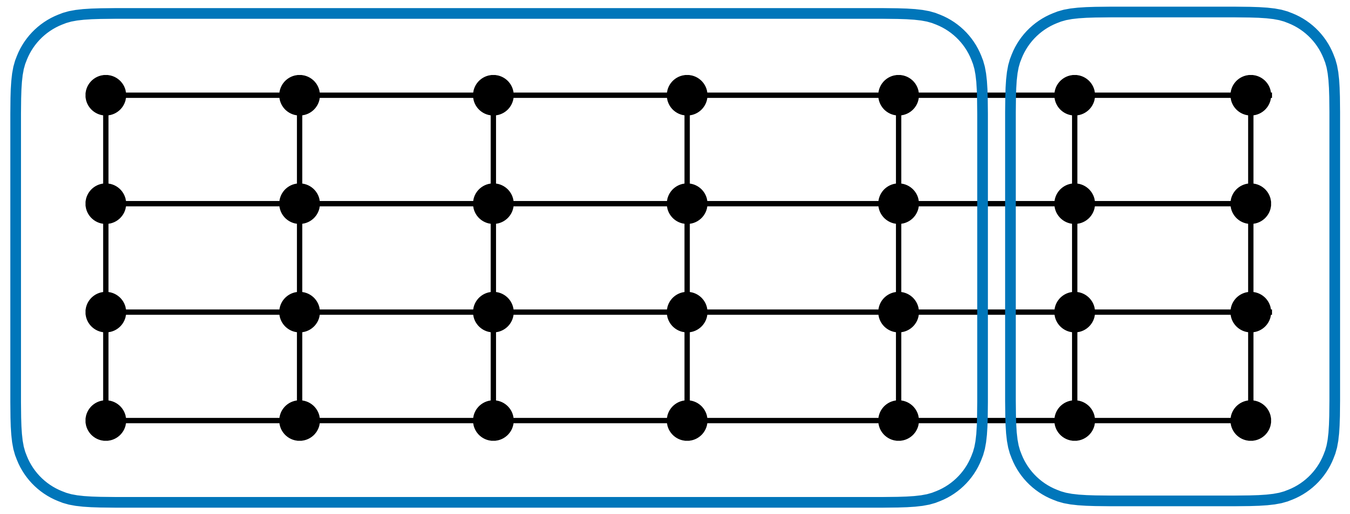

Problem: This only gives us worst-case ⊕L-hardness: $\gamma(g_1, \dots, g_n)$ does not produce "random" instances to first round input

The worst-to-average-case reduction

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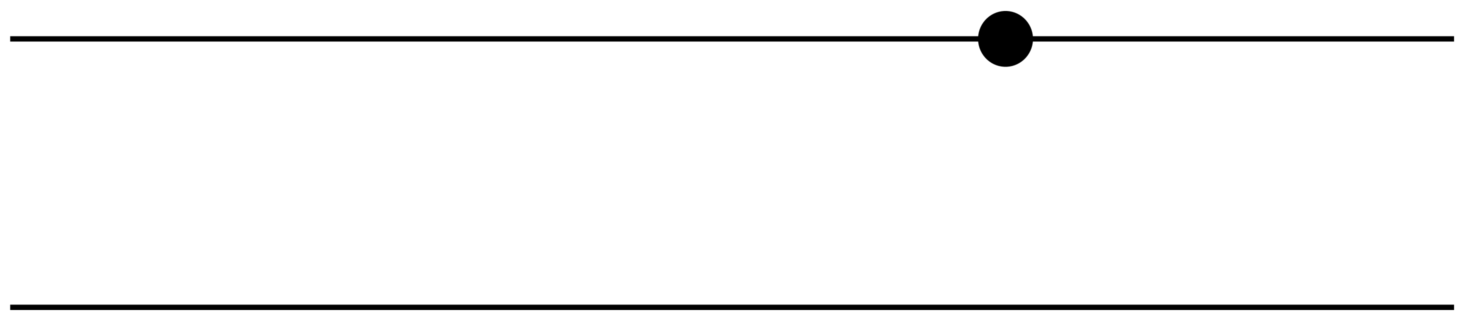

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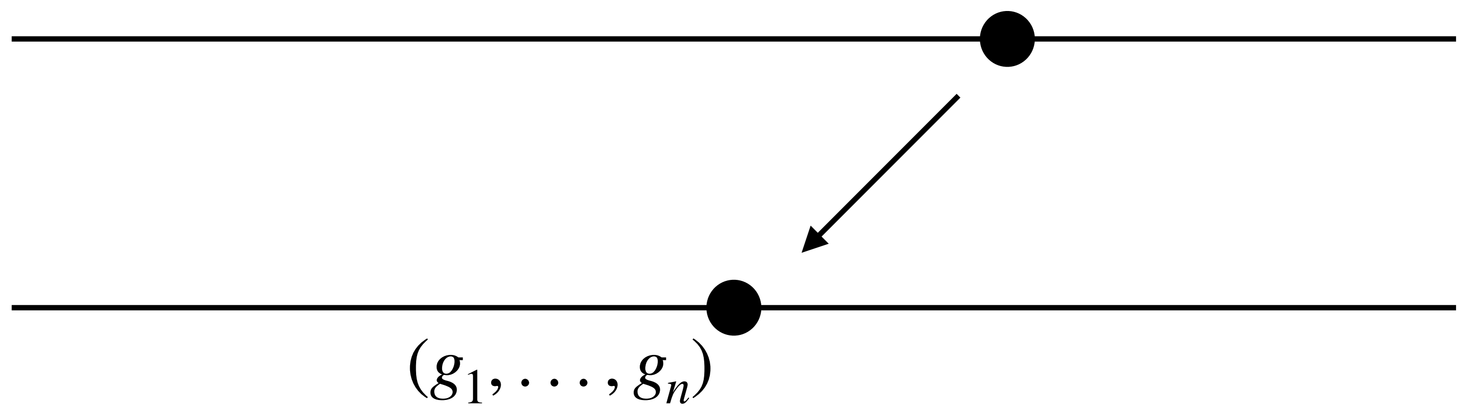

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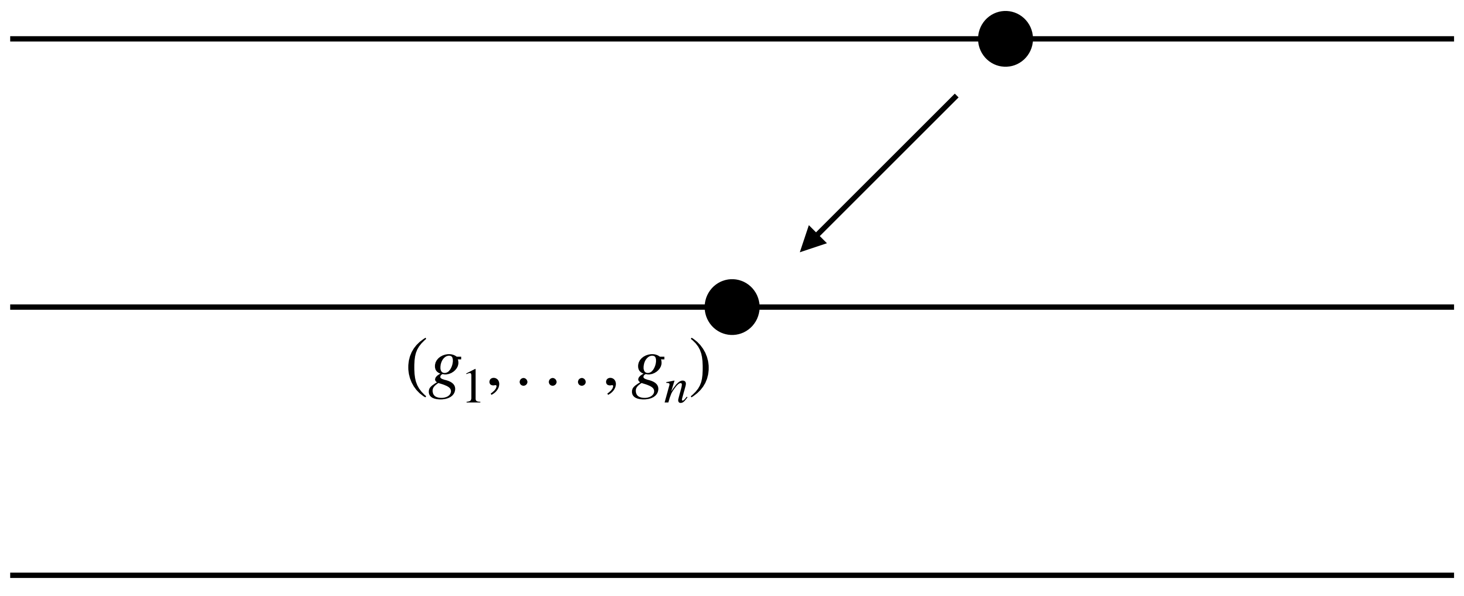

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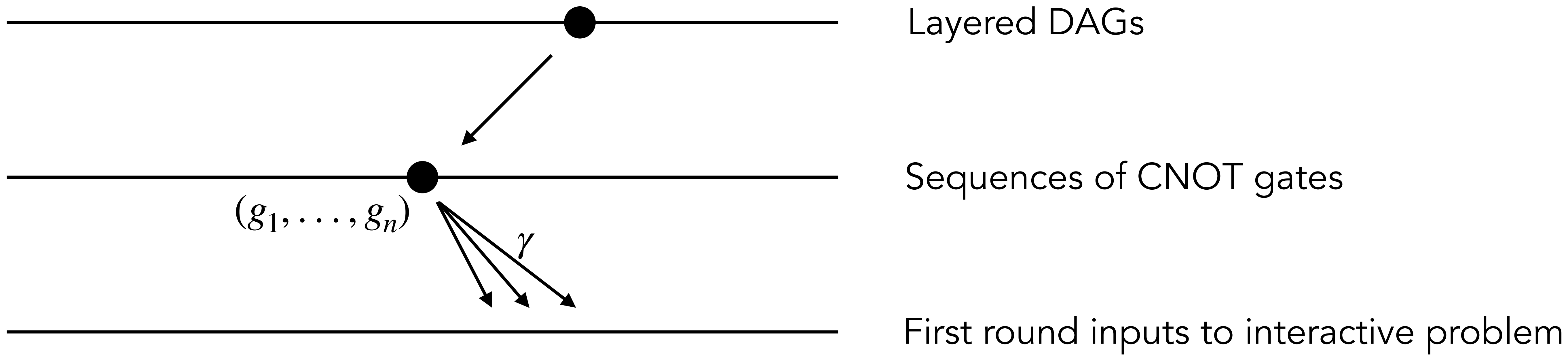

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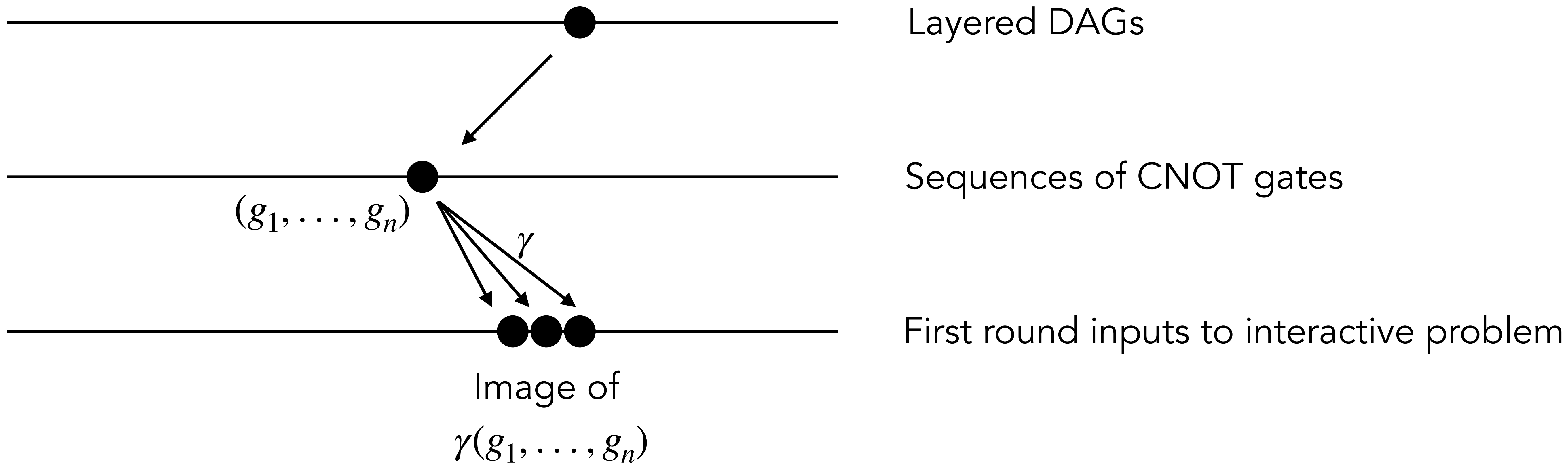

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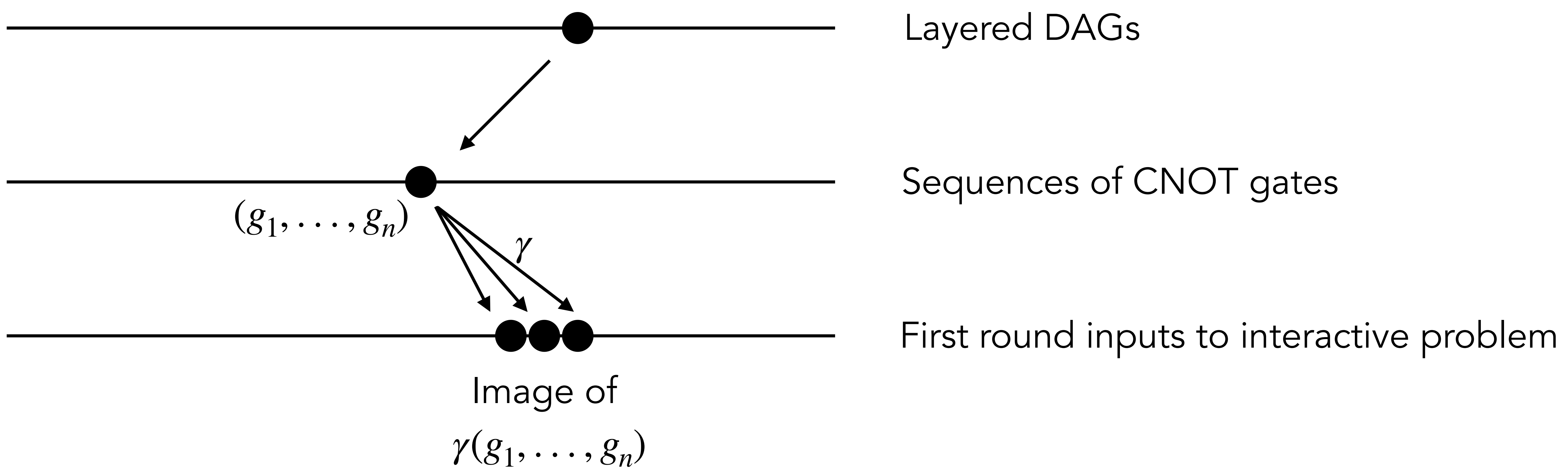

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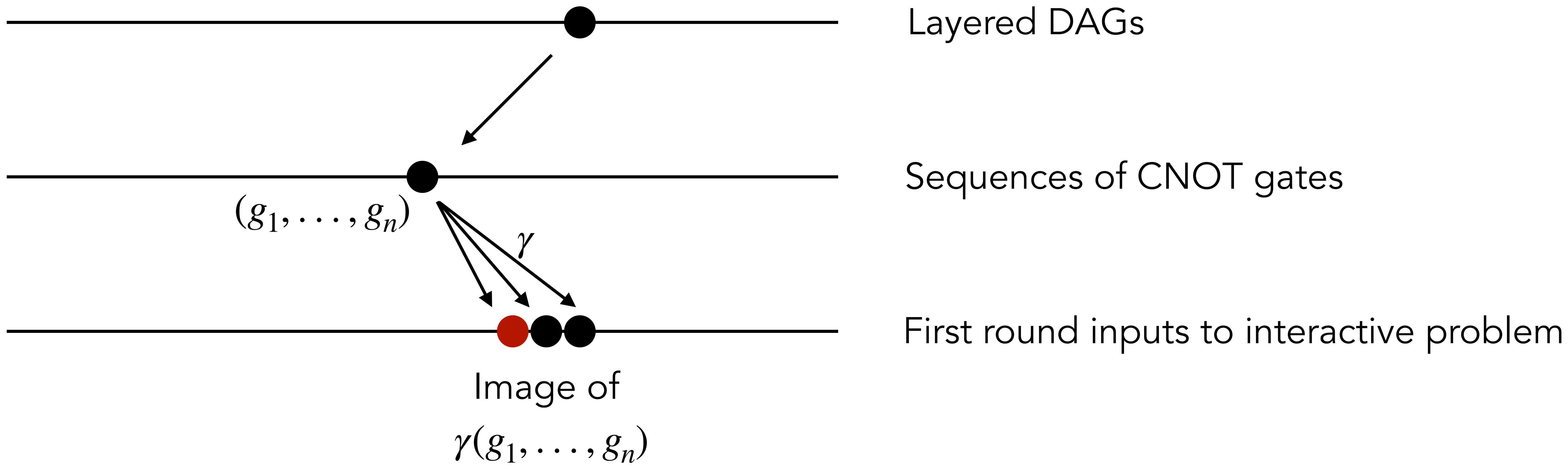

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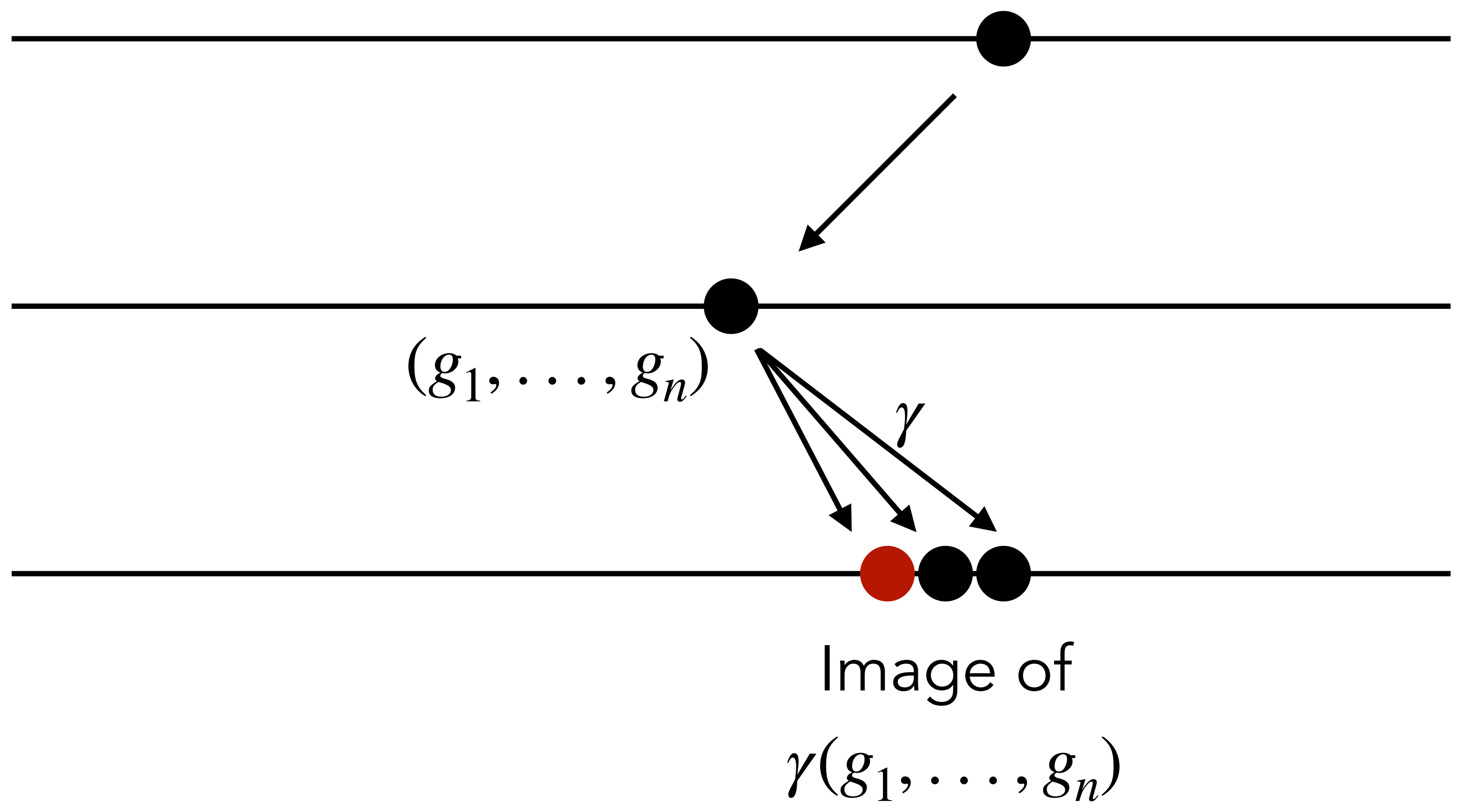

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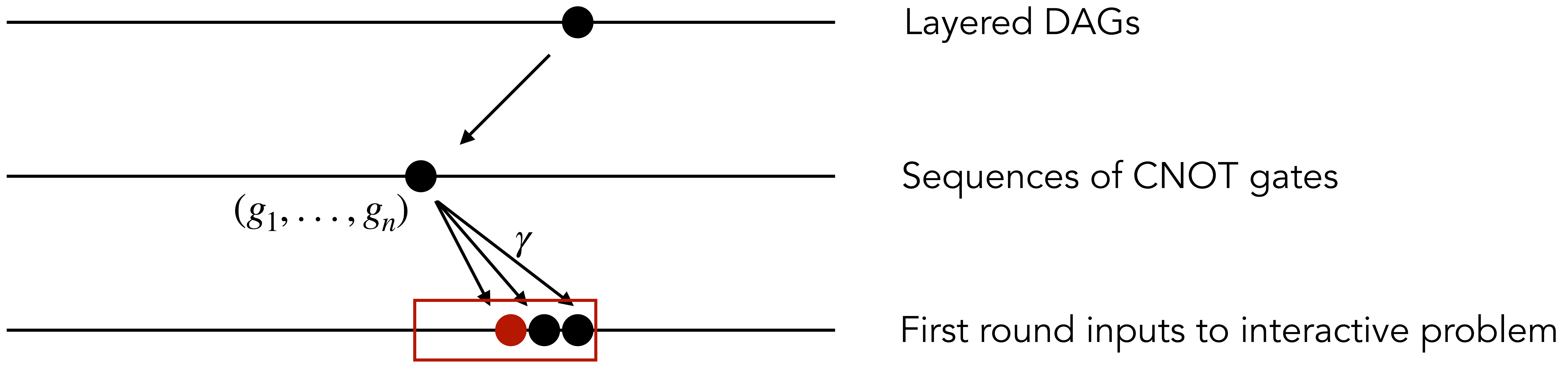

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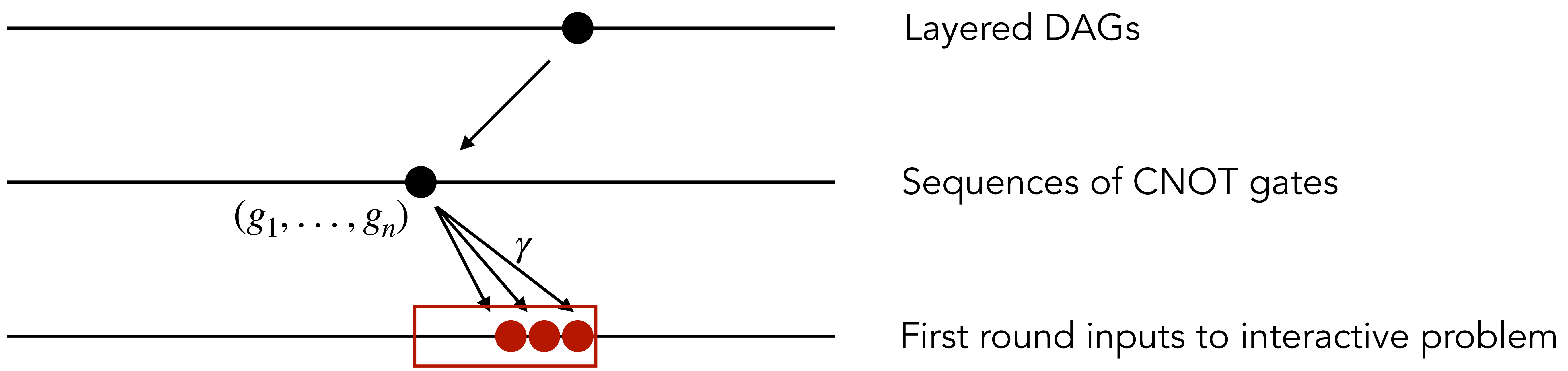

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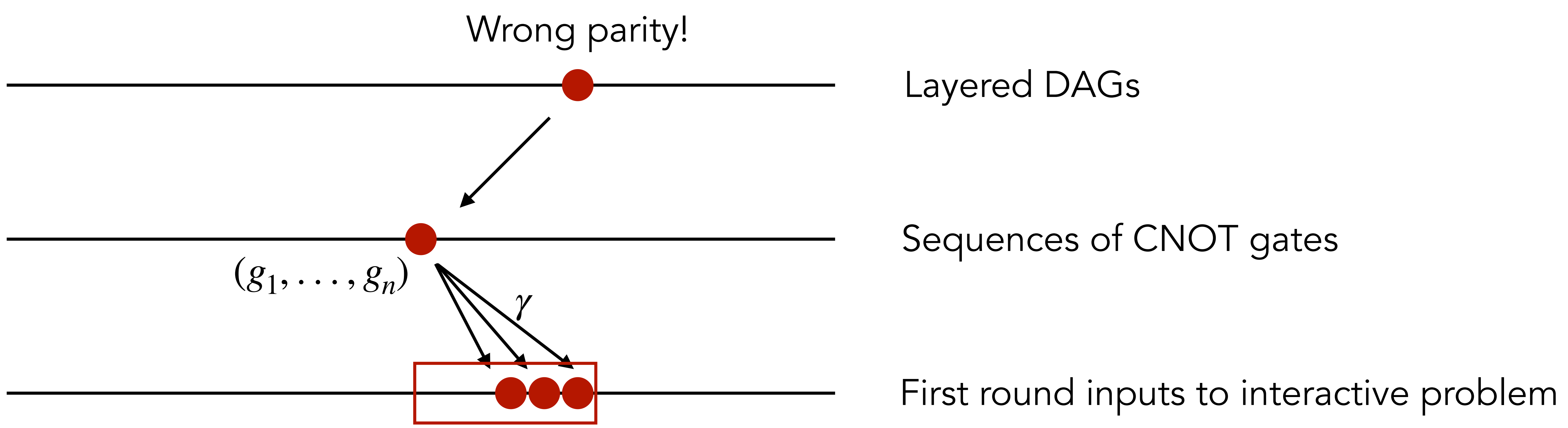

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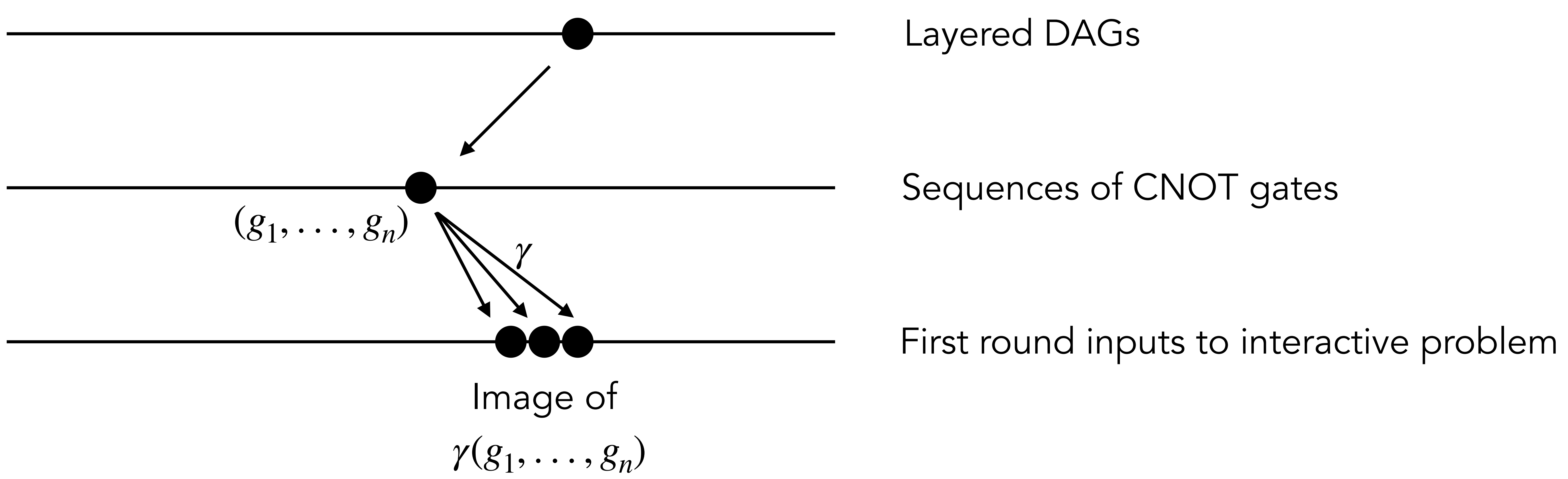

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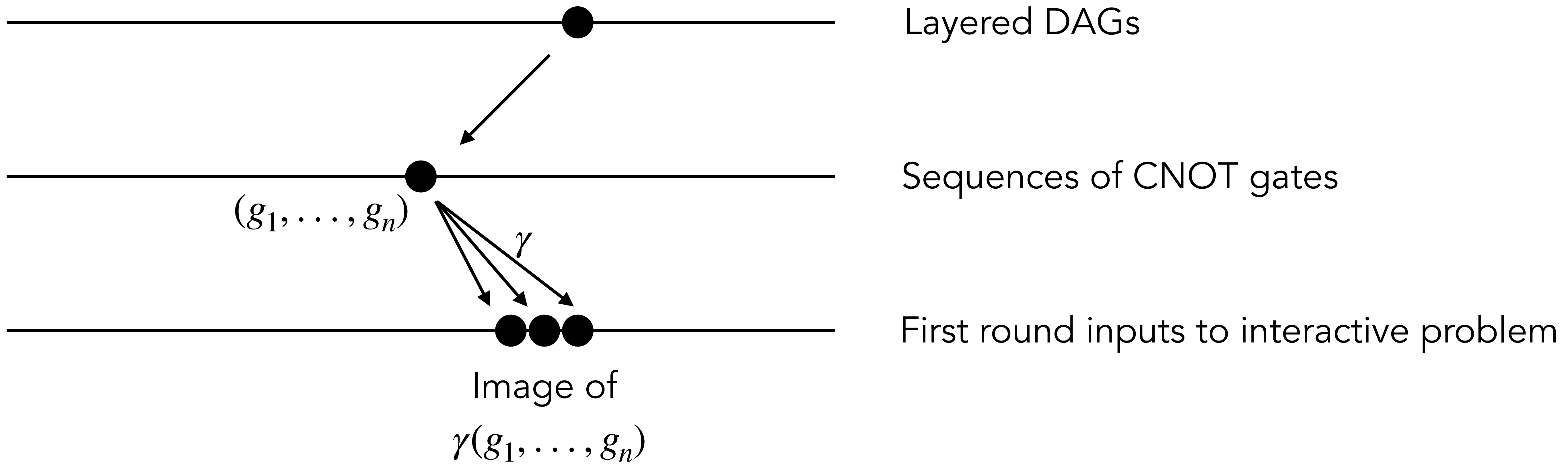

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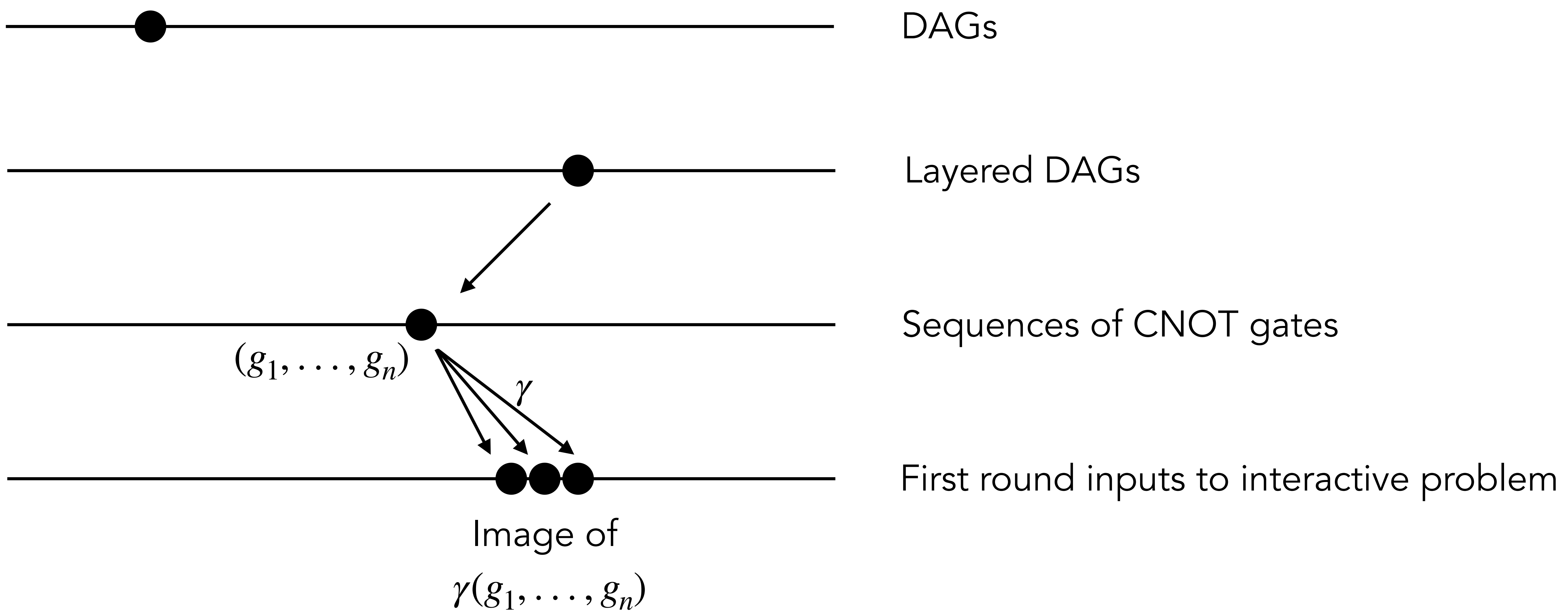

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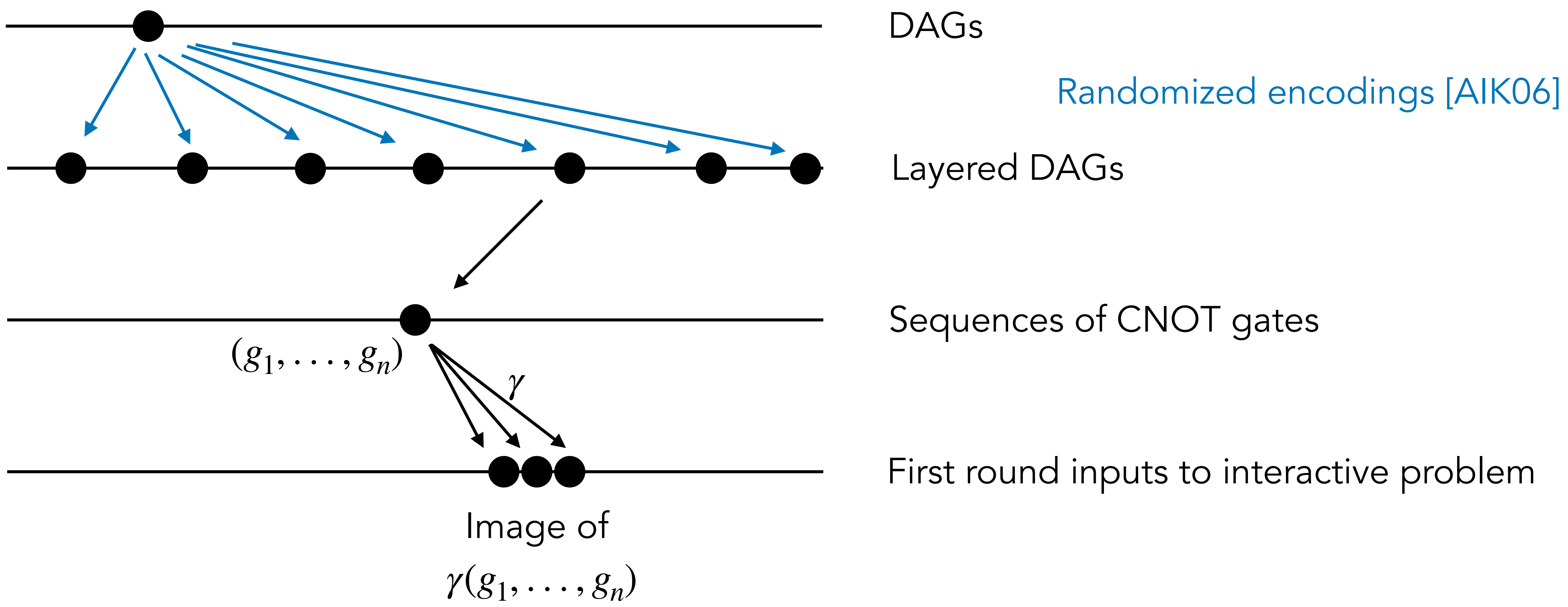

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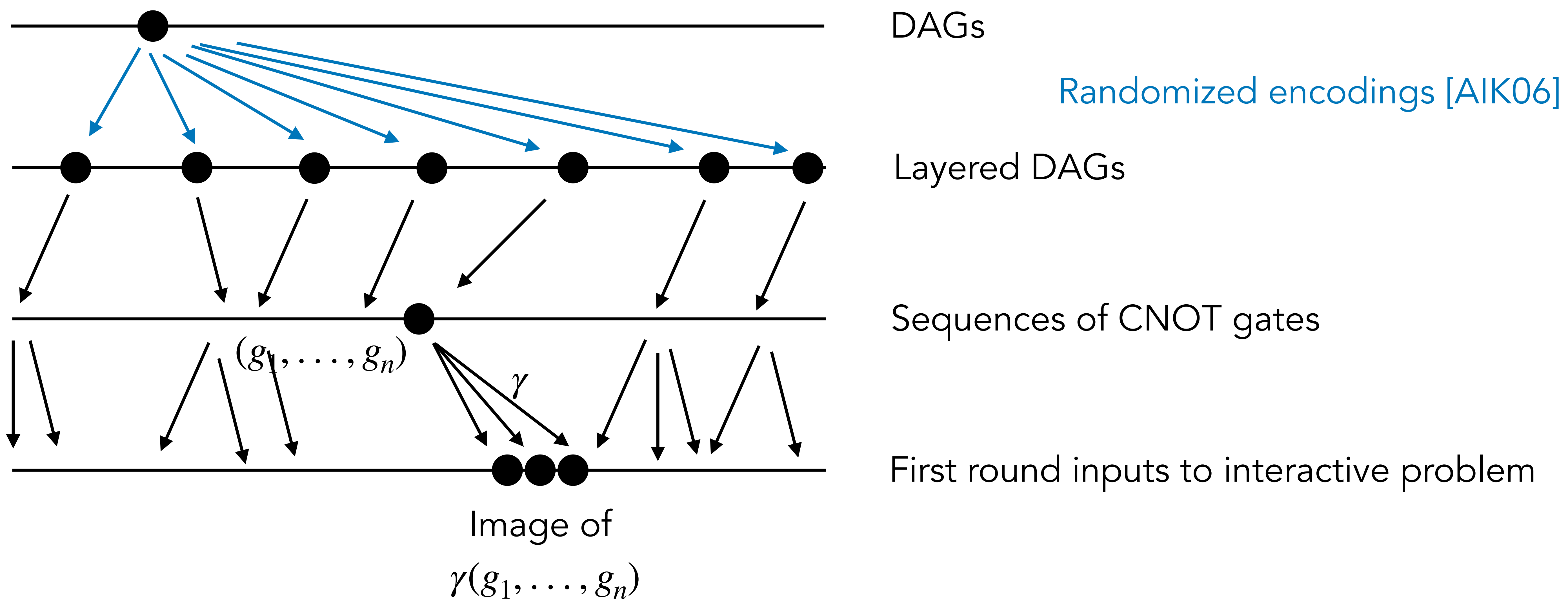

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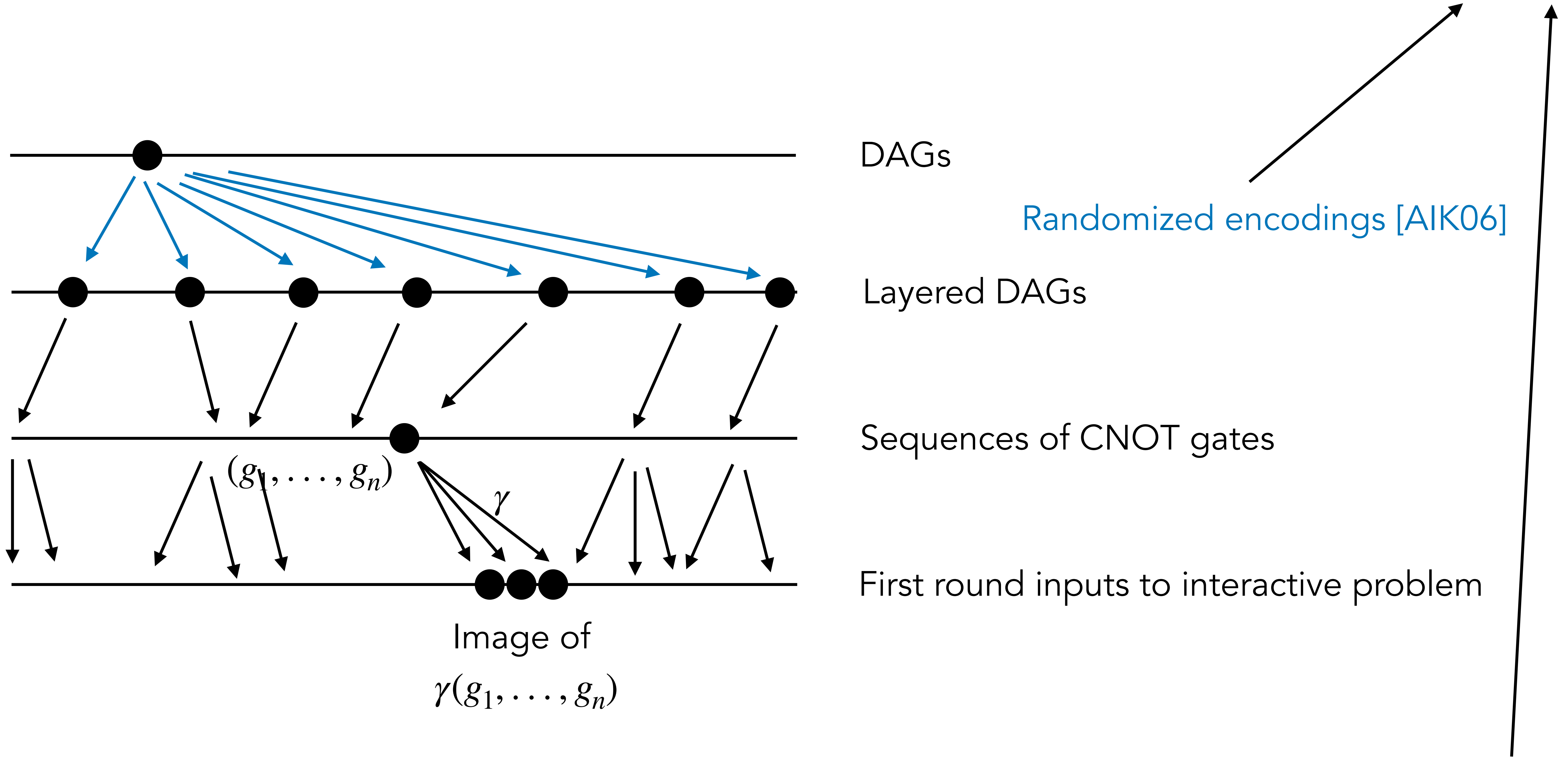
Average-case ⊕L-hard:
Let $\mathscr{R}$ be a classical probabilistic machine that solves the interactive task w/p $420/421$ over uniform input. Then ⊕L $\subseteq (\mathsf{AC}^0)^{\mathscr{R}}$.



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Conclusion

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Unconditional separation between noisy QNC^0 circuits and $\mathrm{AC}^0[p]$ circuits

Conditional separation between noisy QNC^0 circuits and log-space machines

Open problems. Questions?

- We show a $\oplus$L-hardness threshold of $420/421$. Can we go lower than this?
  - Worst-to-average-case reductions with stronger randomization?
  - Direct product theorems?
- We have unconditional, noisy separations for relation and interactive problems against constant-depth classical circuits. What about other types of problems, e.g., sampling?
- Can we base our conditional result on simply $\oplus$L $\neq$ L?