Adaptive Reprogramming in the QROM

QIP 2012
Virtual

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Christian Majenz

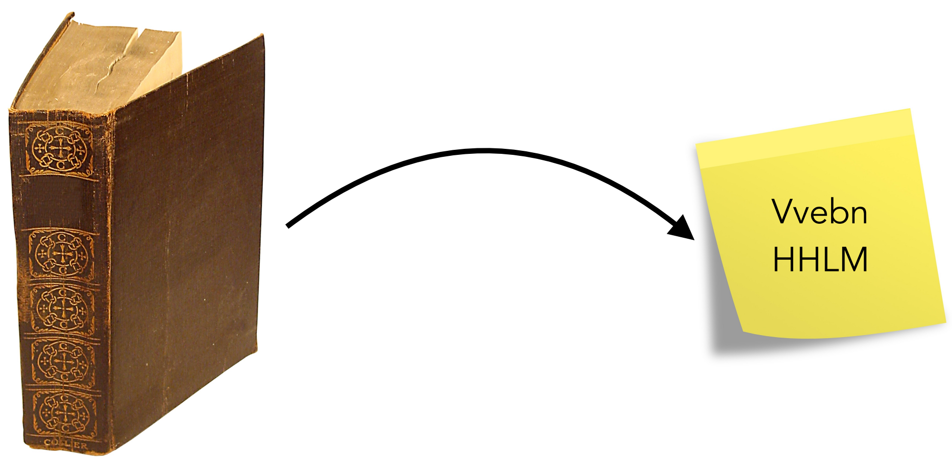
Outline

- Motivation — the quantum random oracle model
- The adaptive reprogramming game
- Results
- Reprogramming superposition oracles
- A matching algorithm

Motivation — The Quantum Random Oracle Model (QROM)

Hash functions

Hash functions are everywhere in crypto



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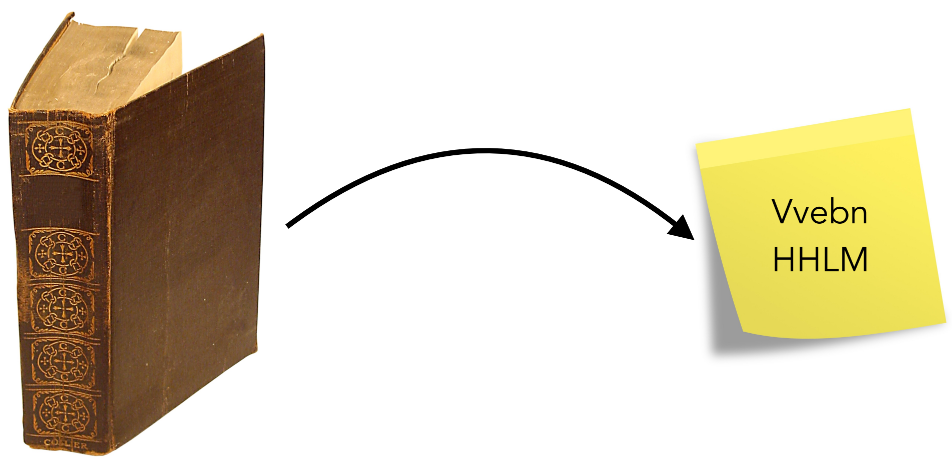
- Digital signatures
- Message authentication
- Chosen-ciphertext security
- Commitments
- …



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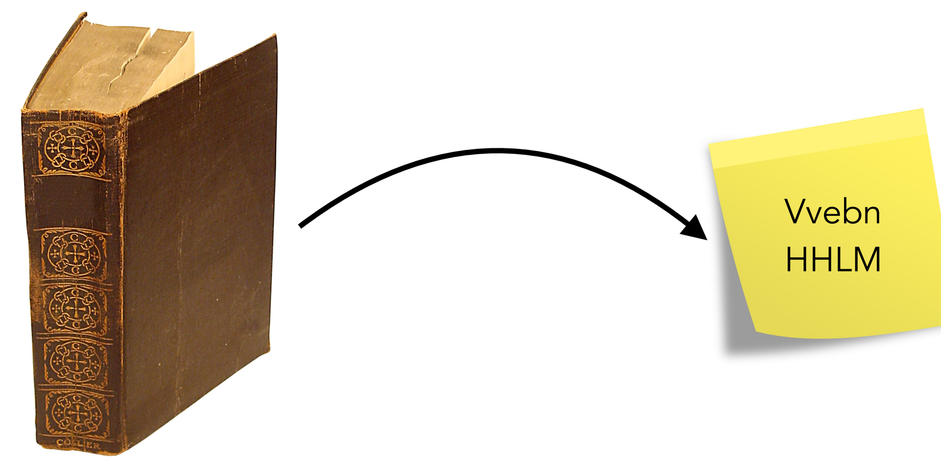

Concept: simple

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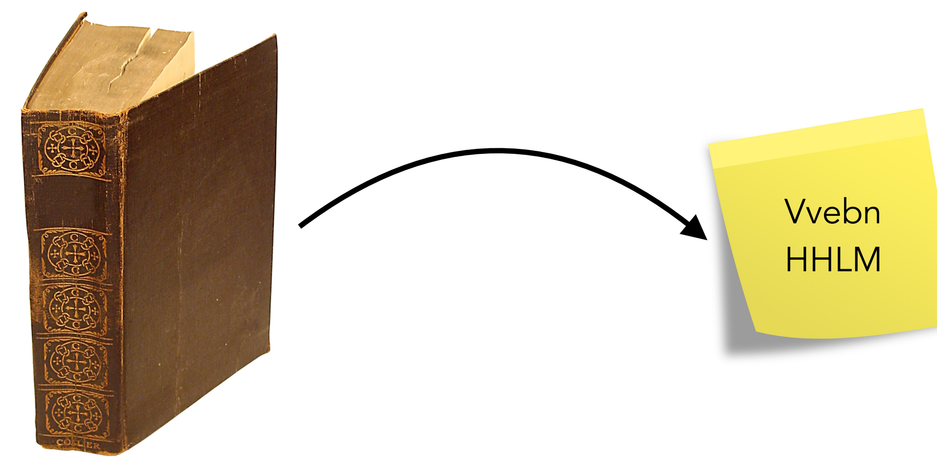

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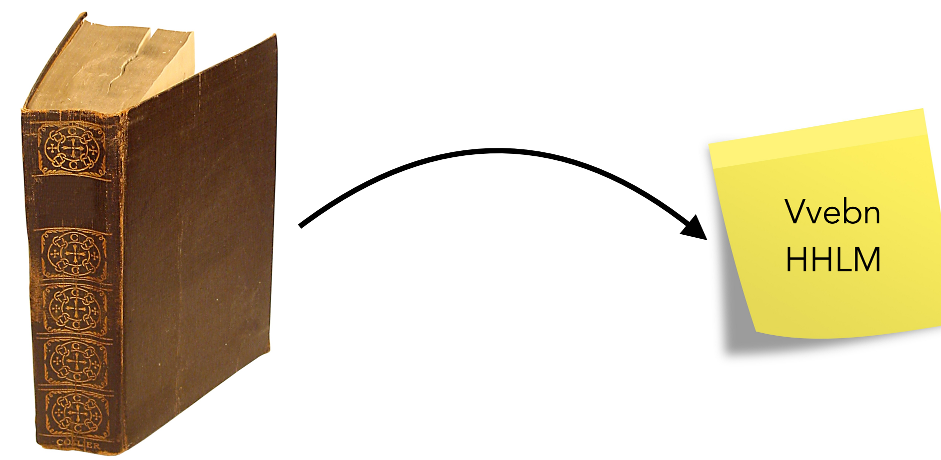

Concept: simple

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Solution:
(Quantum) Random Oracle Model

Random Oracle Model

Idealized model of cryptographic hash functions

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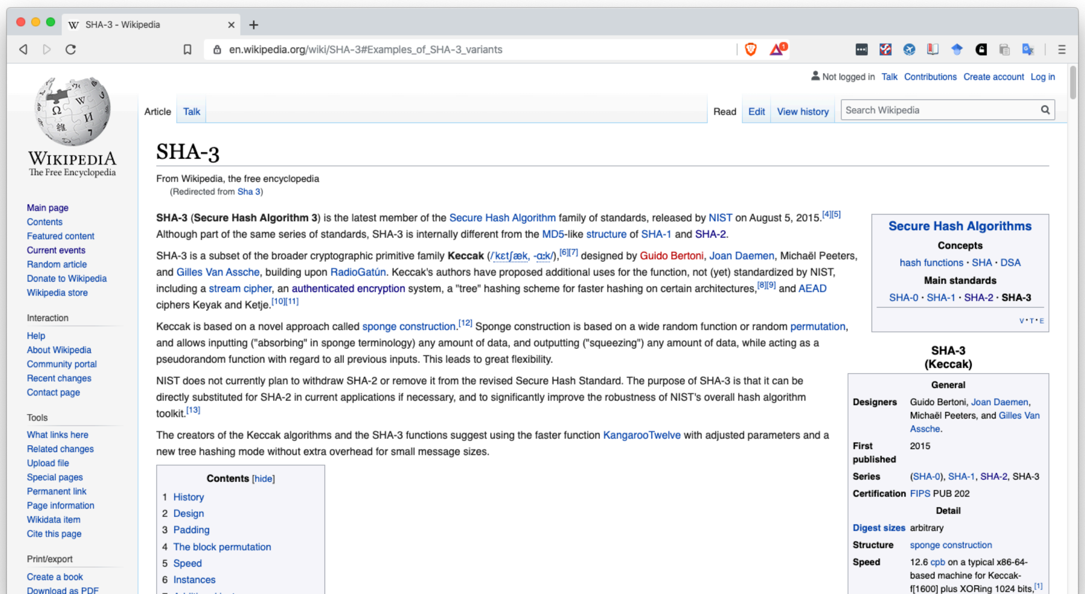
Idealized model of cryptographic hash functions

Reality



Random Oracle Model

Idealized model of cryptographic hash functions

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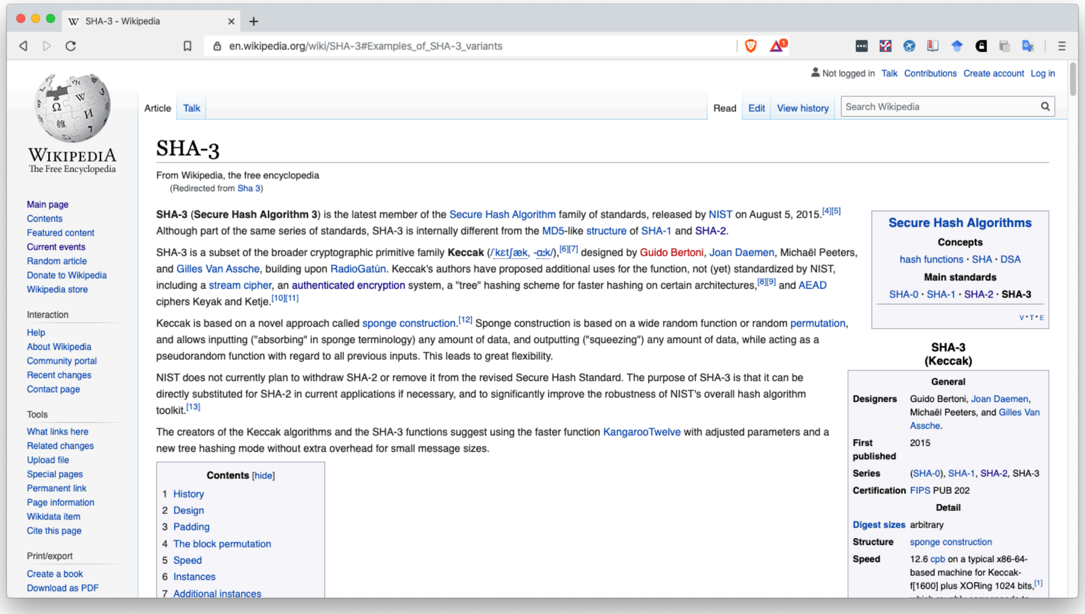	$H : \{0,1\}^* \rightarrow \{0,1\}^n$ Uniformly random All agents have black-box access to H

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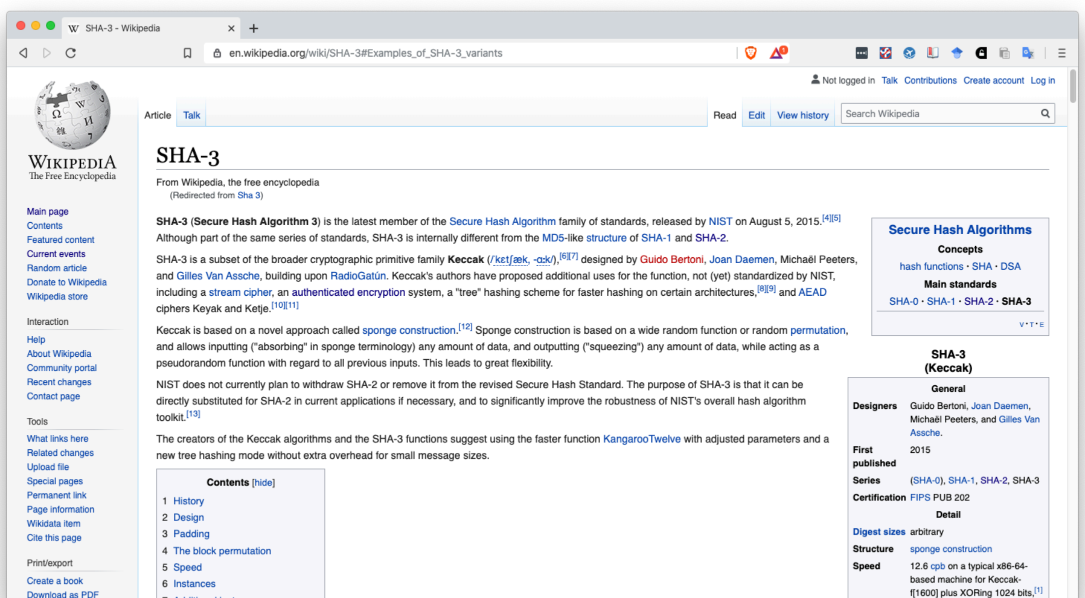	$H : \{0,1\}^* \to \{0,1\}^n$ Uniformly random All agents have black-box access to H

+ Simpler proofs

Random Oracle Model

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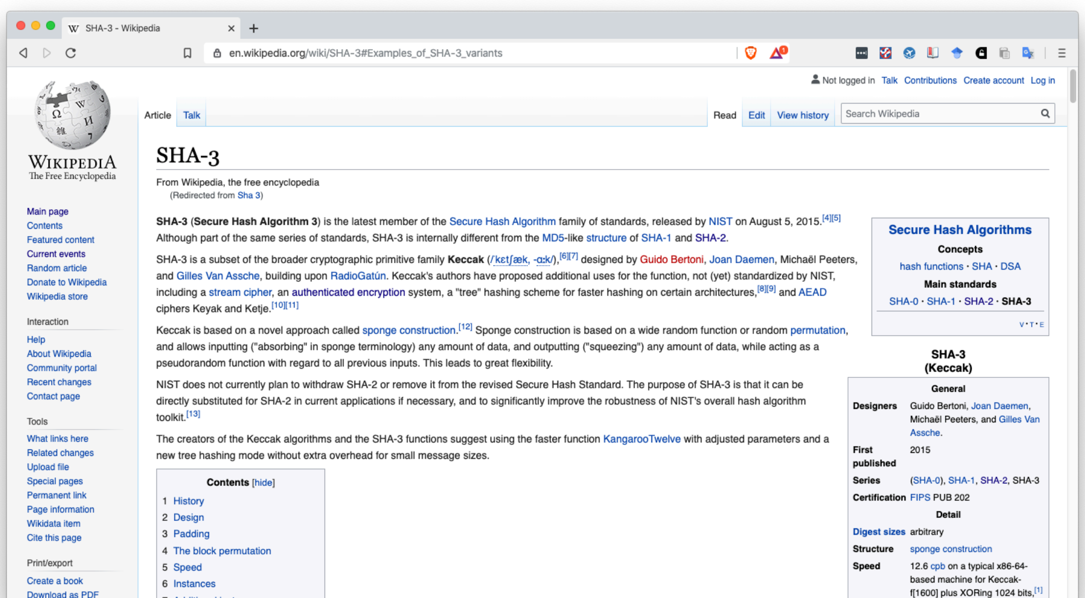


Model

$H : \{0,1\}^* \rightarrow \{0,1\}^n$

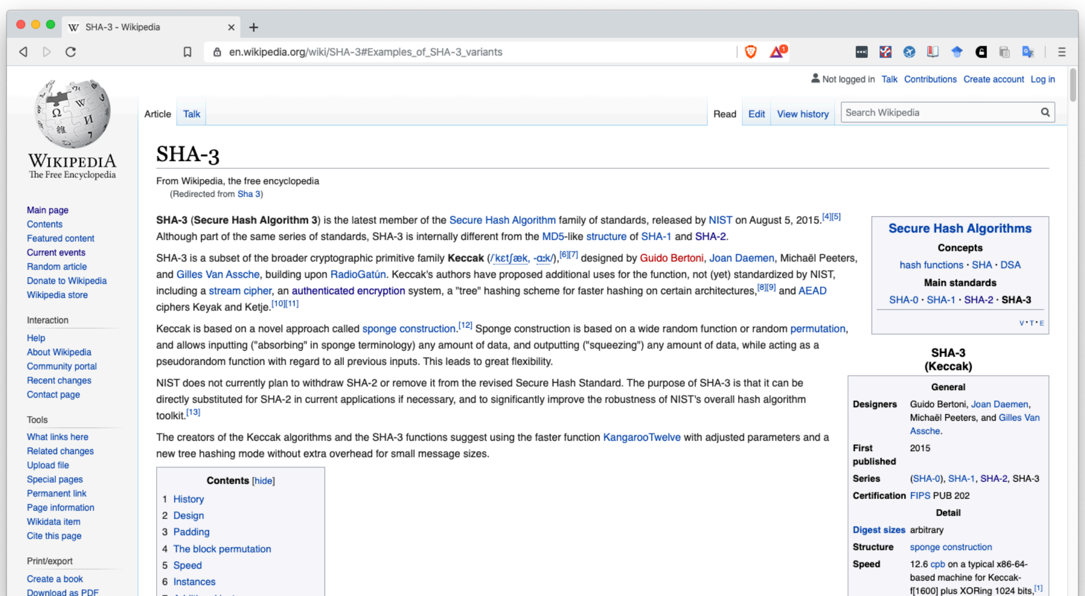
Uniformly random

All agents have black-box access to H

+ Simpler proofs
+ More efficient constructions with provable security

Quantum Random Oracle Model

Attackers with quantum computer can evaluate hash function on it!

Reality	Model
	$H : \{0,1\}^* \to \{0,1\}^n$ Uniformly random All agents have quantum black-box access to H

Quantum Random Oracle Model (Boneh et al. '10)

- Security reductions are quantum algorithms
- Quantum query complexity

The adaptive reprogramming game

The game (simplest version)

Uniformly random function $H : \{0,1\}^n \to \{0,1\}^n$

two-stage oracle algorithm $\mathscr{A} = (\mathscr{A}_0, \mathscr{A}_1)$

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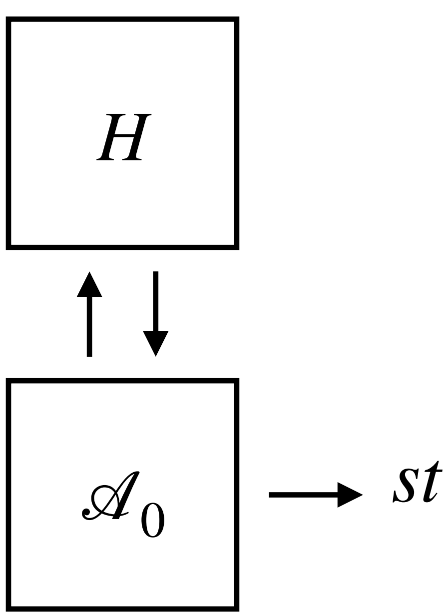

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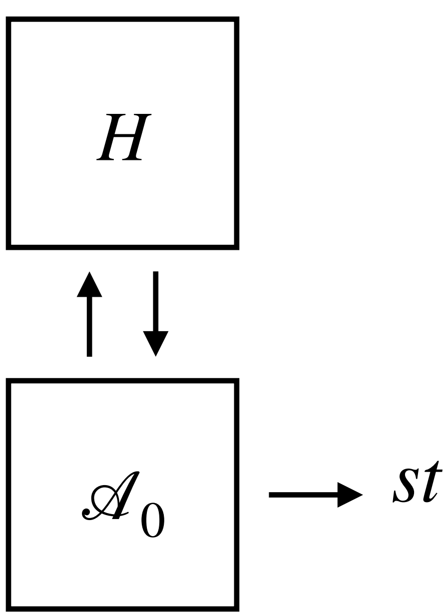

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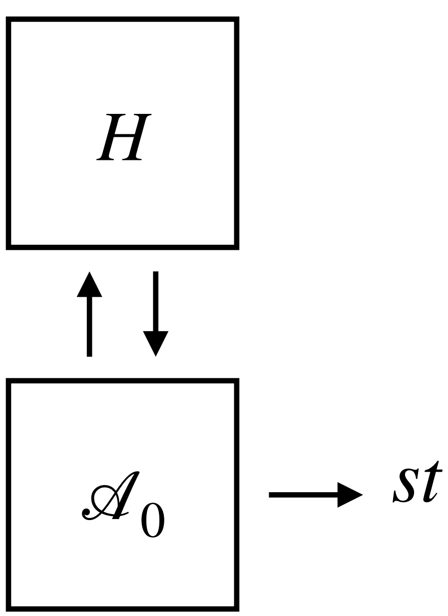

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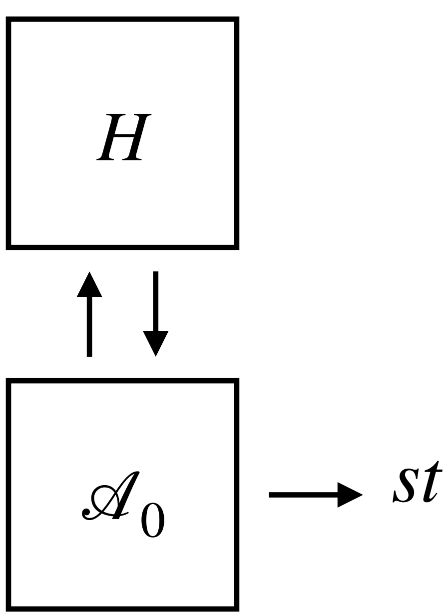

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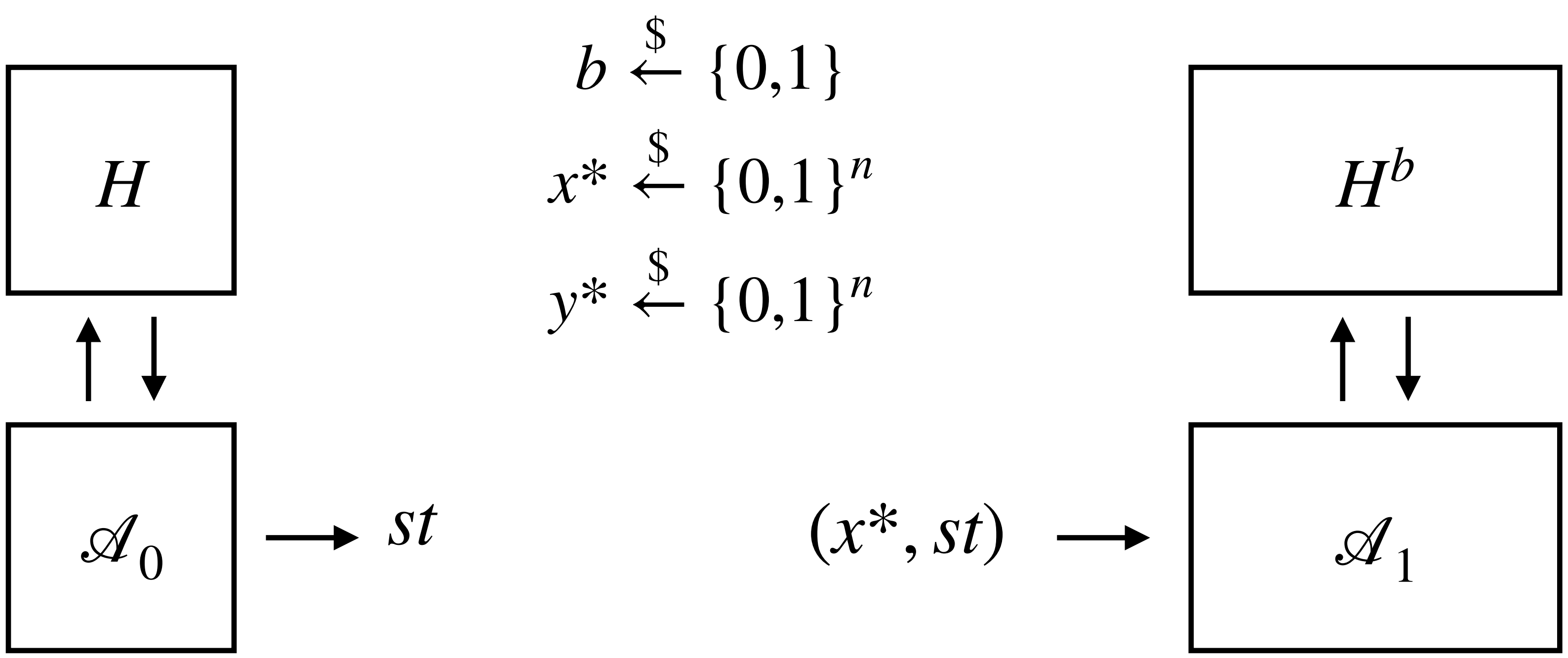

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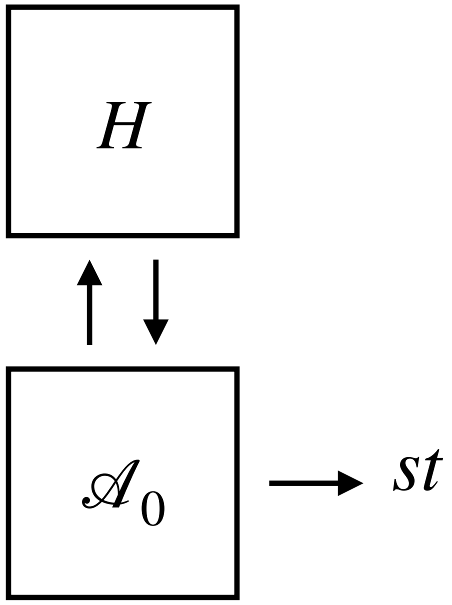

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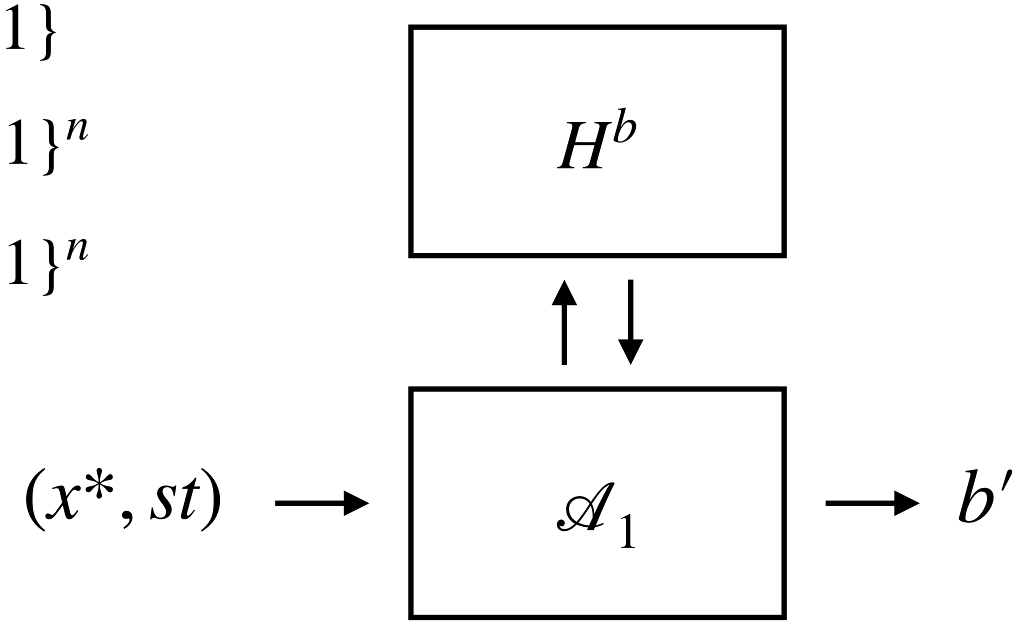

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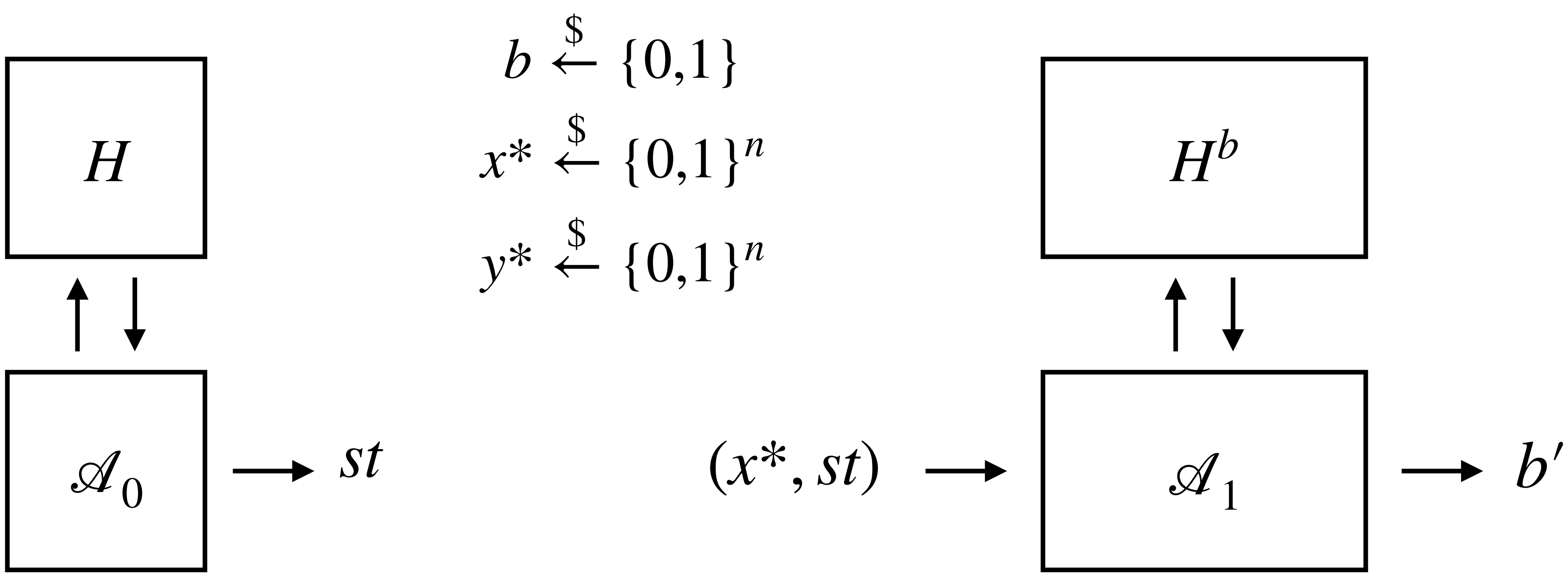
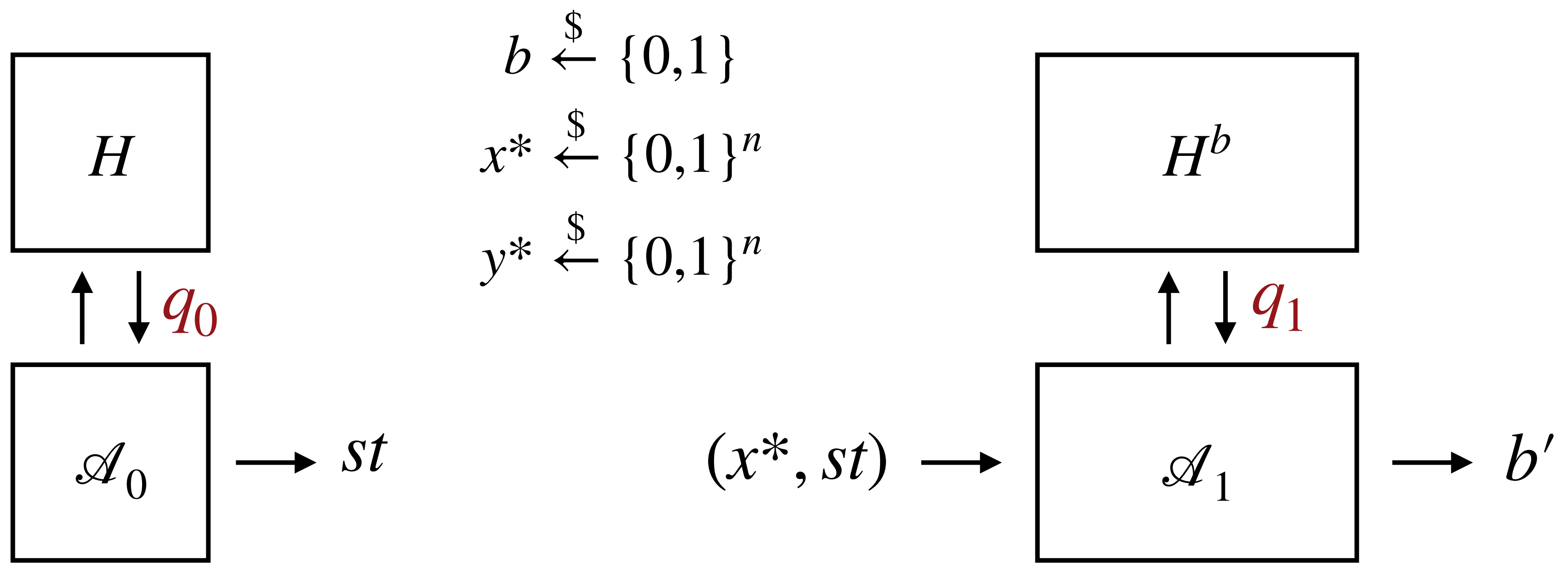


$\mathscr{A}$ wins if $b' = b$

Query lower bound



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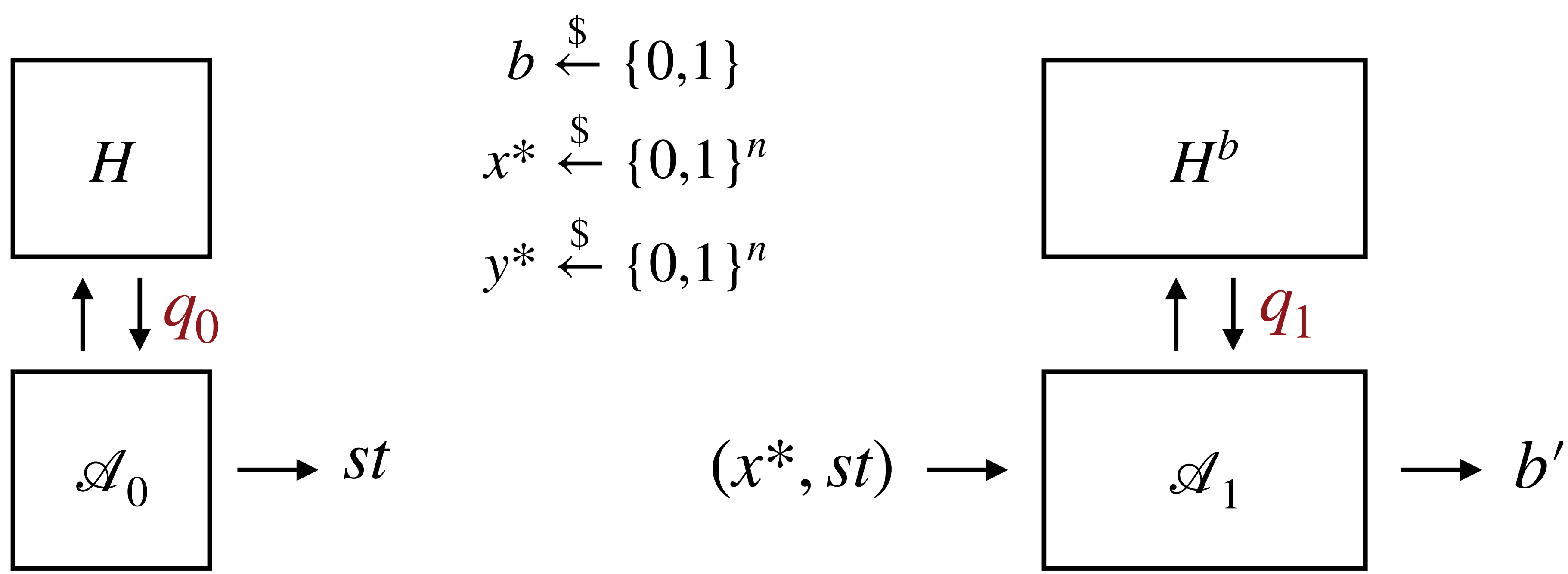

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Theorem: For classical $\mathscr{A}$,

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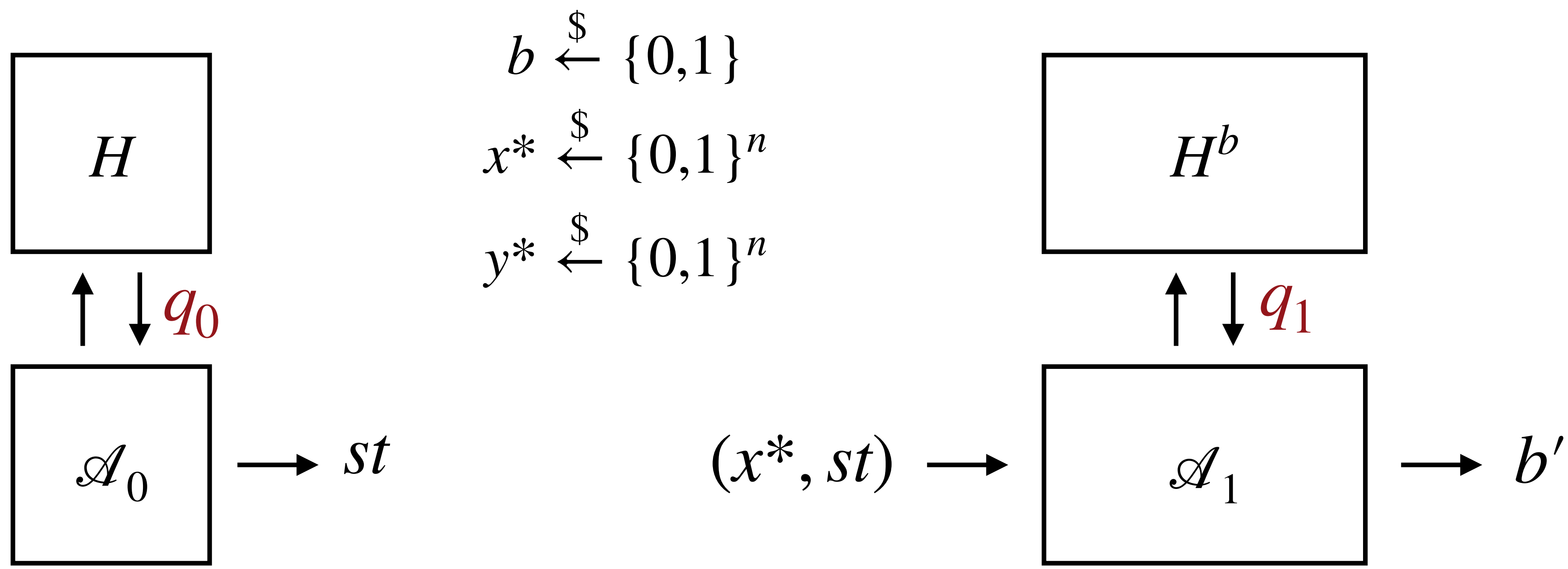

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This is tight, matching algorithm using $O(q_0)$ time, constant space, $q_1 = q_0$

Applications

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Security proofs in the ROM for digital signature schemes:

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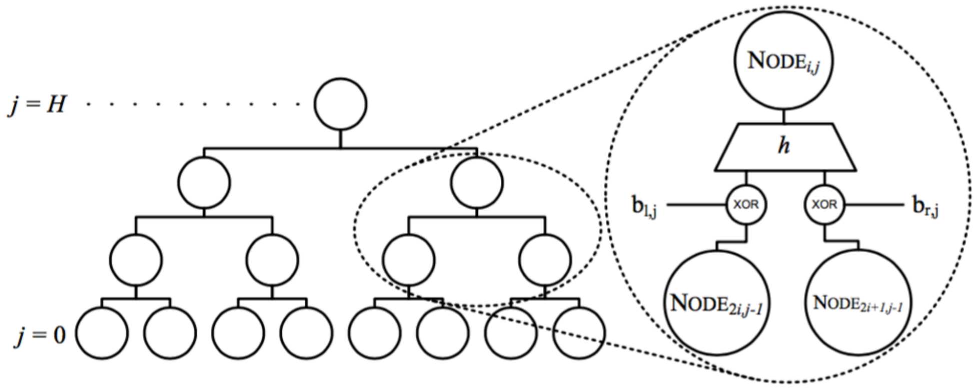
- Hash based signatures (XMSS, standardized as RFC 8391)



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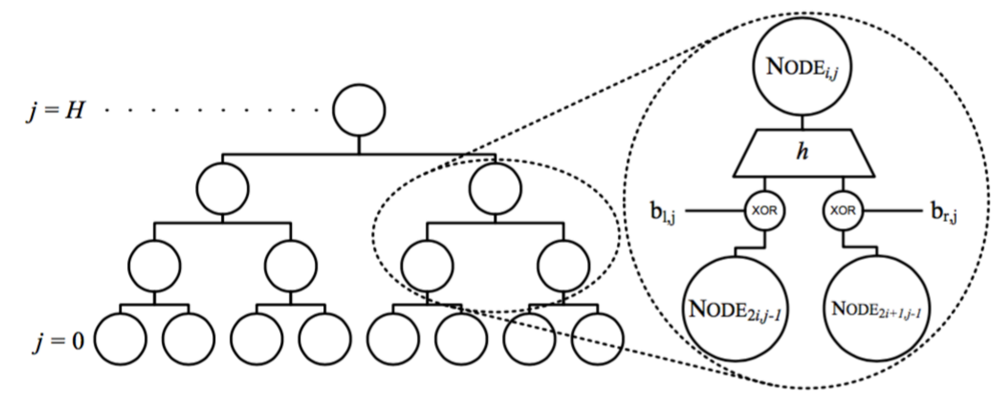

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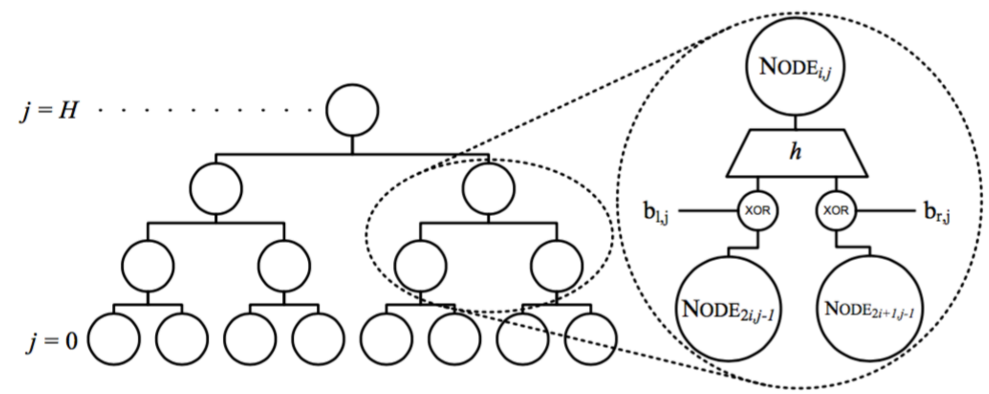
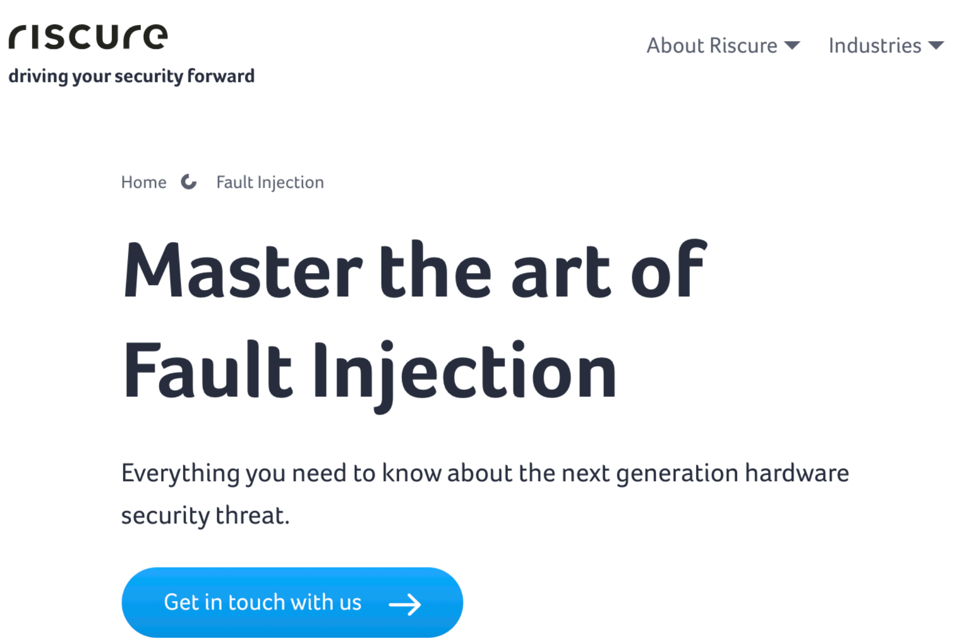
- The hedged Fiat-Shamir transformation
- etc.





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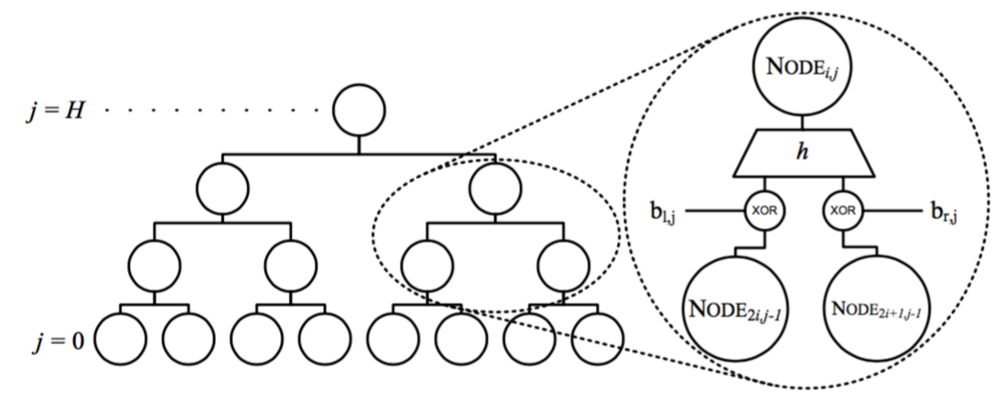
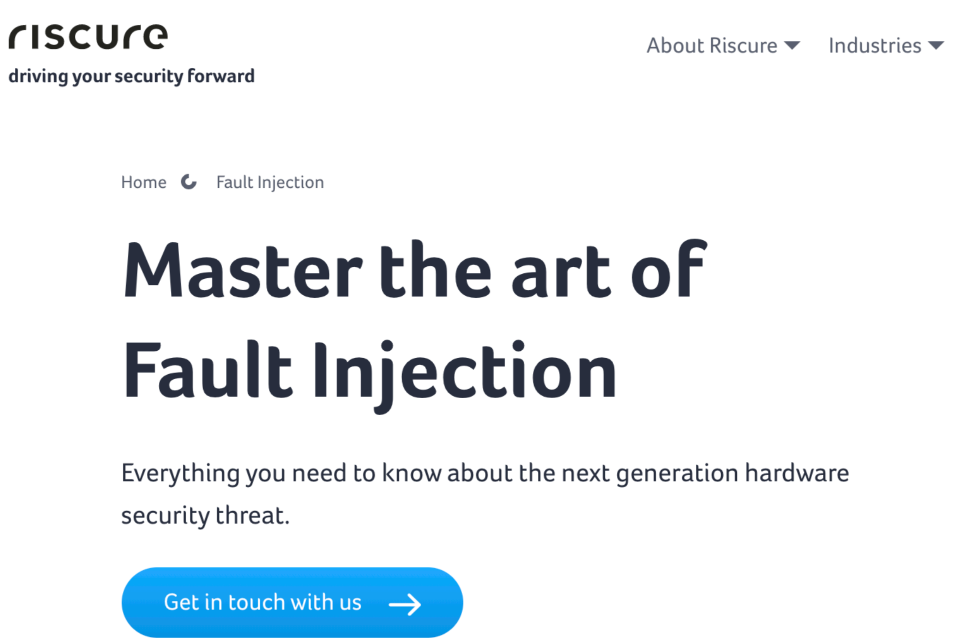
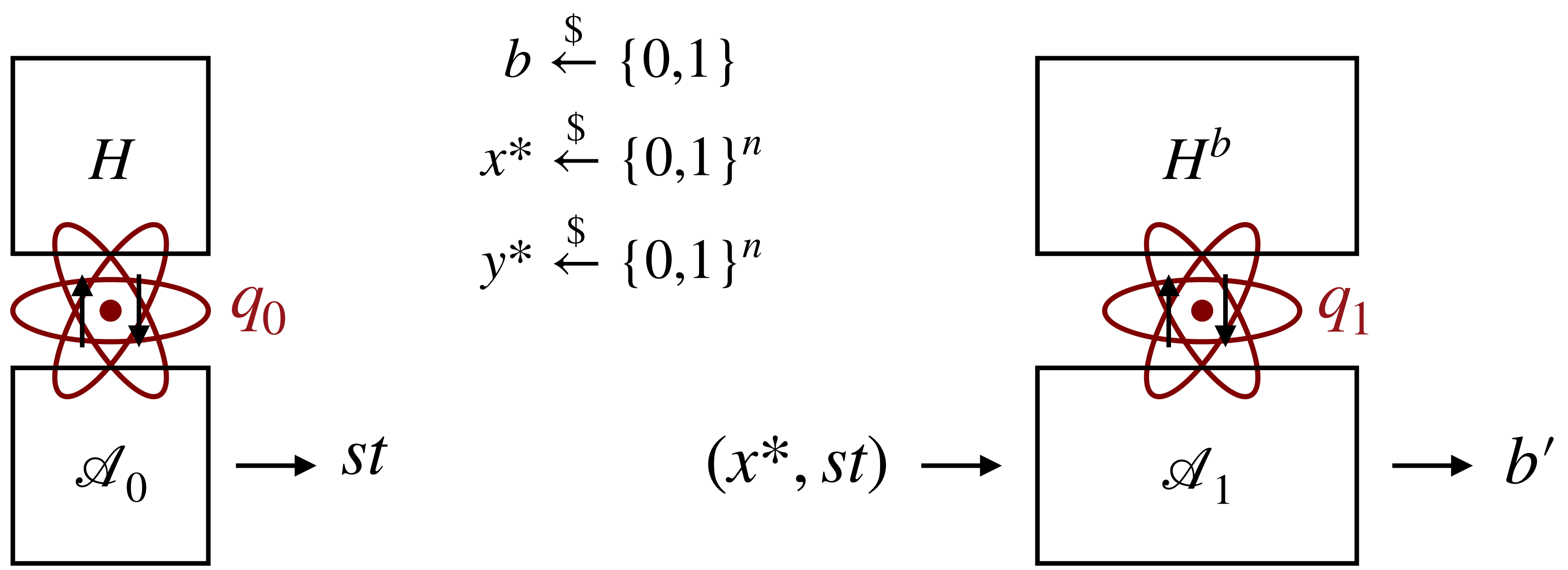




What about post-quantum security?

Quantum query lower bound



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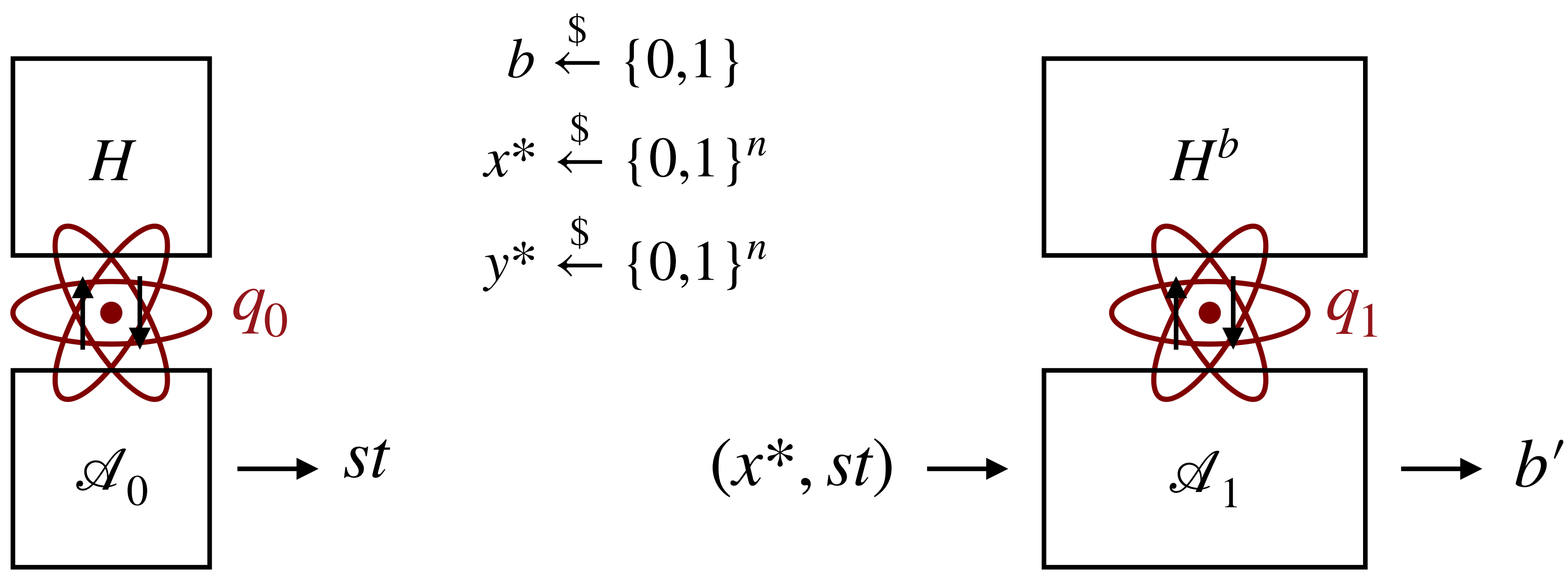

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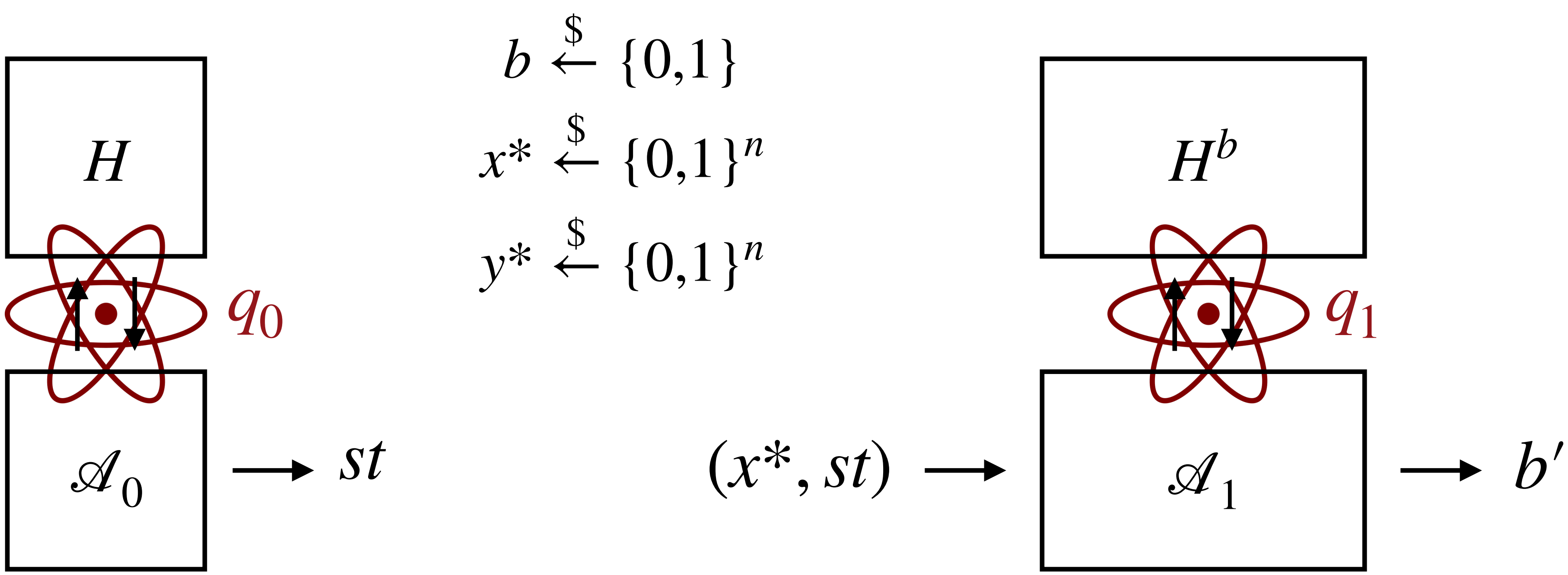

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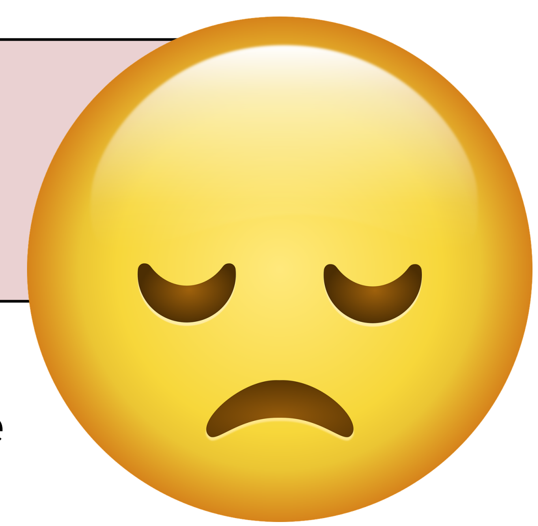

$q_0 = O\left(2^{\frac{n}{2}}\right)$ allows for constant advantage

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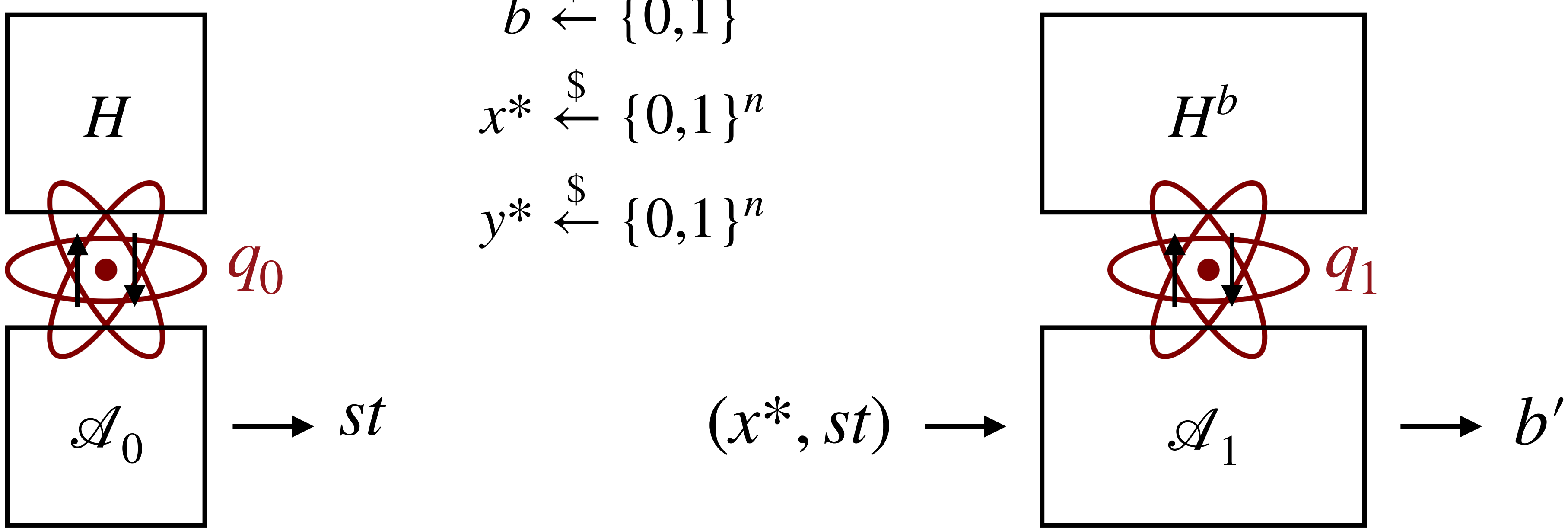

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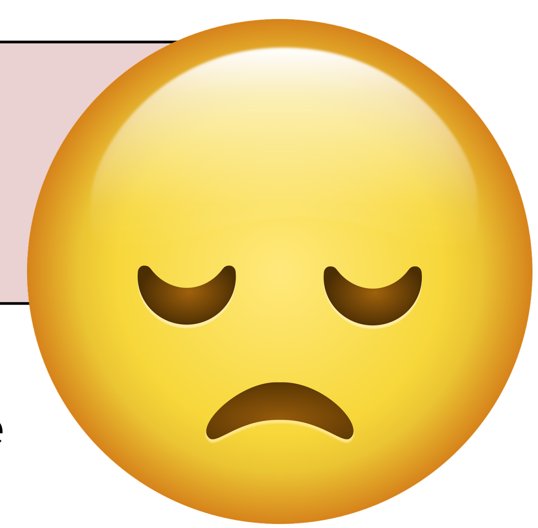

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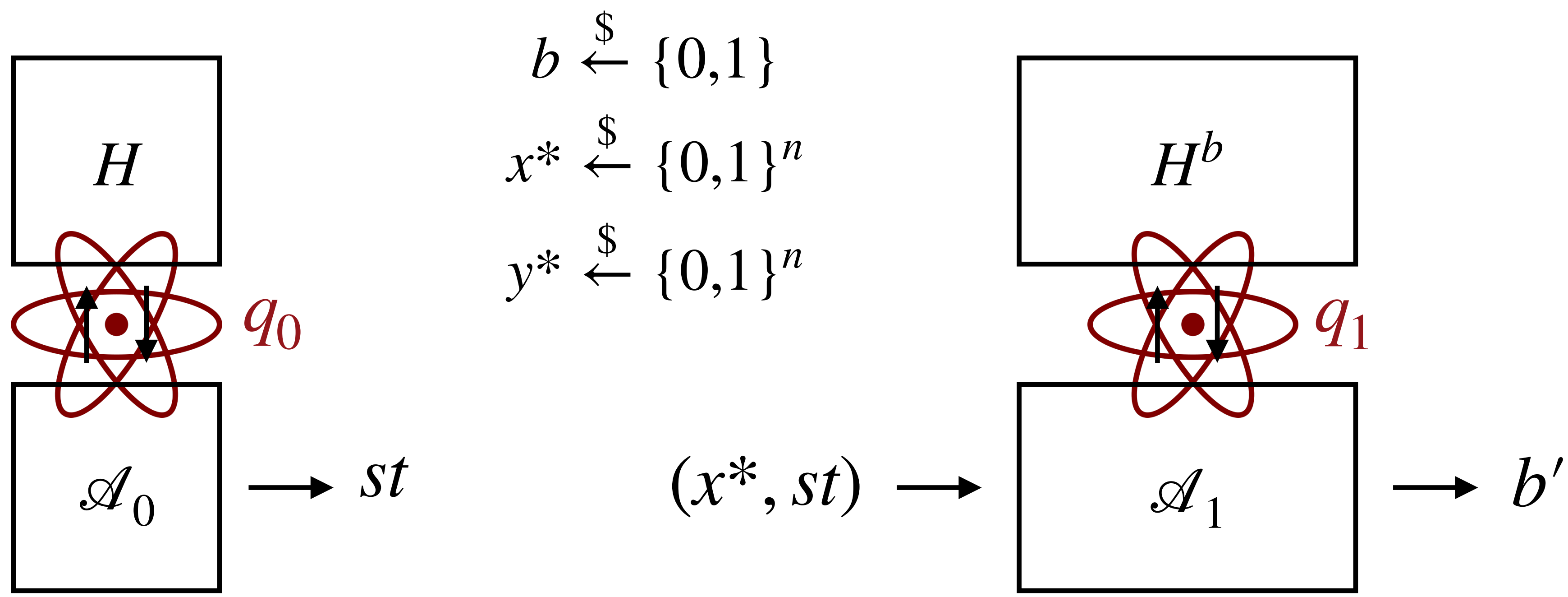
Tightness unlikely: $\mathscr{A}_0$ doesn't know what it is searching for $\Rightarrow$ no Grover!

Results

Tight quantum query lower bound



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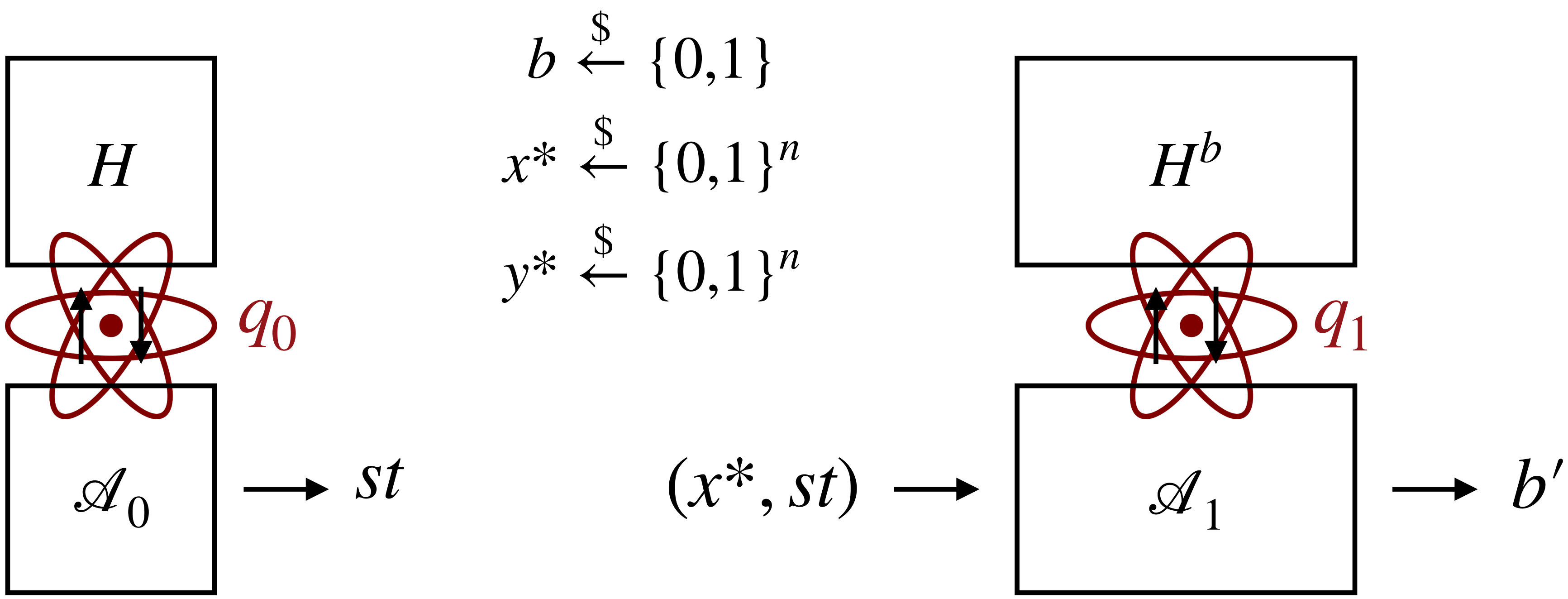

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Theorem (Grilo, Hövelmanns, Hülsing, CM):

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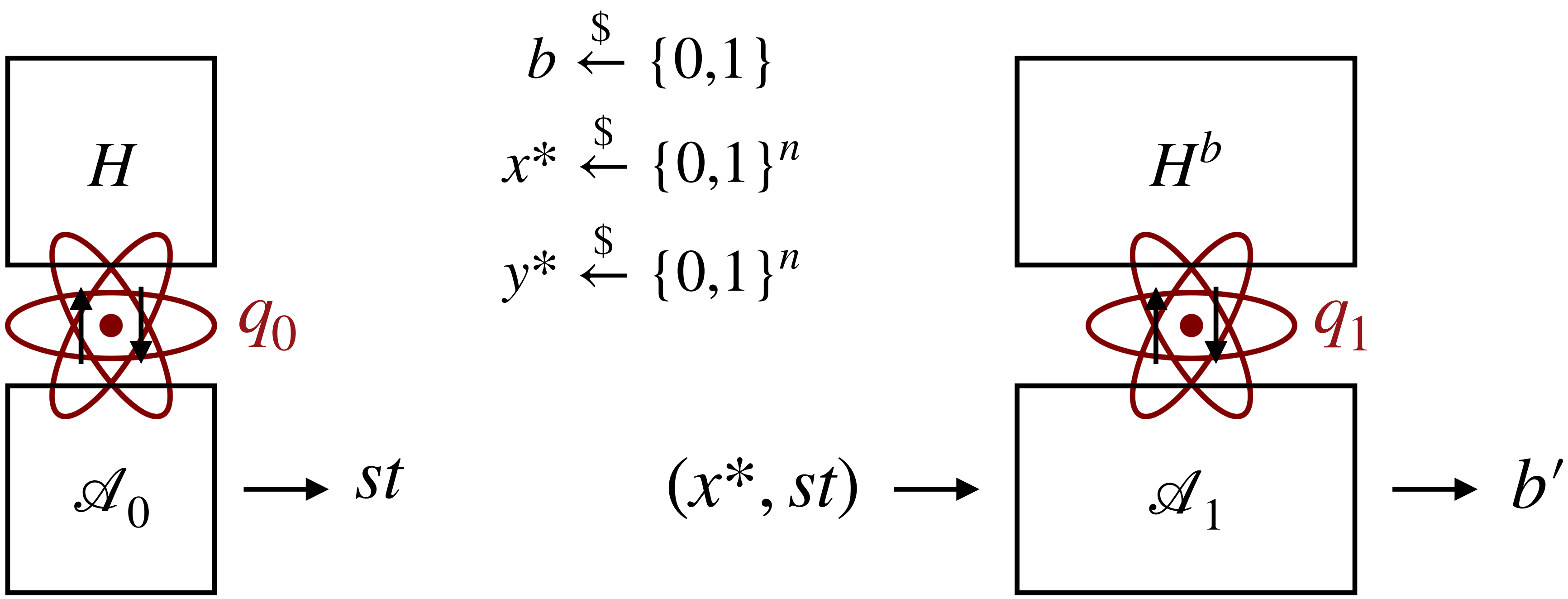

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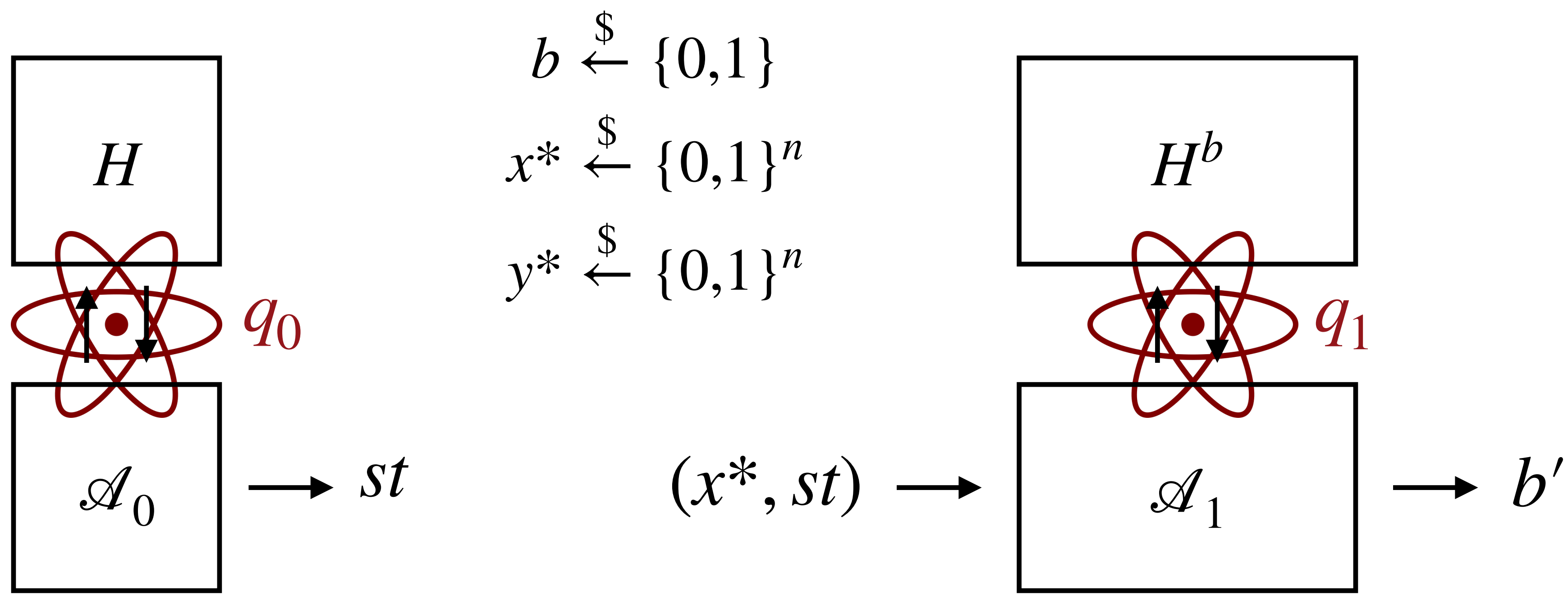

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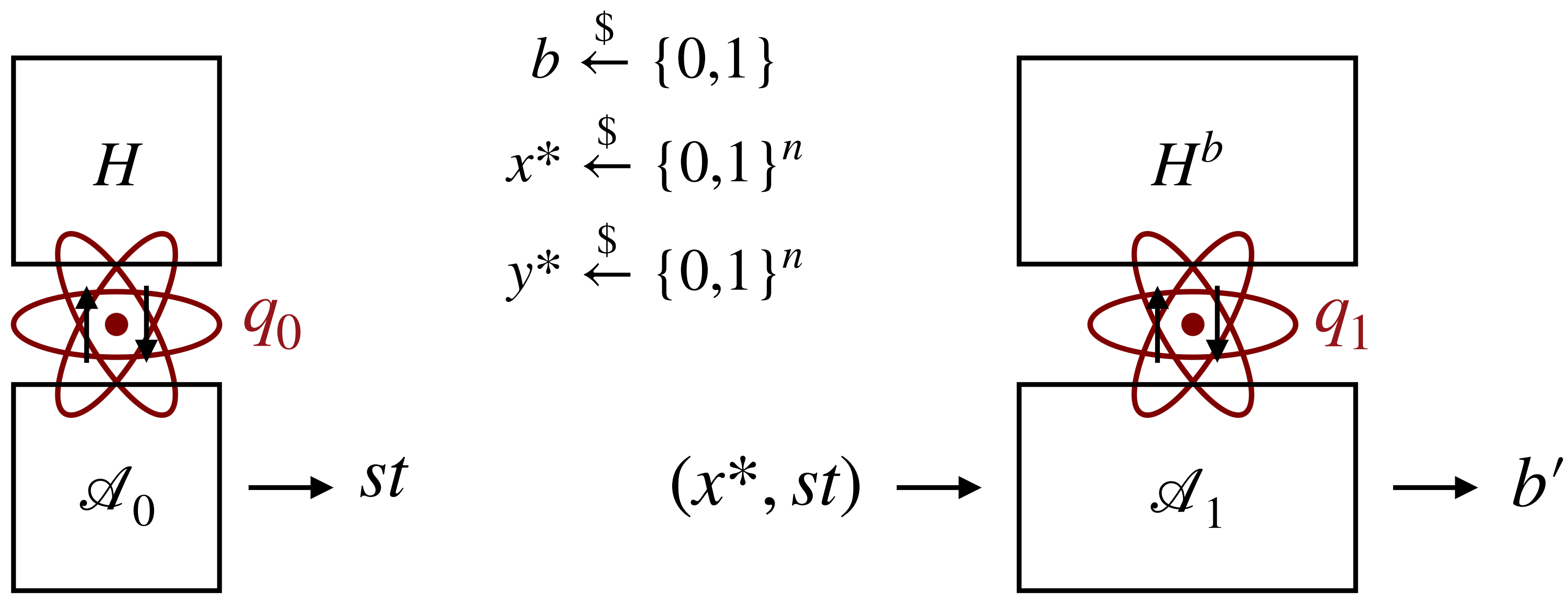

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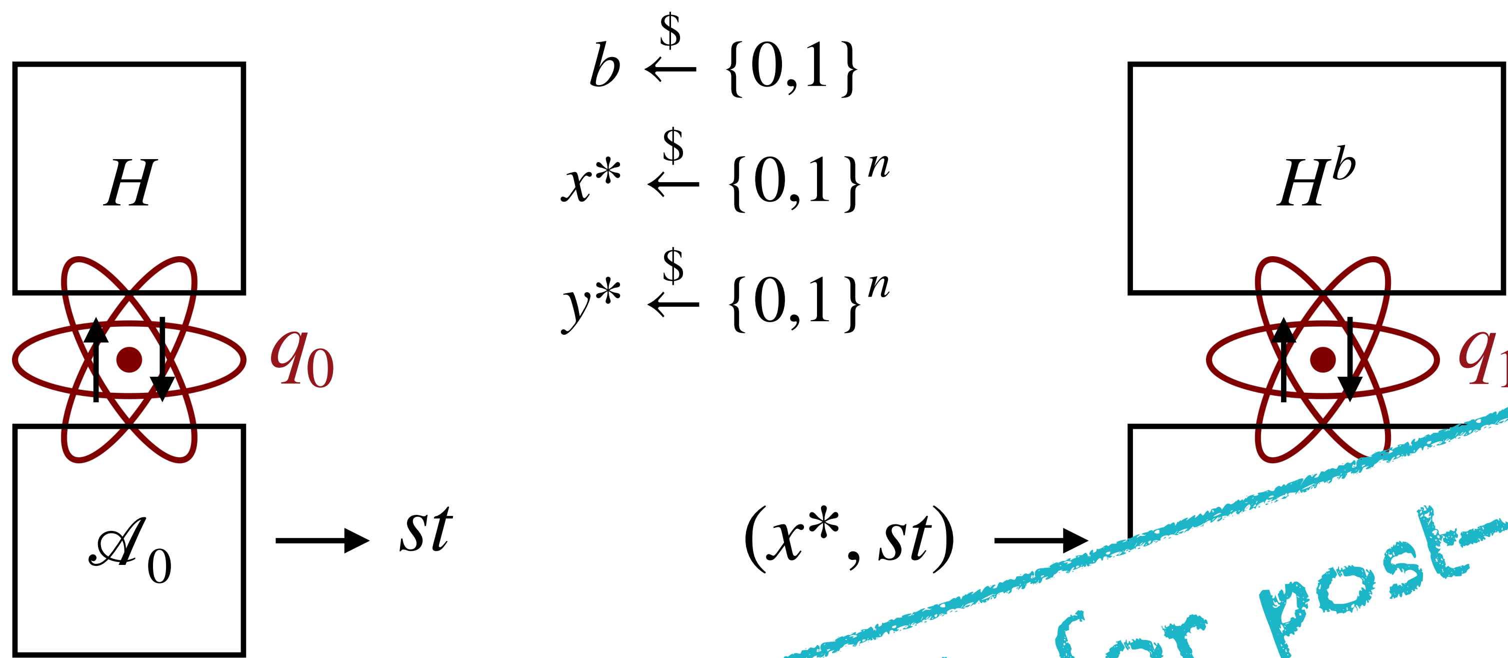

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Tighter security proofs for post-quantum signatures, including NIST candidates.

Th… …n):

… $\leq \frac{1}{2} + \frac{3}{2}\sqrt{q_0 2^{-n}}$

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Reprogramming superposition oracles

The superposition oracle

For simplicity: $H : \{0,1\}^n \to \{0,1\}^n$

Random oracle

Superposition oracle (Zhandry '18)

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Initialize n-qubit register F_x
in state $|\phi_0\rangle = |+\rangle^{\otimes n}$

The superposition oracle

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$U_H|x\rangle_X|y\rangle_Y = |x\rangle_X|y \oplus H(x)\rangle_Y$

Reprogramming at x^*: $y^* \leftarrow \{0,1\}^n$,

$$H'(x) = \begin{cases} y^* & x = x^* \\ H(x) & \text{else} \end{cases}$$

Superposition oracle (Zhandry '18)

For each $x \in \{0,1\}^n$:

Initialize n-qubit register F_x

in state $|\phi_0\rangle = |+\rangle^{\otimes n}$

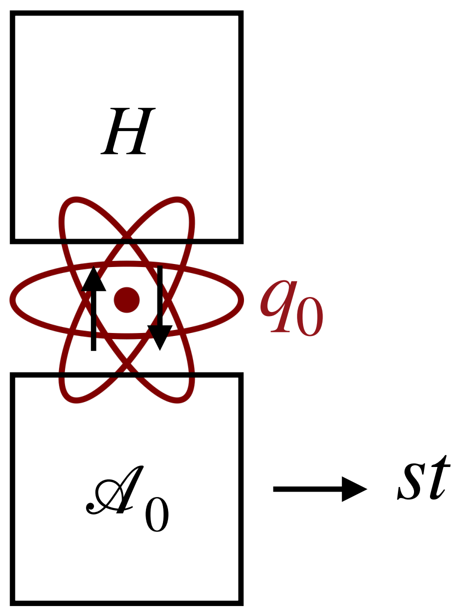
Query unitary:

$U_H|x\rangle_X = \mathrm{CNOT}^{\otimes n}_{F_x:Y}$

Reprogramming at x^*:

- Discard contents of F_{x^*}
- Prepare F_{x^*} in state $|\phi_0\rangle$

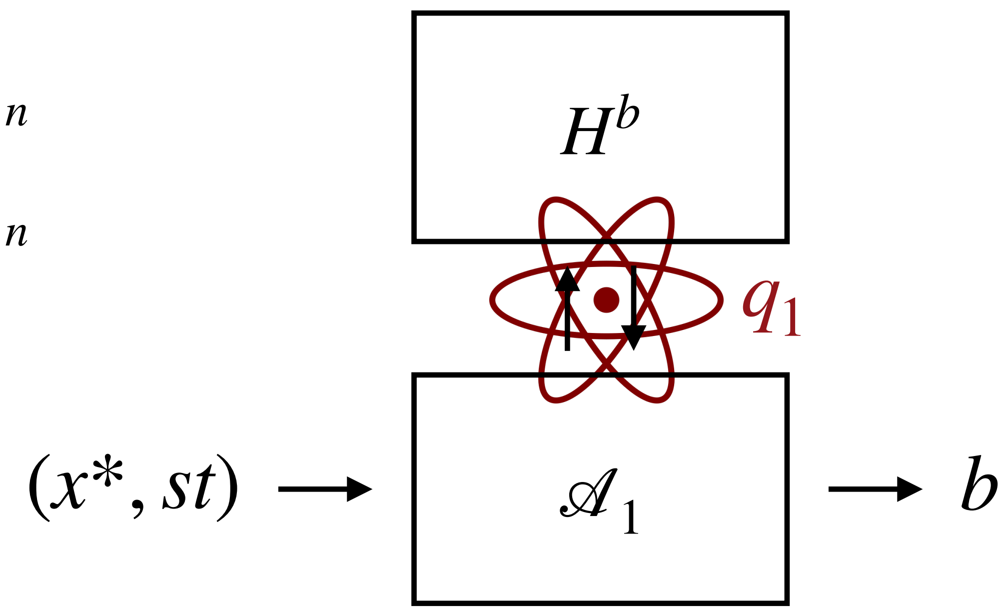
Proof ideas



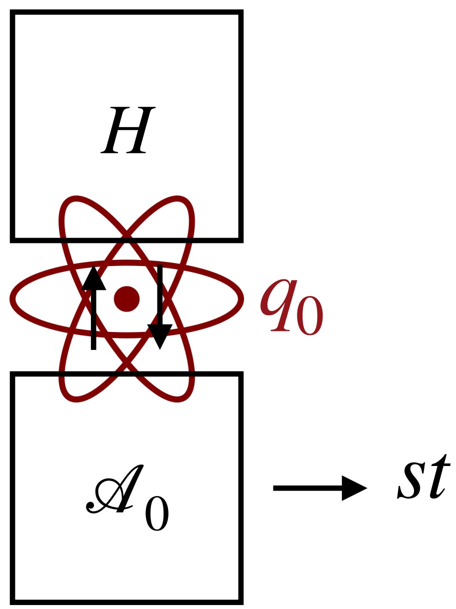
$b \xleftarrow{\$} \{0,1\}$

$x^* \xleftarrow{\$} \{0,1\}^n$

$y^* \xleftarrow{\$} \{0,1\}^n$



Proof ideas



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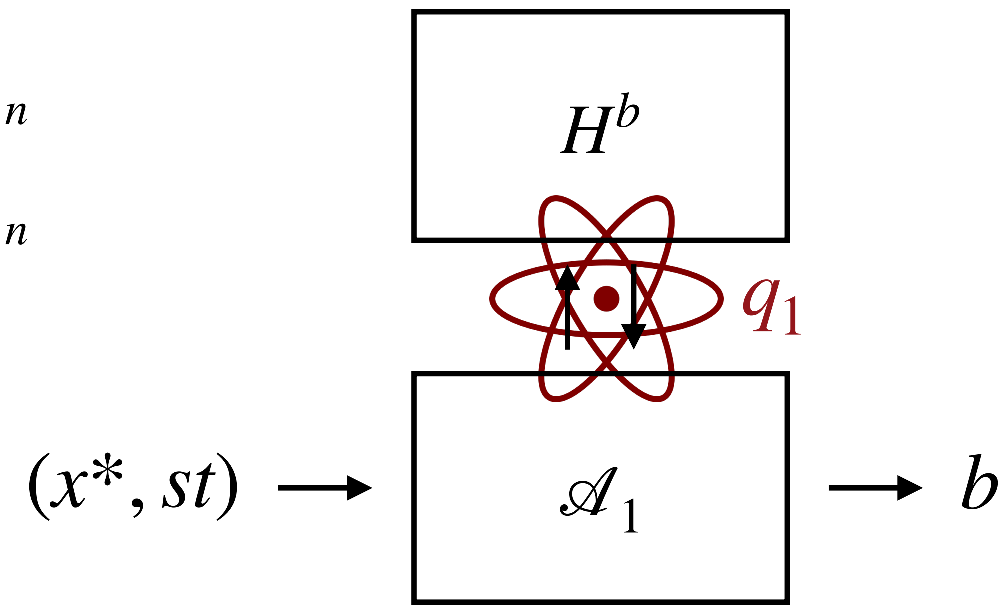
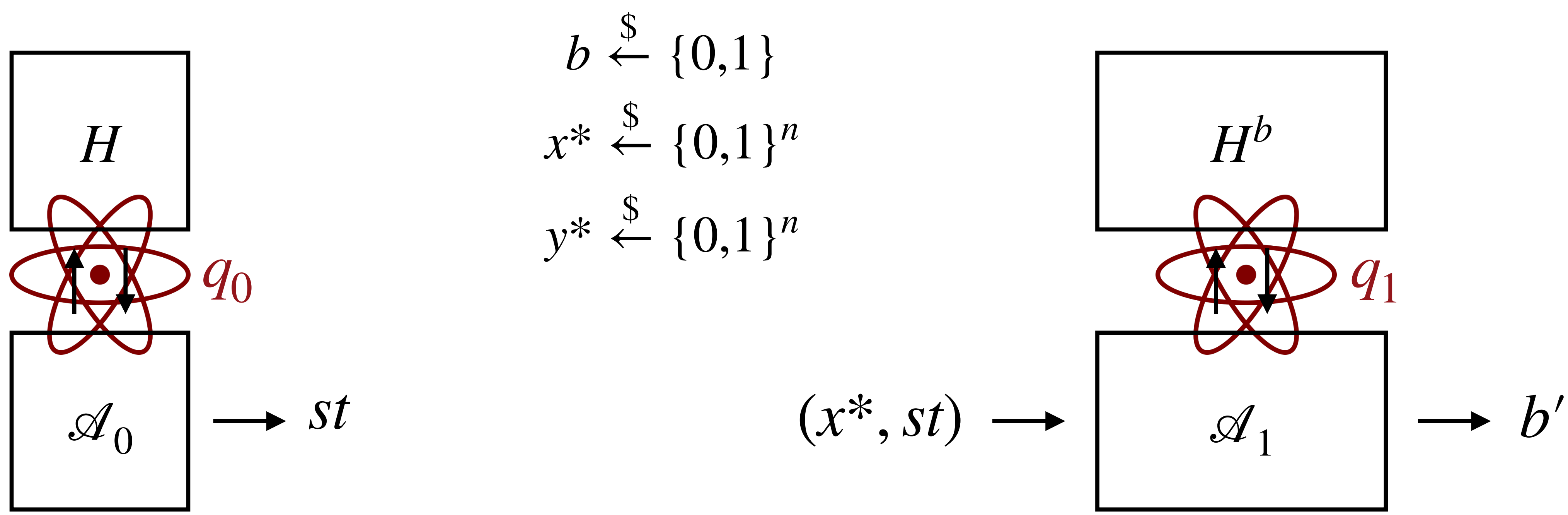


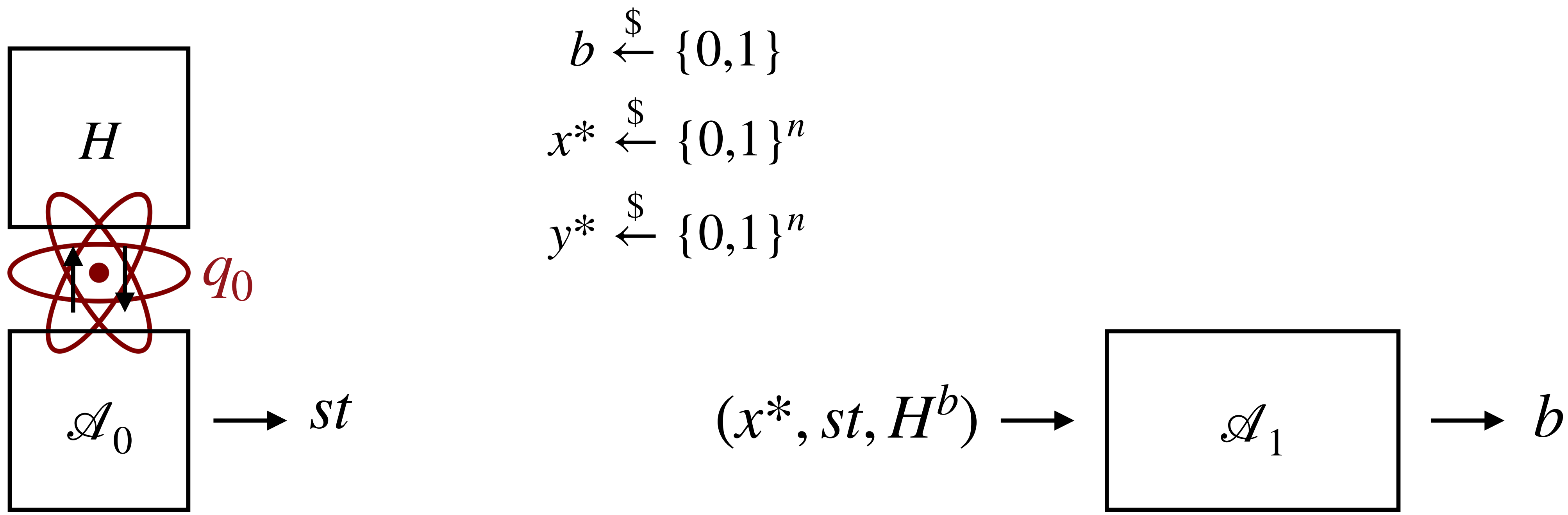
- Intuition: q_0 is limiting quantity

Proof ideas



- Intuition: q_0 is limiting quantity
- simplification: allow $q_1 = 2^n$

Proof ideas



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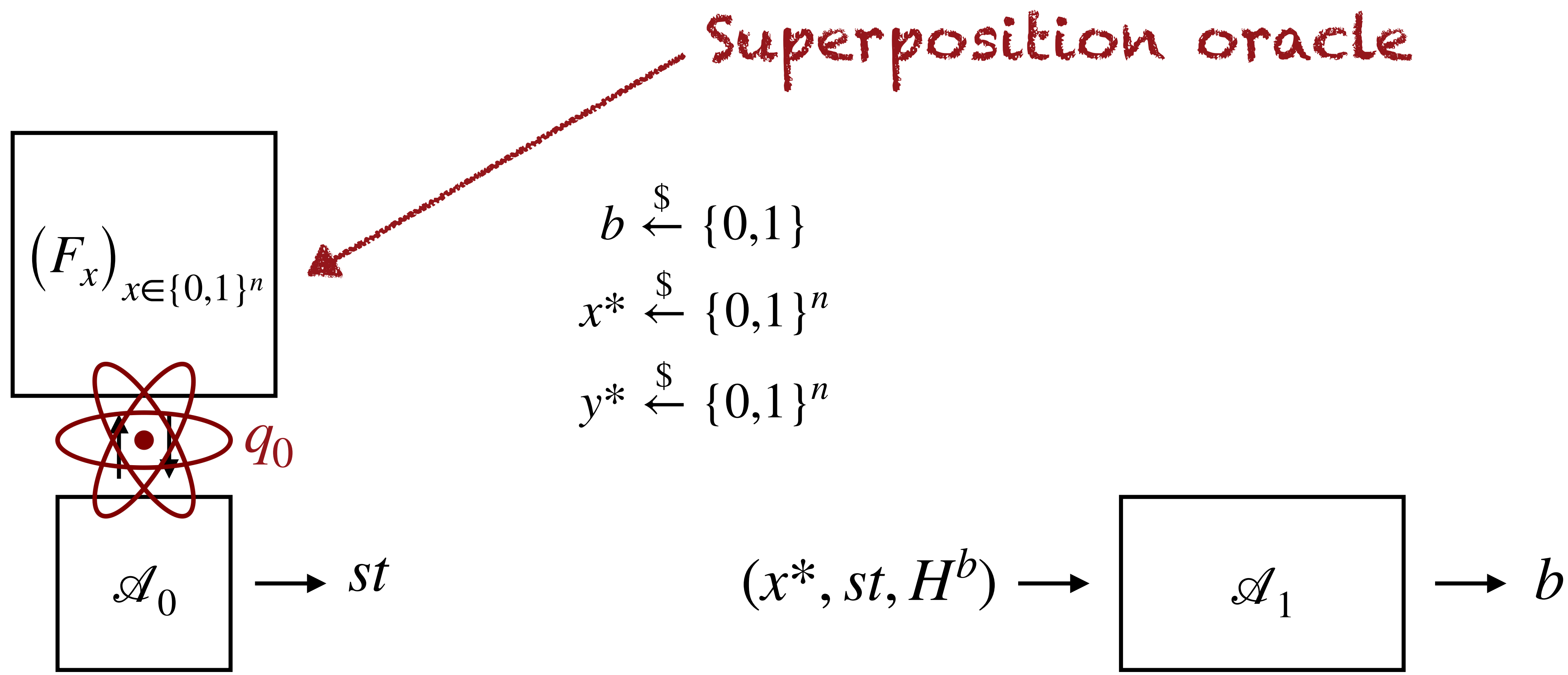

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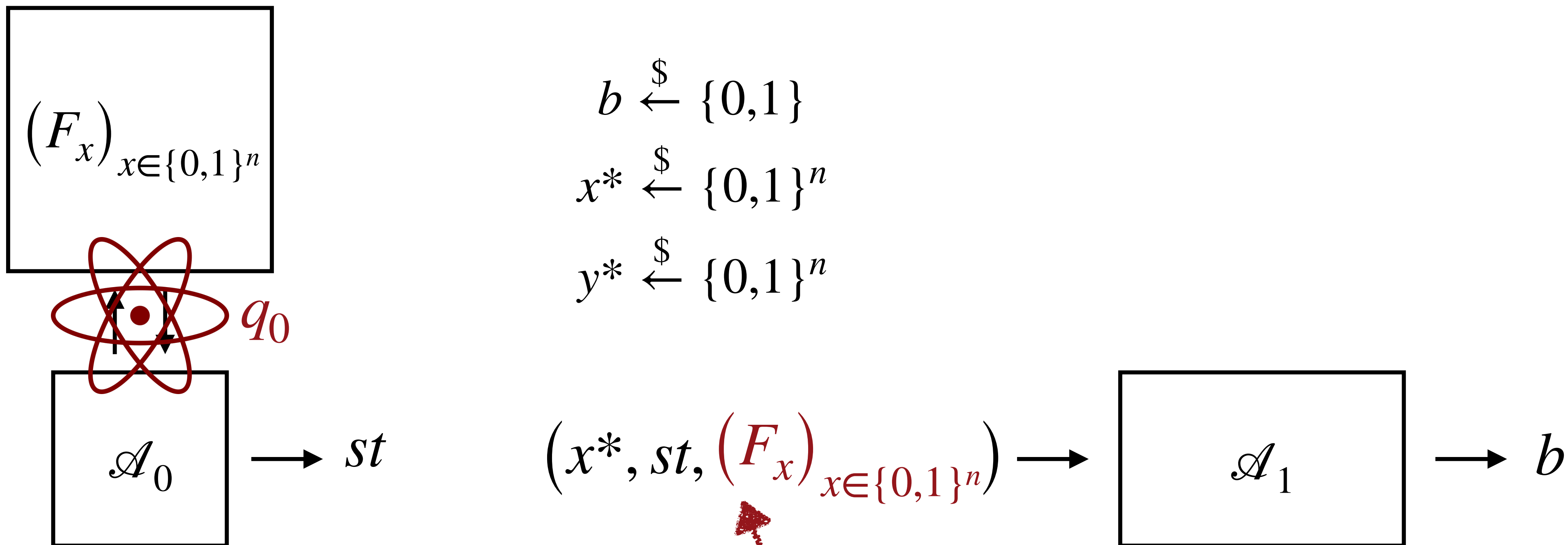

Hand over oracle's internal state after potentially reprogramming:

Reprogramming at x^*:

- Discard contents of F_{x^*}
- Prepare F_{x^*} in state $|\phi_0\rangle$

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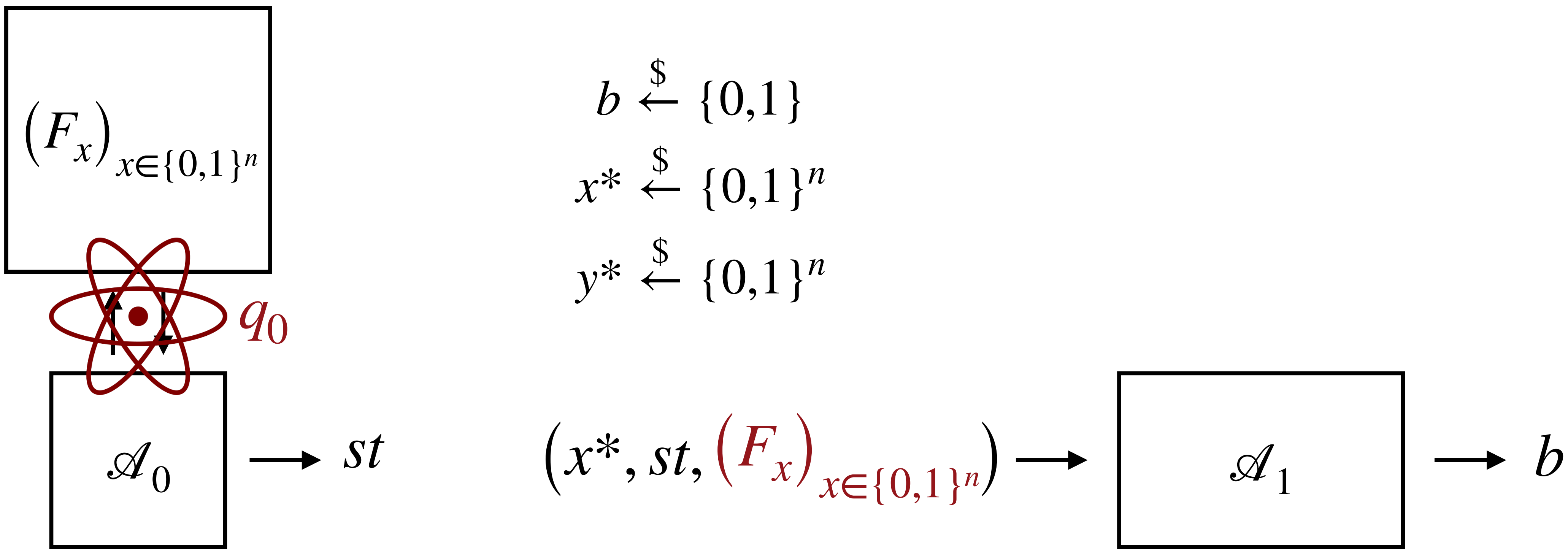
Proof ideas



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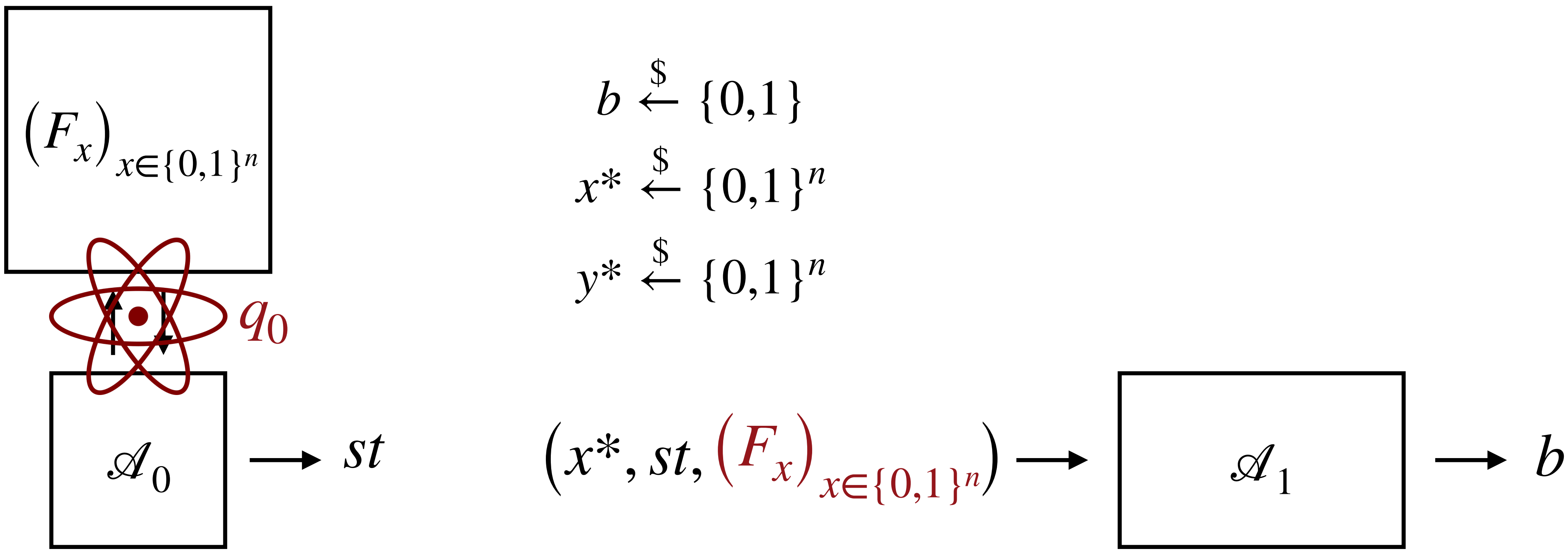
Oracle distinguishing → State discrimination!

Proof ideas



- Intuition: q_0 is limiting quantity
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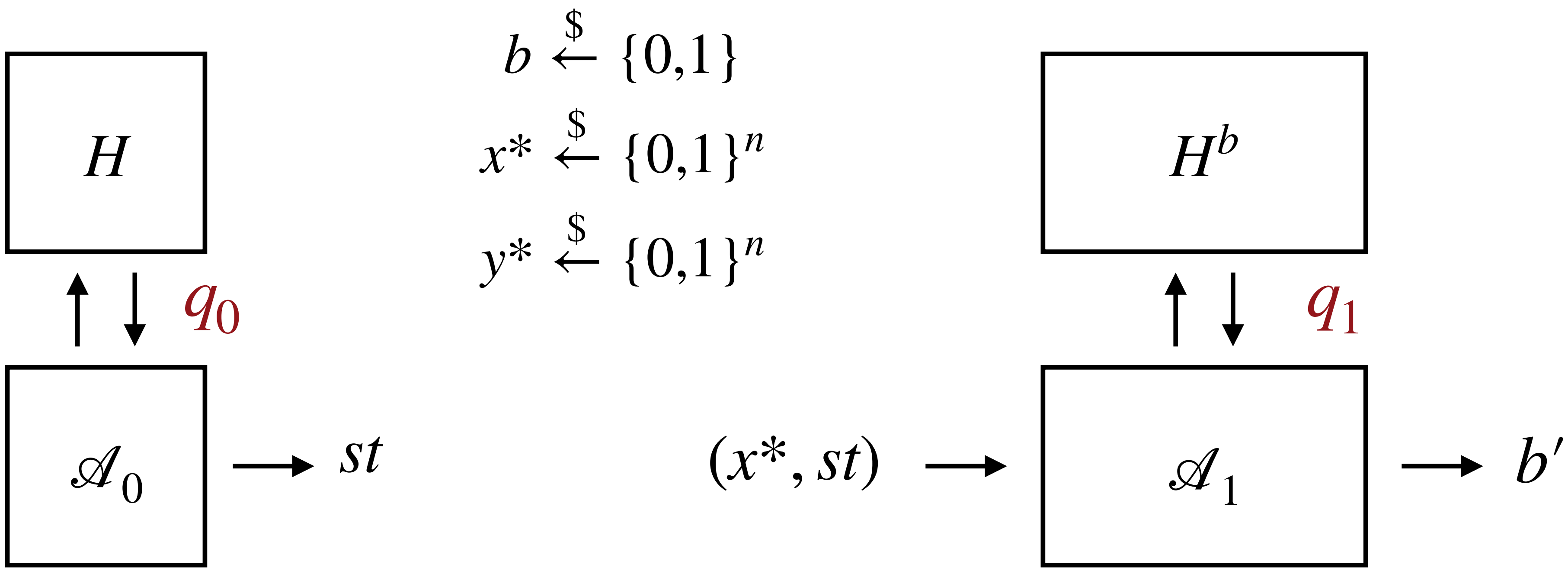
Oracle distinguishing → State discrimination!

Suffices to bound a trace norm distance (for arbitrary $\mathscr{A}_0$).

A matching algorithm

Classical algorithm

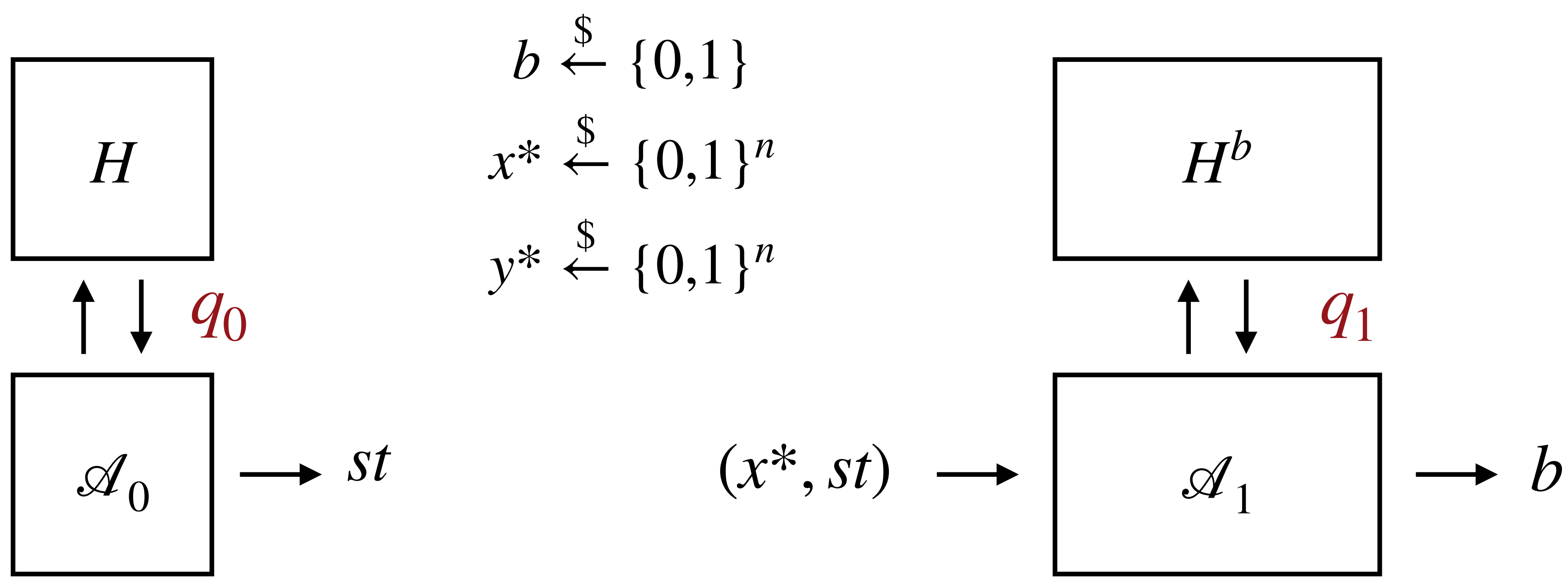


$\mathscr{A}$ wins if $b' = b$

Theorem:

$$\Pr[\mathscr{A} \text{ wins}] \leq \frac{1}{2}\left(1 + q_0 2^{-n}\right)$$

Classical algorithm



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Matching algorithms:

- Simple: query distinct inputs $x_1, \ldots, x_{q_0}$, store result, hope $x^* = x_i$ for some i

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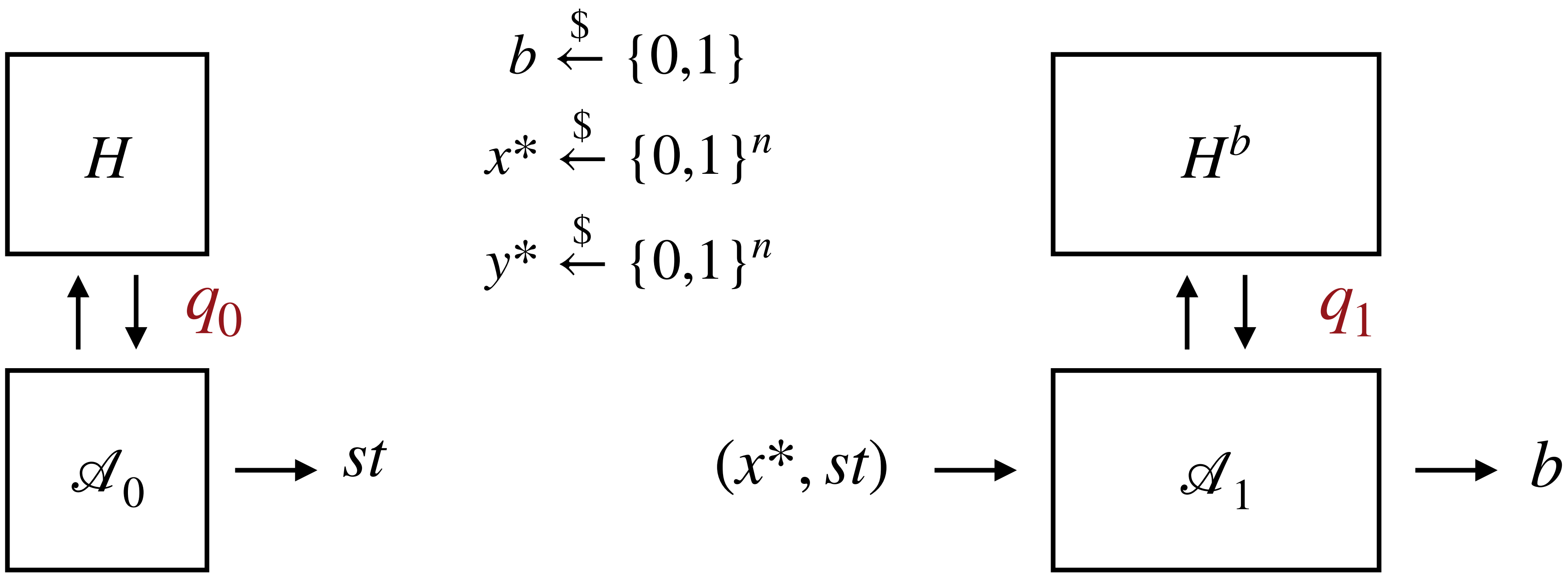

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Matching algorithms:

- Simple: query distinct inputs $x_1, \ldots, x_{q_0}$, store result, hope $x^* = x_i$ for some i
- Constant space: $\mathscr{A}_0$ computes $H(x_0) \oplus H(x_1) \oplus \ldots \oplus H(x_{q_0-1})$, $\mathscr{A}_1$ checks

Quantum algorithm

Theorem: For classical $\mathscr{A}$,

$$\Pr[\mathscr{A} \text{ wins}] \leq \frac{1}{2}\left(1 + q_0 2^{-n}\right)$$

Theorem (Grilo, Hövelmanns, Hülsing, CM):
There exists a quantum algorithm that achieves

$$\Pr[\mathscr{A} \text{ wins}] = \frac{1}{2} + \Omega\left(\sqrt{q_0 2^{-n}}\right)$$

Quantum algorithm

Idea: use classical "checksum algorithm" for a superposition of sets of q_0 inputs

Quantum algorithm

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Classical algorithm:

- $\mathscr{A}_0$ computes $z = H(x_0) \oplus H(x_1) \oplus \ldots \oplus H(x_{q_0-1})$
- $\mathscr{A}_1$ checks z

Quantum algorithm

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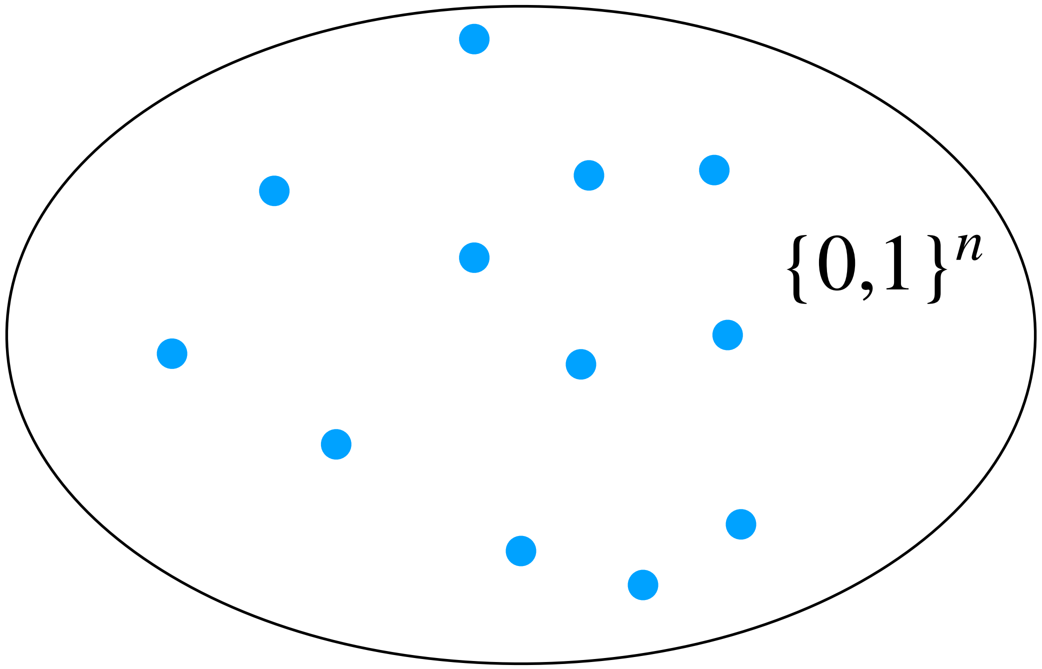

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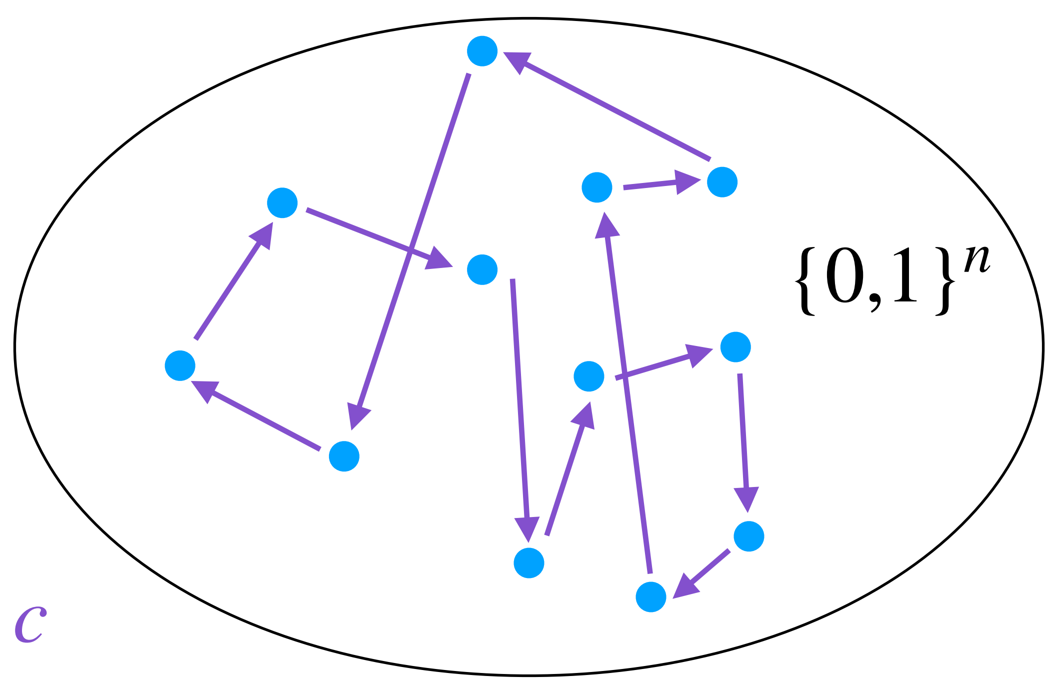
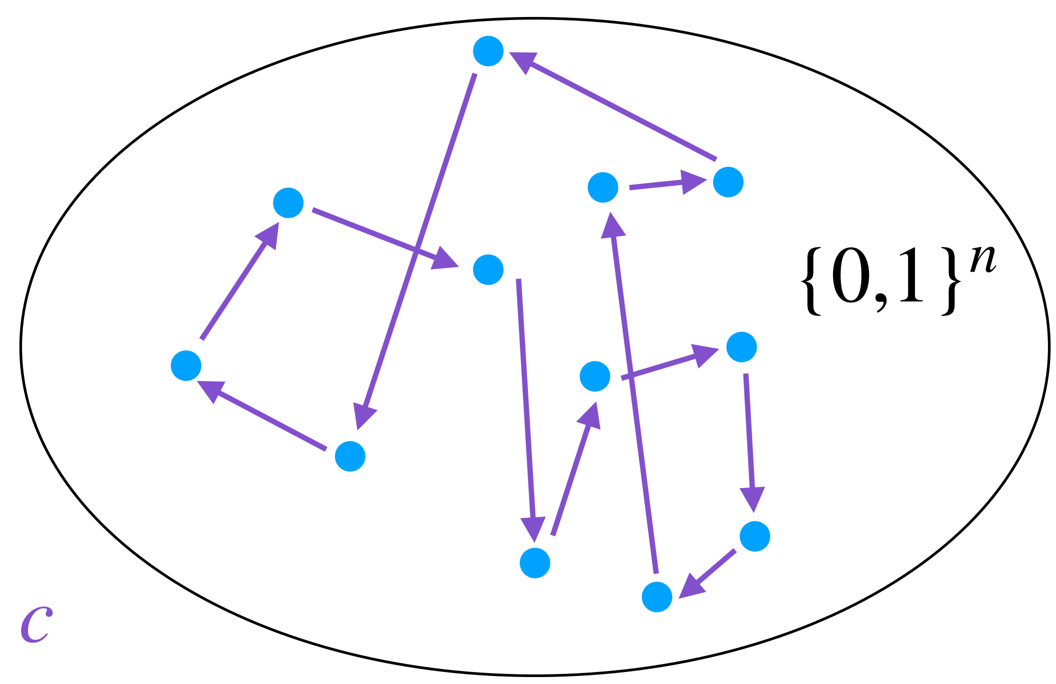


Quantum algorithm

Idea: use classical "checksum algorithm" for a superposition of sets of q_0 inputs

Classical algorithm:

- $\mathscr{A}_0$ computes $z = H(x_0) \oplus H(c(x_0)) \oplus \ldots \oplus H(c^{q_0-1}(x_0))$
- $\mathscr{A}_1$ checks z



Quantum algorithm

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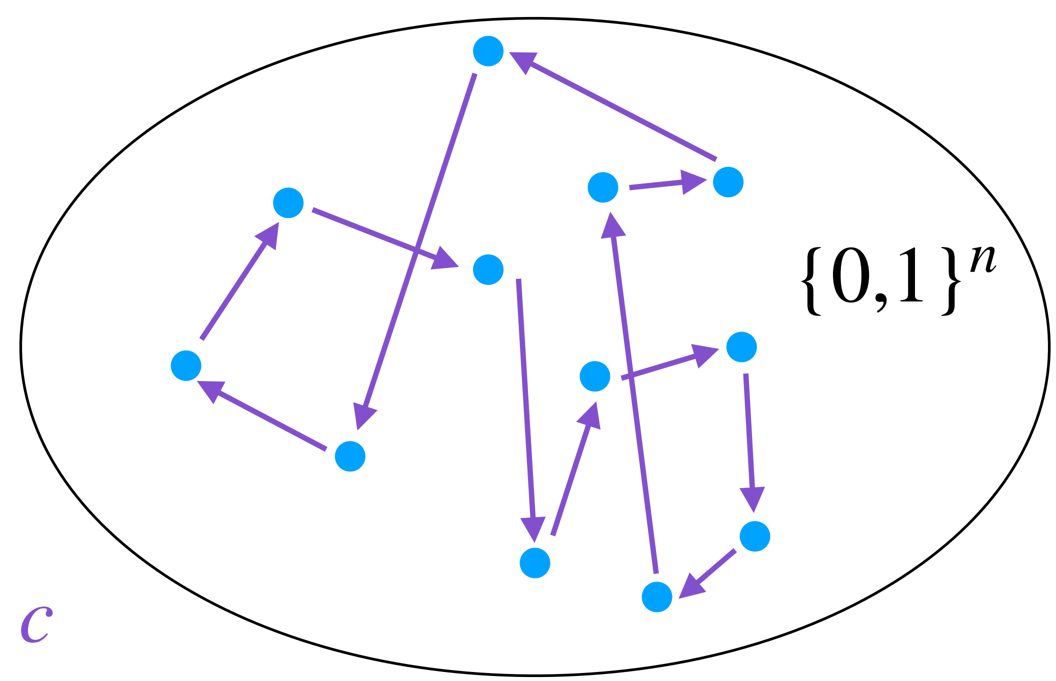
Classical algorithm:

- $\mathscr{A}_0$ computes $z = H(x_0) \oplus H(c(x_0)) \oplus \ldots \oplus H(c^{q_0-1}(x_0))$
- $\mathscr{A}_1$ tries to uncompute z, checks success



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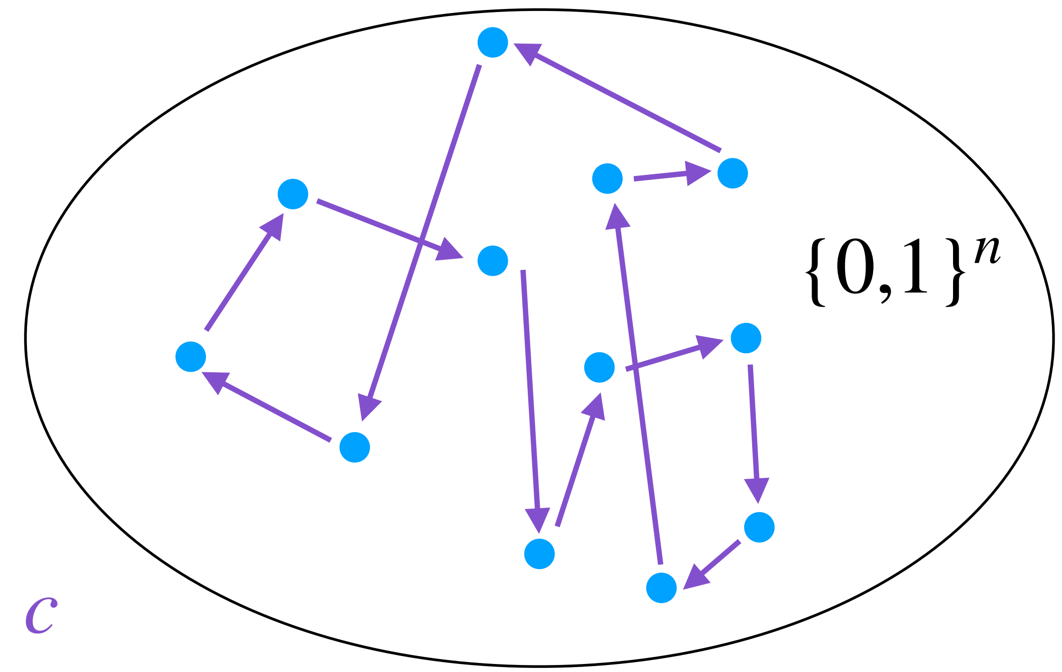

Quantum algorithm

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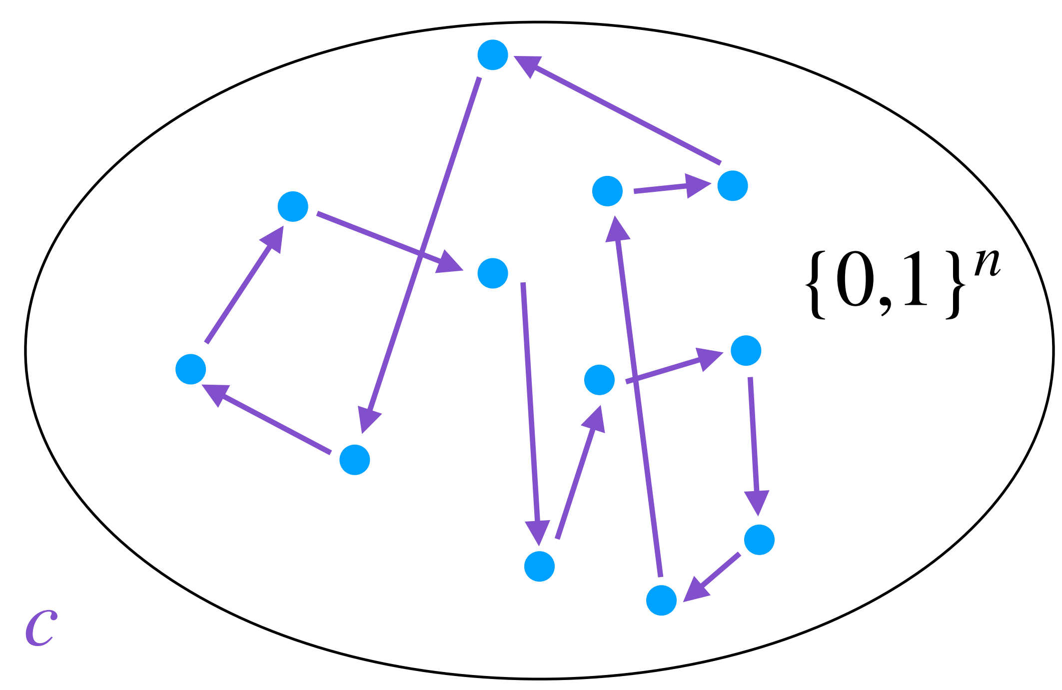
1. $\mathscr{A}_0$ prepares $|\phi_0\rangle = 2^{-\frac{n}{2}} \sum_{x \in \{0,1\}^n} |x\rangle_X |0\rangle_Y$



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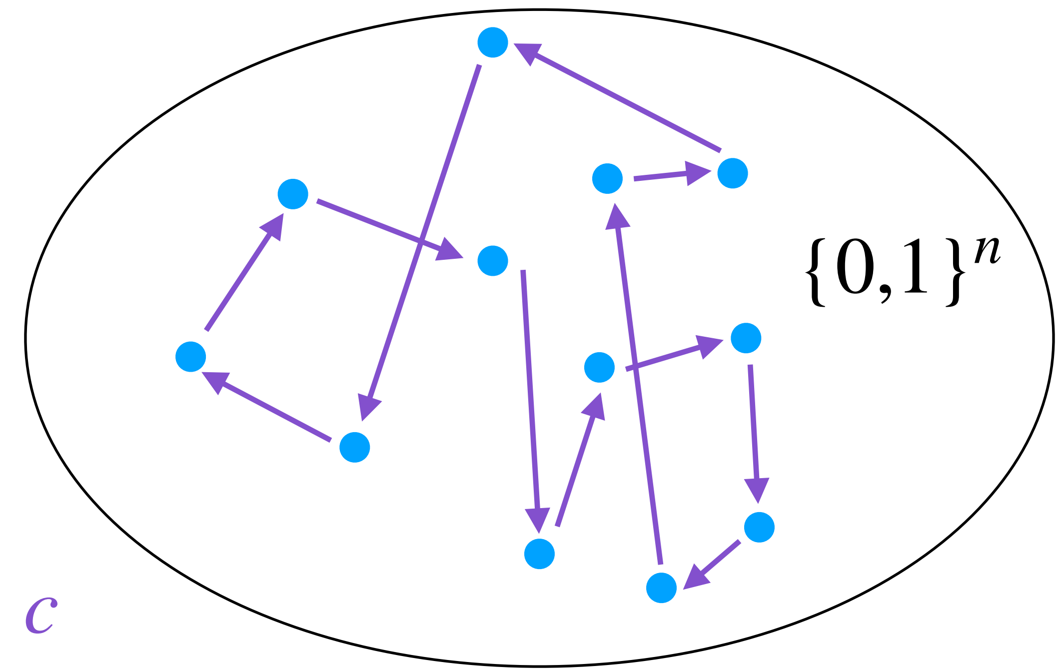

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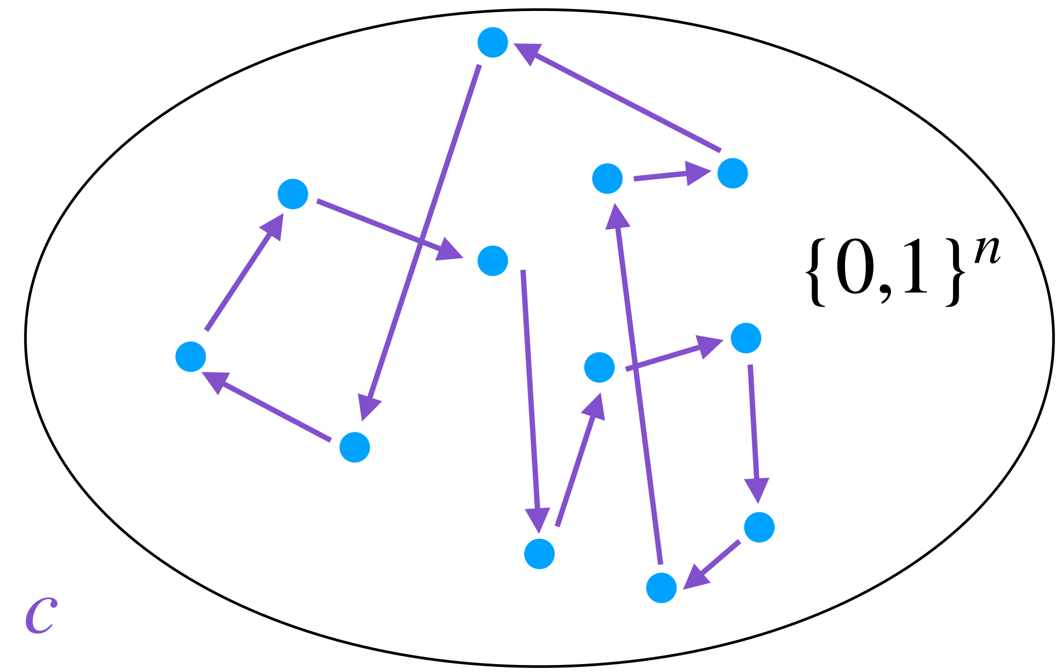
   - query H



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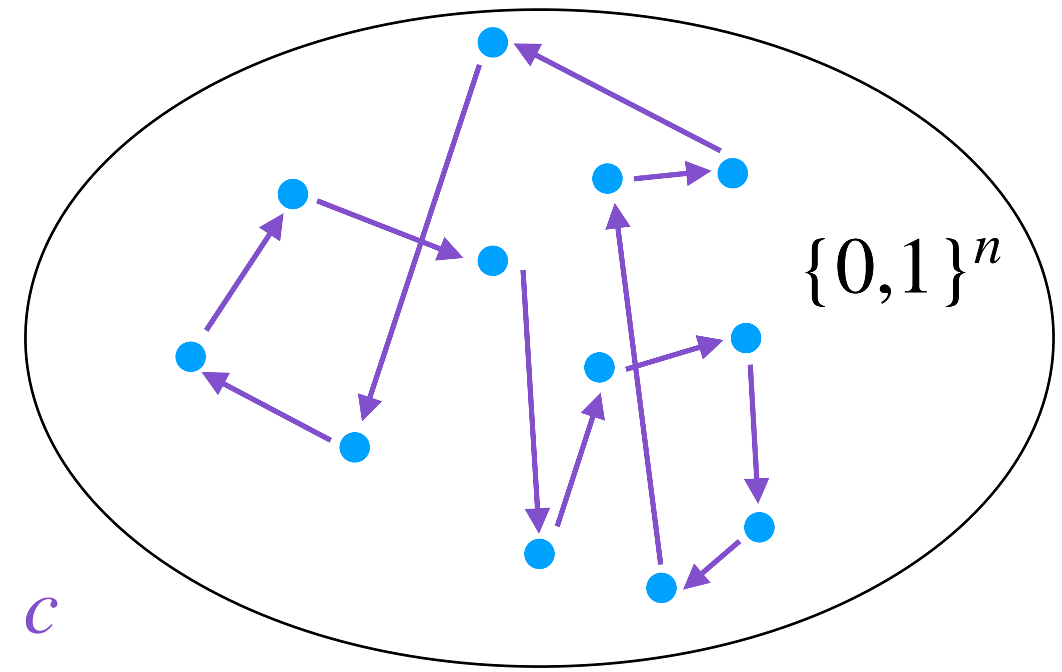

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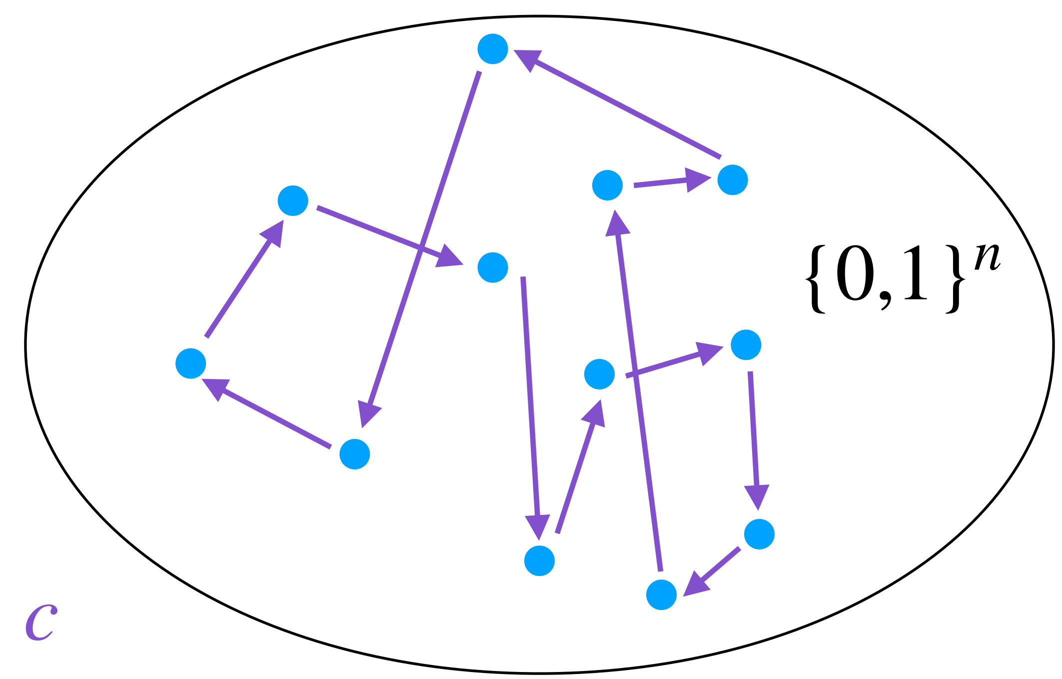
3. $\mathscr{A}_1$ tries to undo 2.



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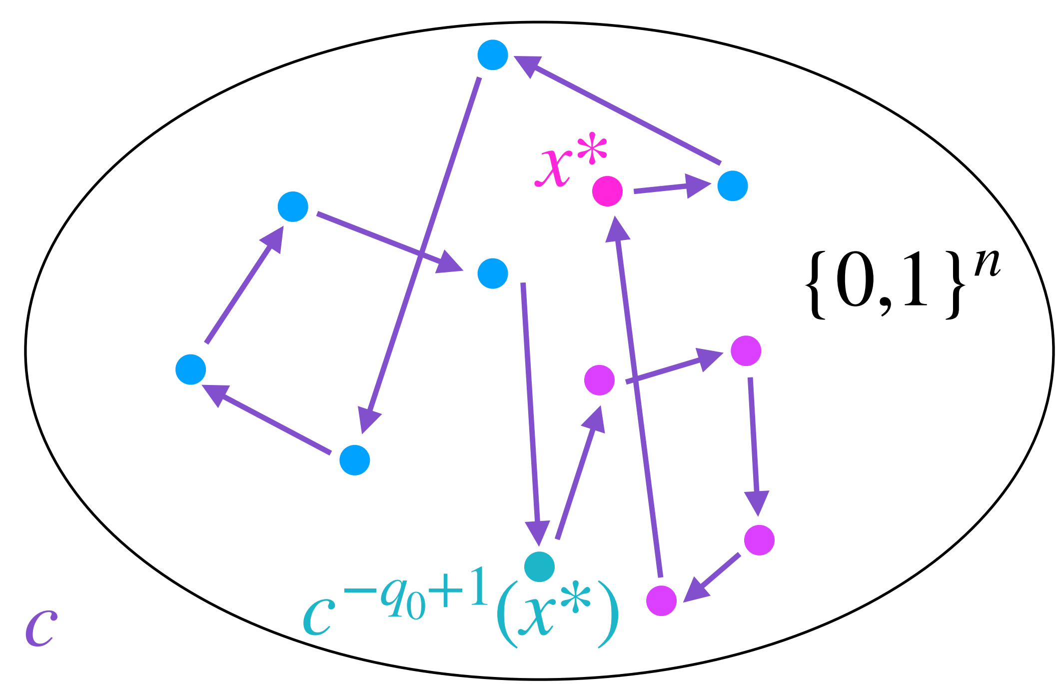

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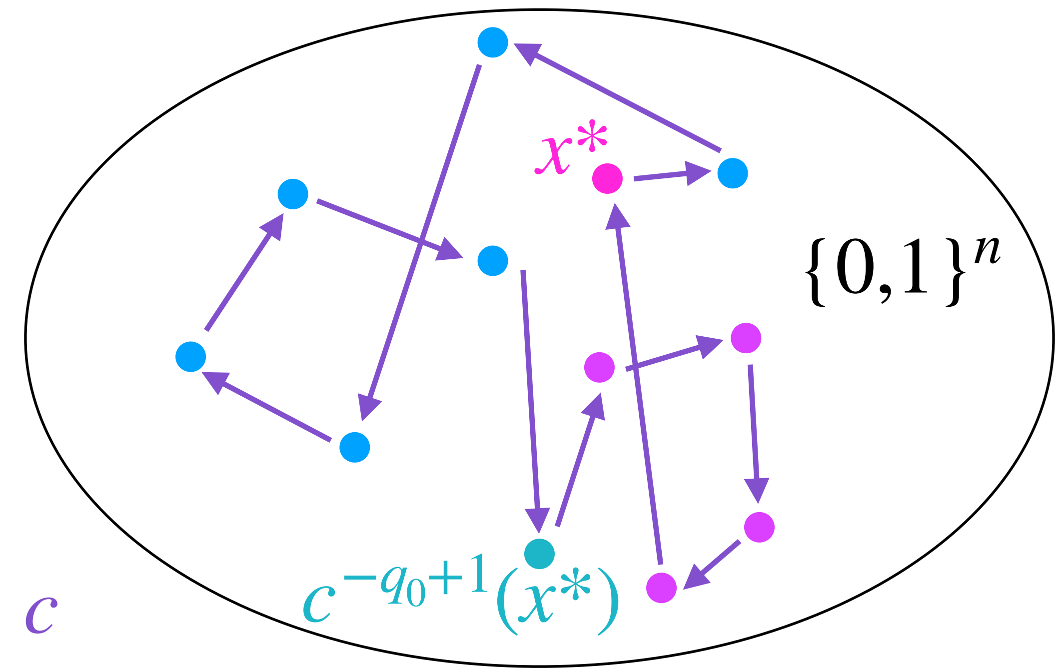

$S = \{ \bullet, \bullet, \bullet, \bullet, \bullet, \bullet \}$

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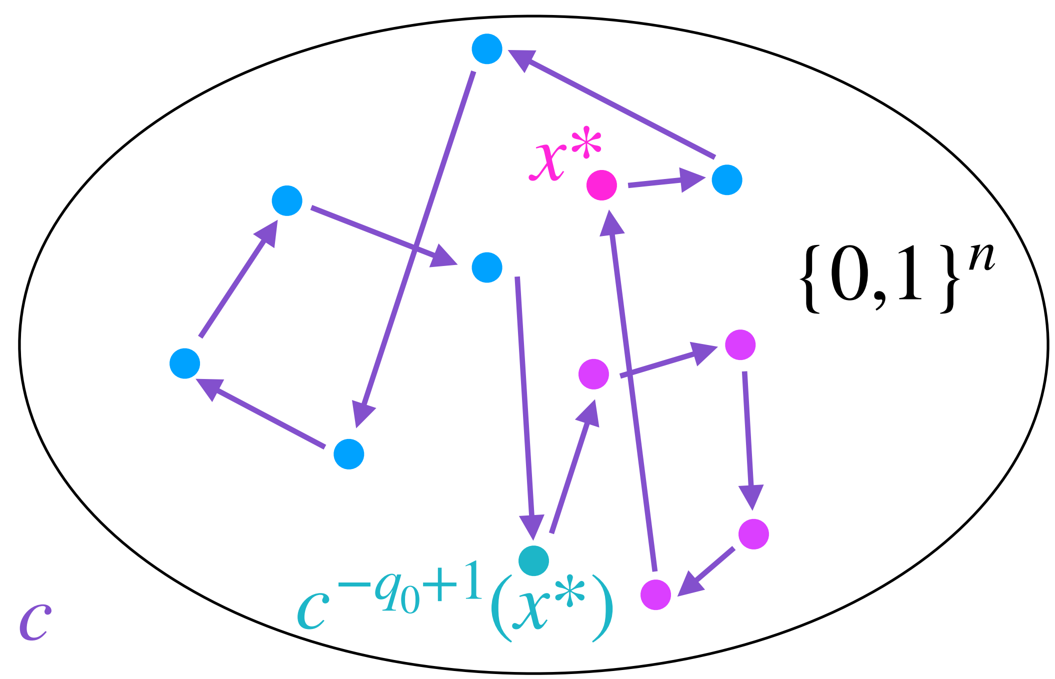

Result: $|\phi_b\rangle$, with

$$|\phi_1 > = 2^{-\frac{n}{2}} \left(\sum_{x\in S} |x\rangle |H(x^*) \oplus y^*\rangle + \sum_{x\notin S} |x\rangle |0\rangle \right)$$

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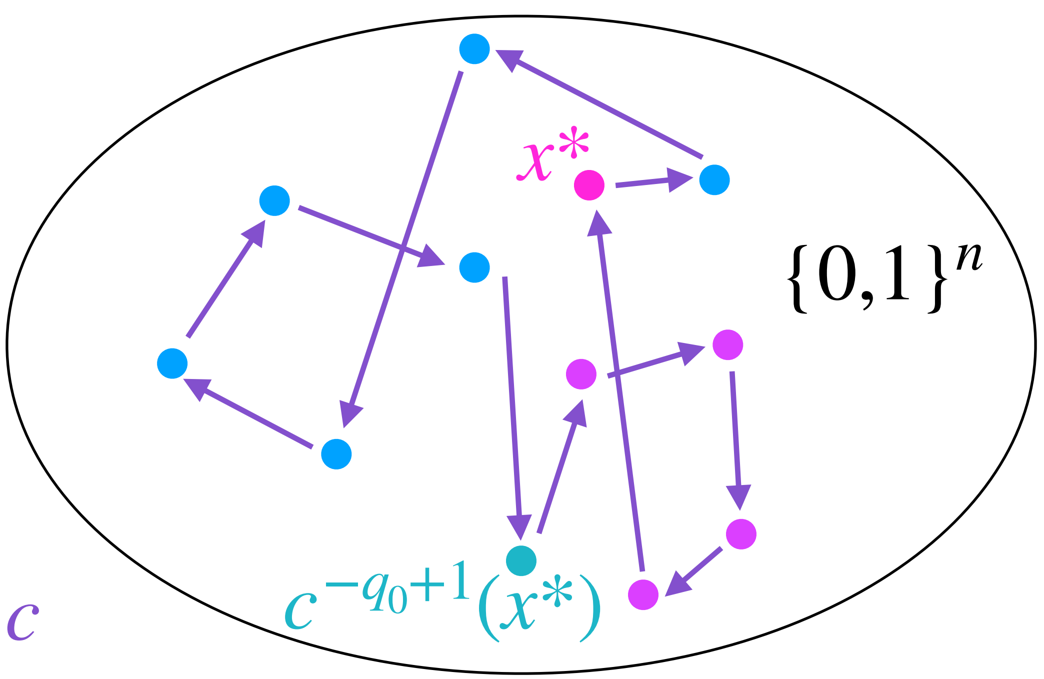

c

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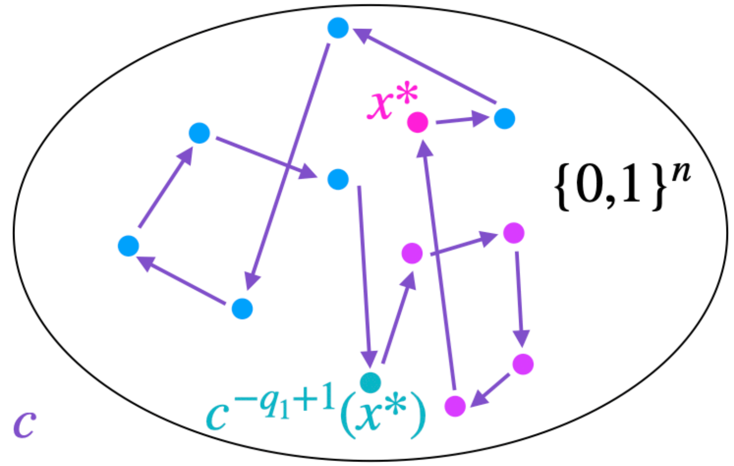
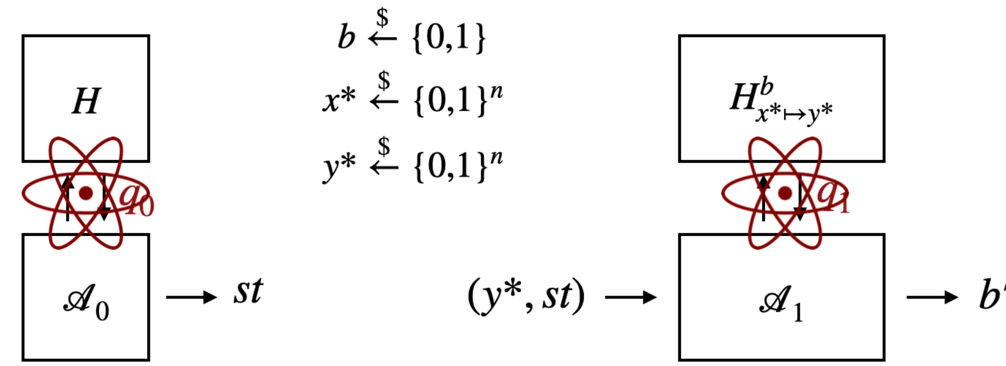
$$\left\| |\phi_0\rangle - |\phi_1\rangle \right\| = \sqrt{2q_0 2^{-n}}$$

$S = \{ \bullet, \bullet, \bullet, \bullet, \bullet, \bullet \}$

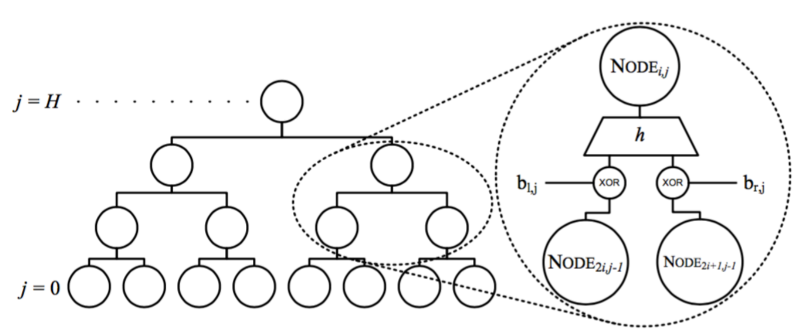
Summary

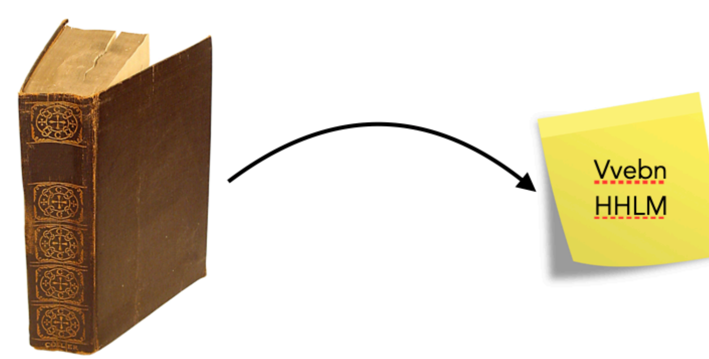
- Tight characterization of "adaptive reprogramming" oracle distinguishing task in the quantum setting
- Informs NIST competition for post-quantum crypto schemes
- Proof based on simplest version of Zhandry's superposition oracle
- Efficient algorithm matching the bound.


x^*
$\{0,1\}^n$
c
$c^{-q_1+1}(x^*)$


H
q_0
$\mathscr{A}_0$
$\to st$
$b \xleftarrow{\$} \{0,1\}$
$x^* \xleftarrow{\$} \{0,1\}^n$
$y^* \xleftarrow{\$} \{0,1\}^n$
$H^b_{x^* \mapsto y^*}$
q_1
$(y^*, st) \to$
$\mathscr{A}_1$
$\to b'$

Thanks!


$j = H$
$j = 0$
NODE$_{i,j}$
h
$b_{l,j}$
XOR
XOR
$b_{r,j}$
NODE$_{2i,j-1}$
NODE$_{2i+1,j-1}$


Vvebn
HHLM