Implementation of arbitrary quantum measurements using classical resources and only single ancilla

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The implementation of quantum operations becomes more noisy as the underlying system size increases. The conventional method to implement a general quantum measurement on a d dimensional system is via Naimark's dilation theorem. Using a generalization of this theorem, it was shown that one requires at least $\log_2 d$ ancillary qubits to implement the most general measurement on a d dimensional system [1]. Not only is this overhead significant from the view point of near-term quantum devices, the implementation is noisy. With this in mind, **we propose a scheme to implement a general quantum measurement using only a single ancillary qubit and classical resources**. The classical resources employed are (see fig 1).

(i) Classical randomisation: given a set of measurements, one can implement a probabilistic mixture of these. This is done over the course of a certain number of trials, by implementing the measurements with frequencies, depending on the desired target probability distribution. In doing so, on average, another measurement gets implemented. We then say that this averaged measurement is implemented by randomisation over the set of measurements, using the relevant probability distribution.

(ii) Post-Processing: upon implementing a given measurement and obtaining measurement outcomes, one can coarse-grain over these outcomes. This is also done over the course of a certain number of trials. In doing so, on average, one implements another measurement. This averaged measurement is said to be obtained from the former measurement via post-processing.

The randomness appearing in randomisation and post-processing has classical origins. This randomness is, thus, a classical resource. In this work, we're interested in reducing quantum resources (in the form of ancillary qubits) to implement a target measurement. Thus classical resources are free, and the aforementioned operations, i.e., randomisation and post-processing are classified as free operations.

Another operation that we employ in our scheme is post-selection. In post-selection, one implements a measurement and samples only from an a priori chosen subset of measurement outcomes, while ignoring the remaining outcomes. Under certain conditions, one can simulate a target measurement by implementing another measurement and post-selecting appropriately, on the latter's outcomes. Unlike classical operations, post-selection has a cost: the target measurement is implemented with a probability of success, which is less than one. Physically, the success probability less than one signifies that one cannot obtain a sample of the target measurement in every trial of the scheme, but only over the course of a certain number of trials. The average number of trials needed obtain a sample of the target measurement is $1/\mathrm{p}_{\mathrm{succ}}$, where we denote the success probability by $\mathrm{p}_{\mathrm{succ}}$.

We next explain our scheme. This scheme works for any general measurement on a d dimensional system. Given the target measurement, we show that one can construct another set of other measurements, each of which can be implemented using only a single ancillary qubit. Additionally, this set of measurements is such that the following sequence of steps implements the target measurement. The four-step scheme is schematically depicted in fig 1. (i), (ii) There is an appropriate choice of randomisation over this set of measurements using which the probabilistic mixture is implemented. (iii) The outcomes of this randomized measurement are post-processed, which results in a measurement with larger number of outcomes. (iv) Then, the outcomes of this measurement can be appropriately post-selected upon, which yields our target measurement with some success probability. The merit of this scheme is, thus, determined by the value of this success probability: the higher, the better. Particularly to understand the experimental feasibility of this scheme, we're interested to know how this success probability scales with the dimension of the underlying quantum system.

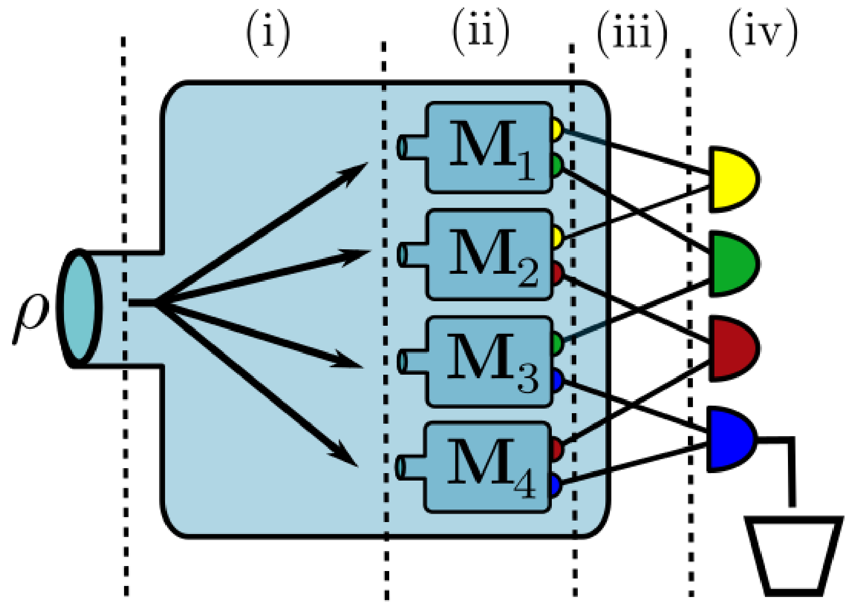

FIG. 1. Schematic depicting the four-step scheme to simulate general measurements using classical resources and post-selection: (i) classical randomisation of measurements, (ii) action of measurements, (iii) post-processing of measurement outcomes, and (iv) post-selection on outcomes. Randomisation and post-processing are classical resources, and thus regarded as free operations.

In an earlier work [2], a similar scheme to implement any measurement with no ancillary qubits, was proposed. The worst-case success probability for this scheme scales as $1/d$. This renders the earlier scheme infeasible for use in large dimension. **Our scheme is a generalisation of this previous scheme. We find that the success probability of our scheme scales significantly better. We expect that, for all general measurements, it is bounded below by a constant, i.e., a lower bound independent of dimension. We obtain strong evidence for this**, which is as follows:

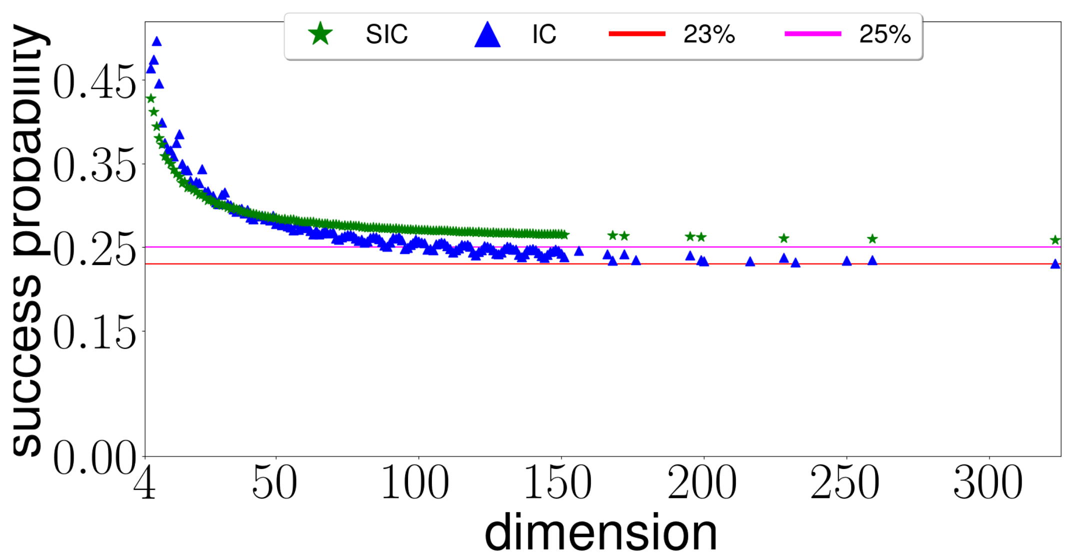

FIG. 2. Plot of success probability for SIC measurements and IC measurements.

(i) Analytic results: to study the success probability for measurement in a d dimensional system, we convert out target POVM into a random variable. To understand how this is done, note that a general POVM can be implemented by (i) attaching our d dimensional system with an ancillary. To implement the most general POVM, this ancilla needs to be at least d dimensional. (ii) Then we rotate this composite system by a global unitary. (iii) Finally, we perform a projective measurement on this composite system. When the global unitary is Haar-random, the target POVM are typical Haar-random POVMs. The maximum number of outcomes of these POVMs is d^2. **Using concentration of measures, we prove, that for large but finite dimension, with overwhelming probability, the success probability is bounded below by a constant of 2.7%**. This contrasts with the worst-case success probability of the earlier scheme, which scales as $1/d$. Separately, using free probability theory, in the asymptotic limit of dimension, we have analytical backing that the lower bound is as high as 25%.

(ii) Numerical evidence: we compute the success probability for symmetric informationally complete measurements (SIC-measurements) [3], and for non-symmetric informationally complete measurements [4] (IC-measurements) for dimensions upto 323. As the dimension increases, **the success probability of SIC-measurements and IC-measurements tend to 25% and 23%** respectively, as shown in the fig 2.

Conjecture: Based on this evidence, **we conjecture that the success probability is bounded from below by a constant for all measurements, independent of dimension**.

Finally, we also investigate the effects of noise. For a chosen noise model, we study how noise compounds in circuits which implement our scheme for a target measurement. This is then compared to the noise compounding in circuits which implement the target measurement using the conventional Naimark method. To implement Haar-random POVMs, we choose our noise to be global depolarising, which is a commonly adopted noise model to study random circuits. In particular, this is the same model used in Google's recent demonstration of quantum computational advantage [5]. We make the following comparison between our scheme and the standard Naimark scheme for implementing typical Haar-random POVMs: the noise compounding in circuits which implement our scheme is significantly lower than for Naimark's.

Context and relevance: From the perspective of implementation in near-term devices, it is imperative to find schemes to reduce the qubit overhead to perform quantum operations. This is not only to reduce the noise, on account of having to control fewer qubits, but also because of geometrical limitations of the device (for instance, inter-connectivity among qubits). Aside from such practical aspects, we believe that our work is also relevant from a theoretical view point for many reasons. Firstly, our work fits squarely into the larger study of simulation of measurements with restricted classes of measurements, a topic which has received some attention off late. There are many questions of interest in this topic, for instance, to understand the relative power of these restricted classes of measurements, whether the gap between these classes diminishes to zero in the asymptotic limit or not, etc. Related to the study of simulability, is the study of resource theory of measurements, to which a lot of attention has been paid in the last few years. We know that the success probability of implementation is related to the robustness of said measurement with respect to corresponding free measurements [6, 7]. A potential question worth pursuing would be to study what other properties (like entanglement cost, visibility, etc) could be related to the success probability as well. Simulability of measurements by other measurements, in itself, also holds significance for other areas of study. For instance, from the perspective of Bell-nonlocality, one would like to know if it maybe possible to violate Bell inequalities with various restricted classes of measurements. Another example is that of randomness generation. Aside from these, our work is significant to those studying General Probabilistic Theories as well, particularly to understand what relevance post-selection has from foundational perspective.

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