Perfect Value 3XOR Games

Based on work by **Adam Bene Watts**, J. William Helton

arXiv:2010.16290.

Nonlocal Games

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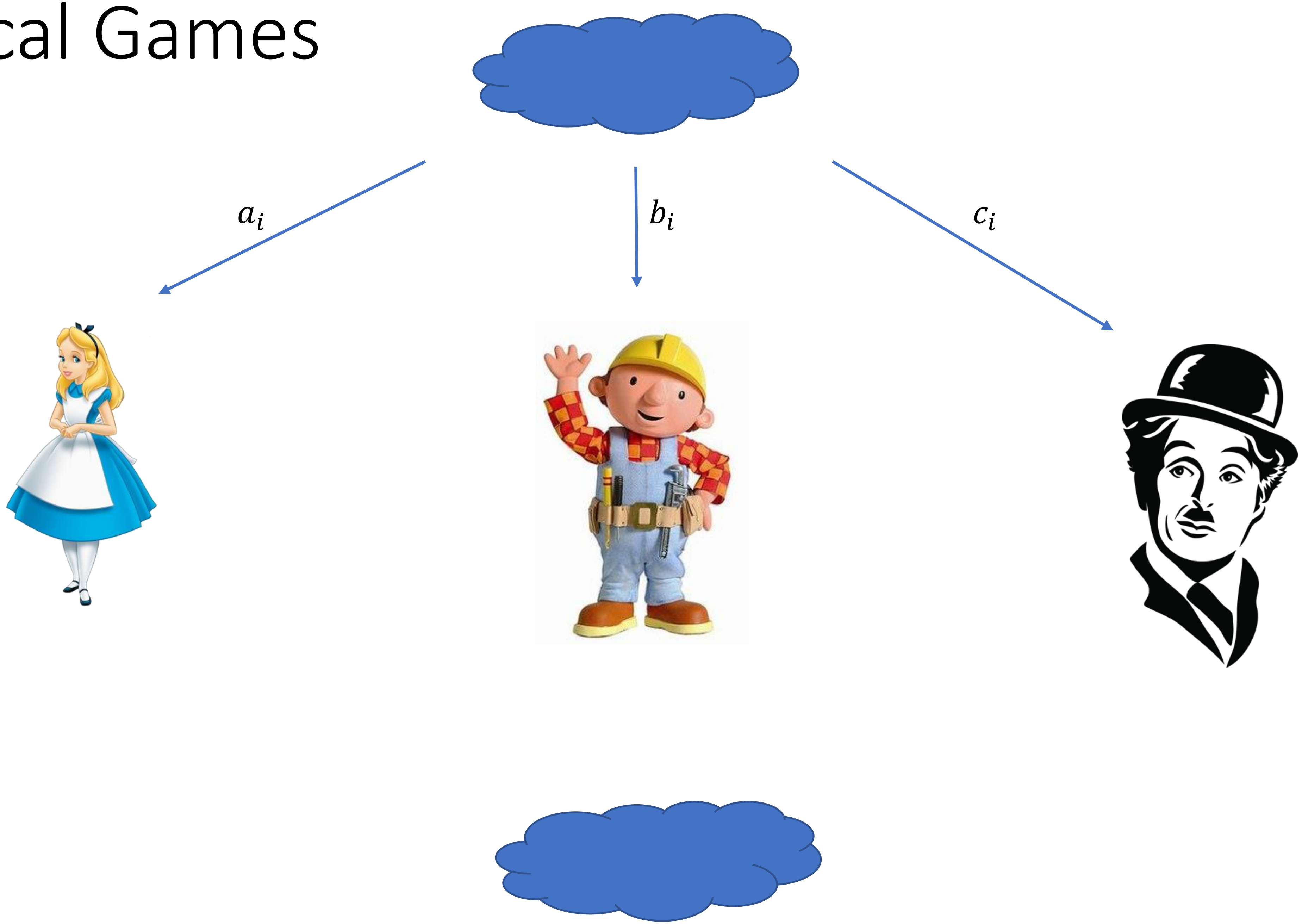

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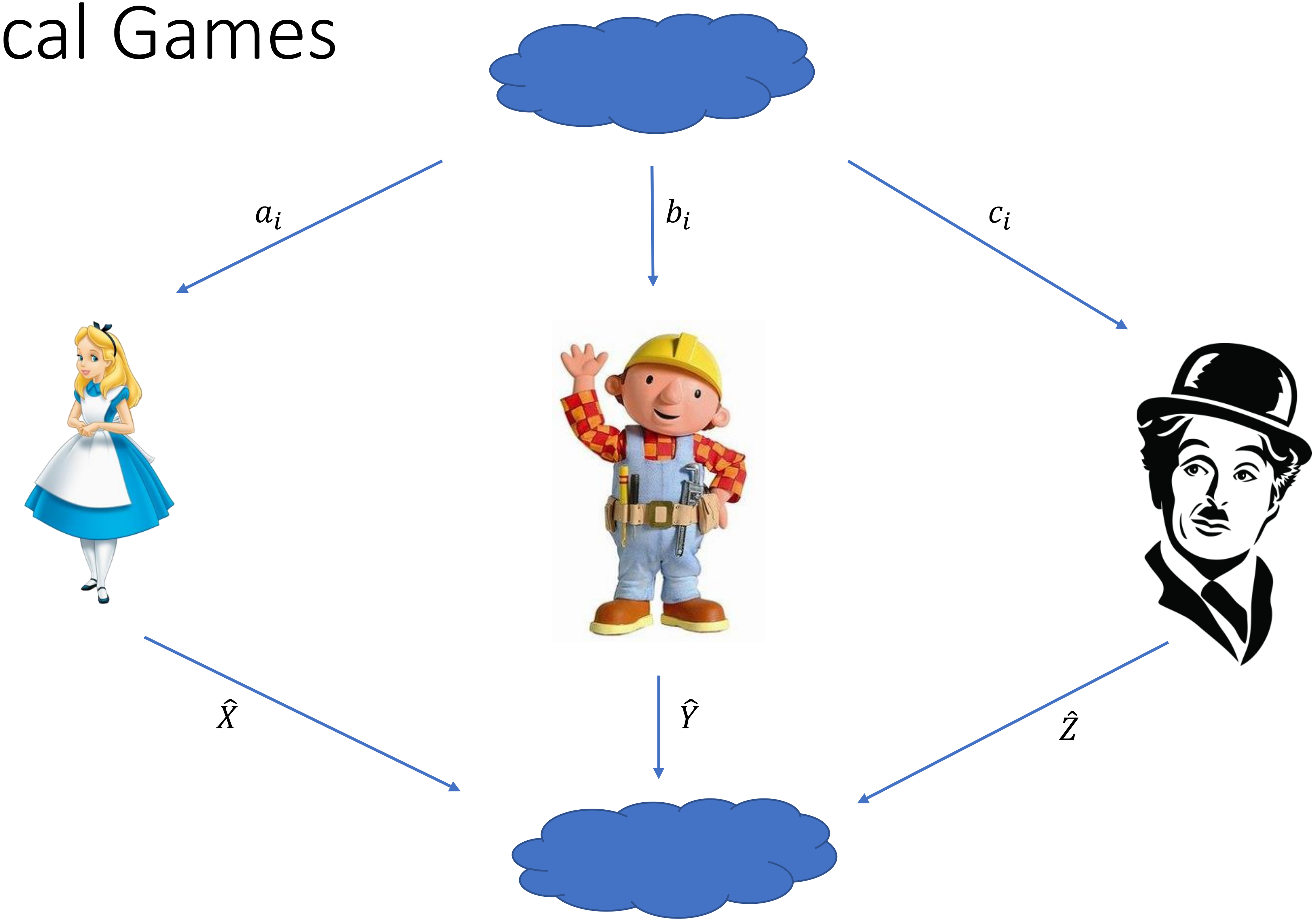

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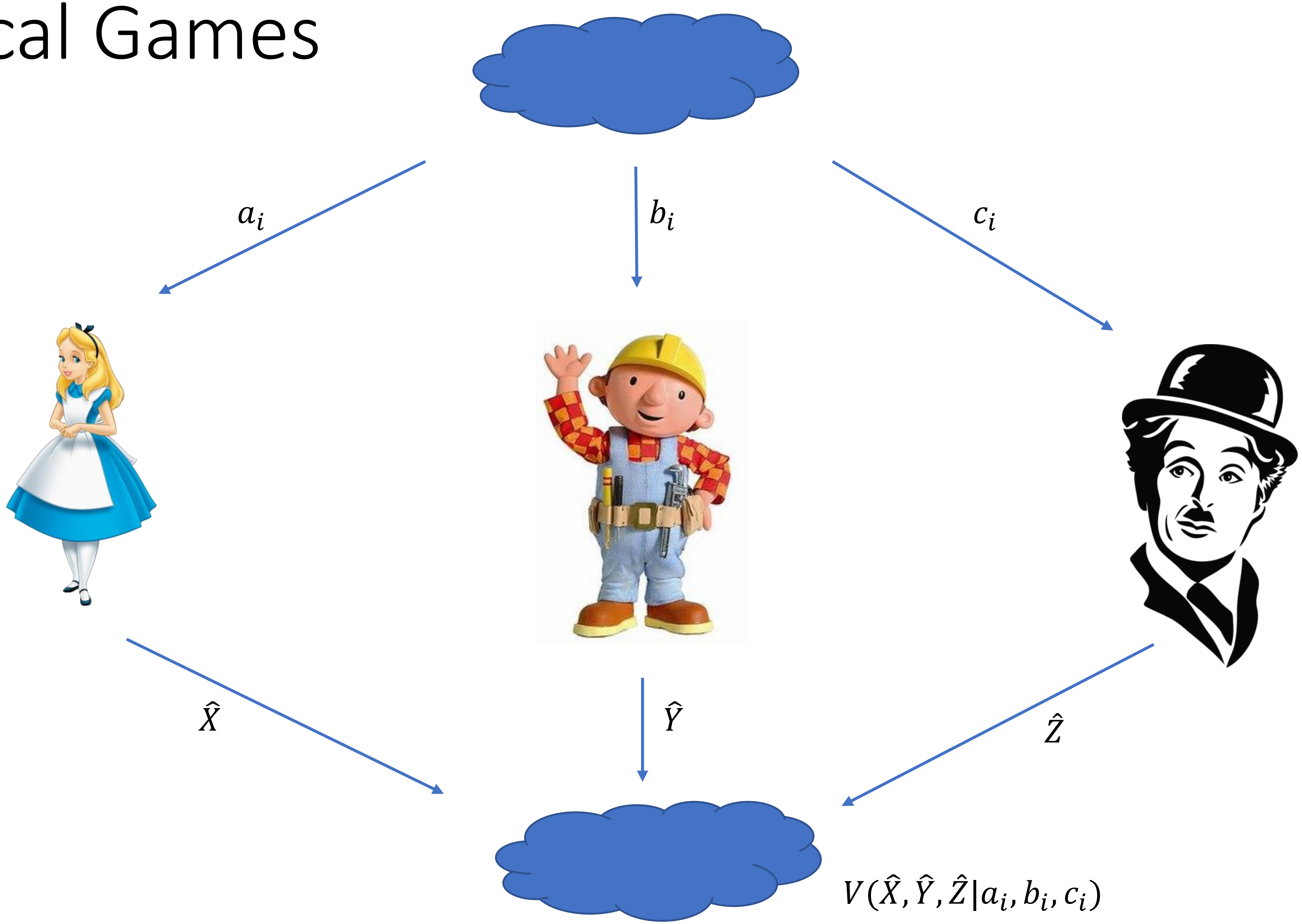
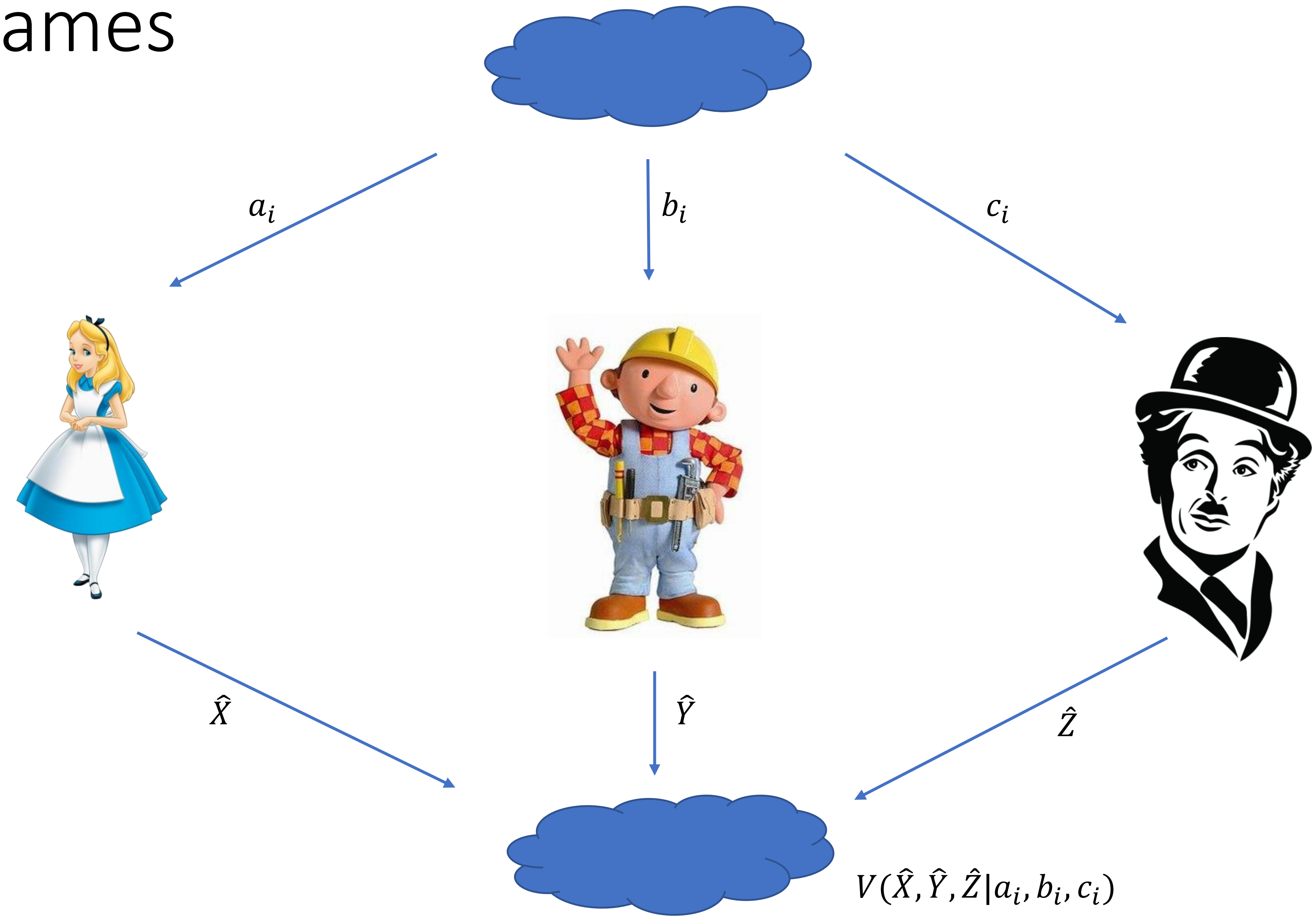


XOR Games



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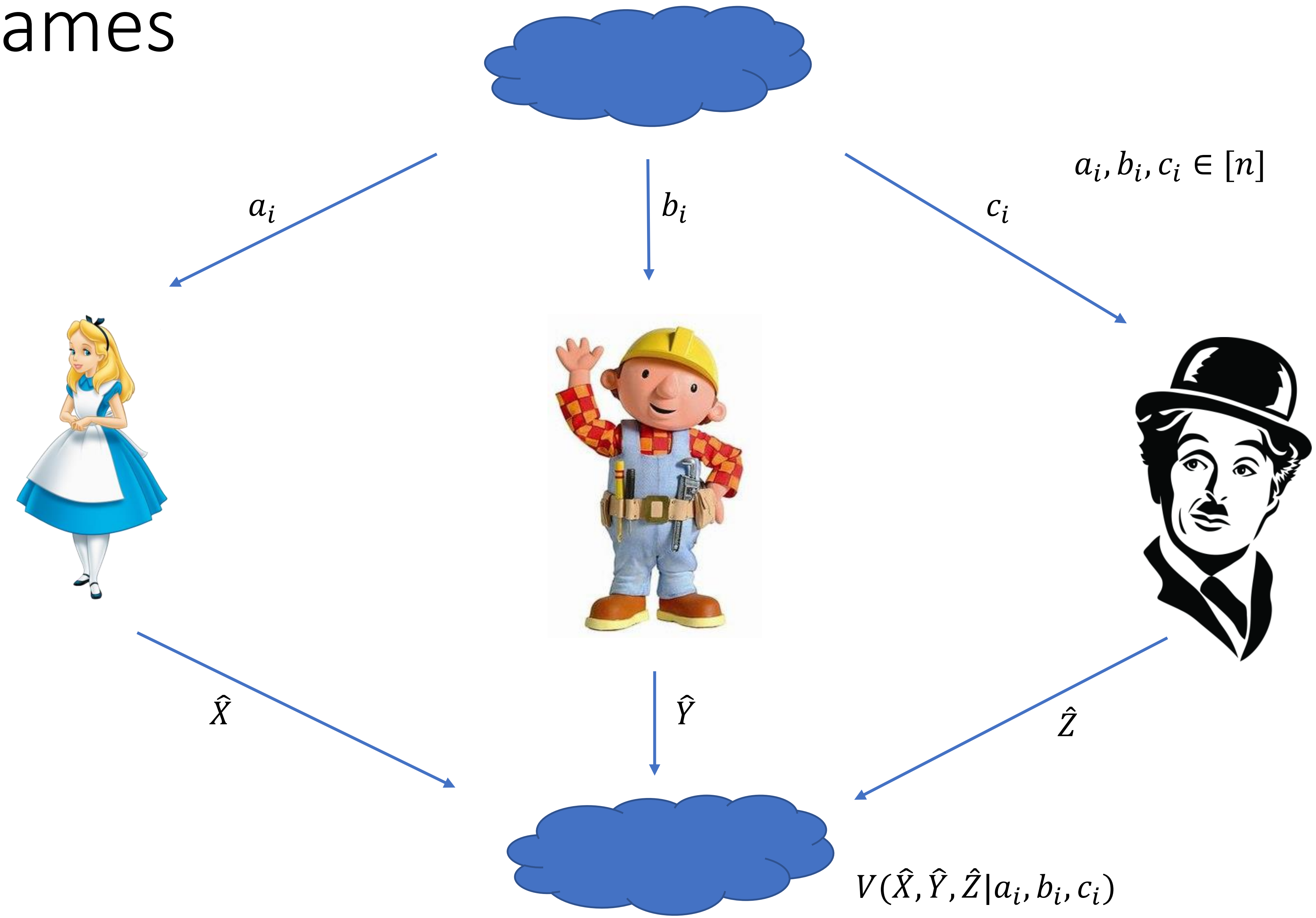

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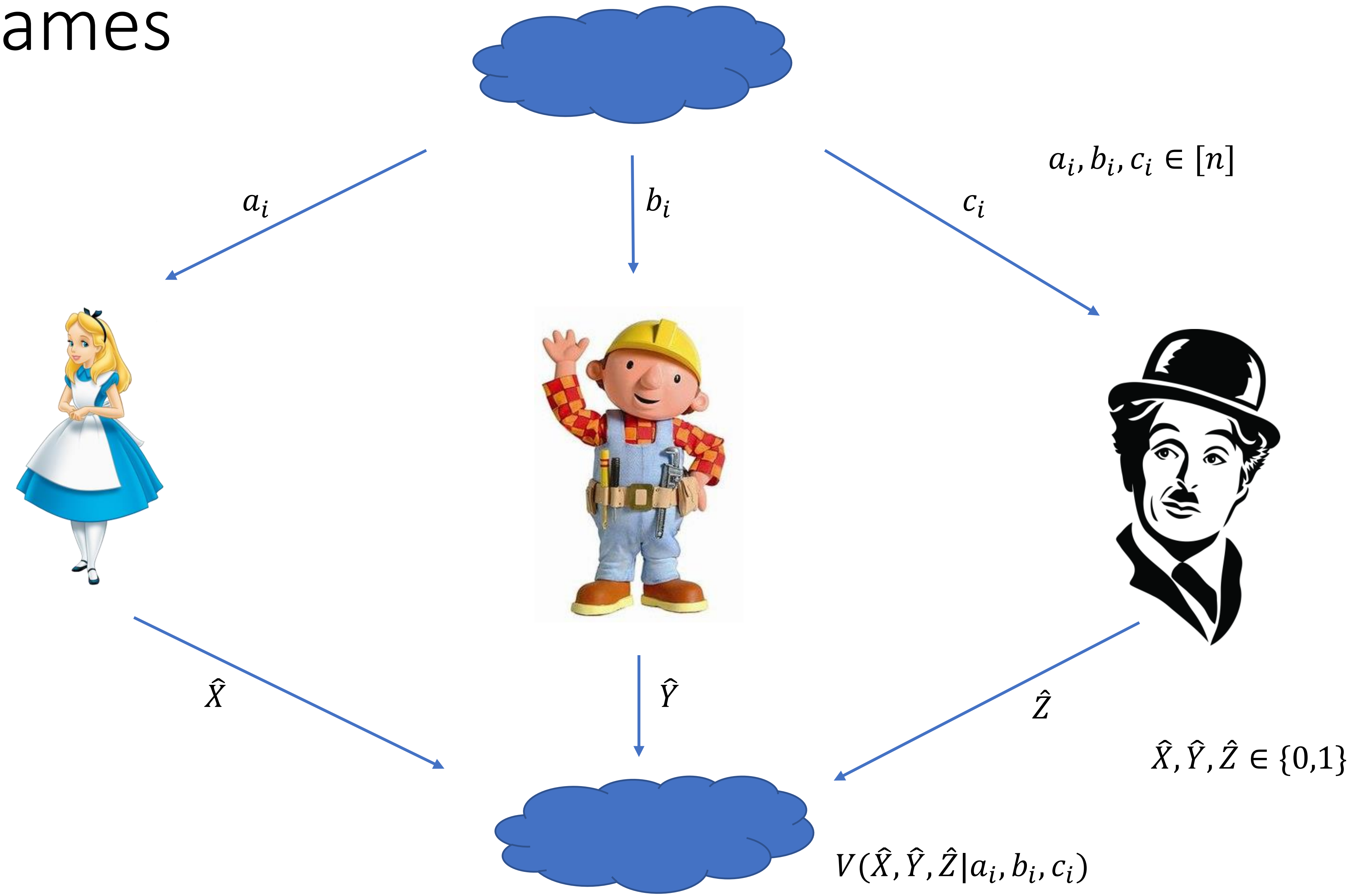

a_i
b_i
c_i
$a_i, b_i, c_i \in [n]$
$\hat{X}$
$\hat{Y}$
$\hat{Z}$
$\hat{X}, \hat{Y}, \hat{Z} \in \{0,1\}$
$V(\hat{X}, \hat{Y}, \hat{Z} | a_i, b_i, c_i)$

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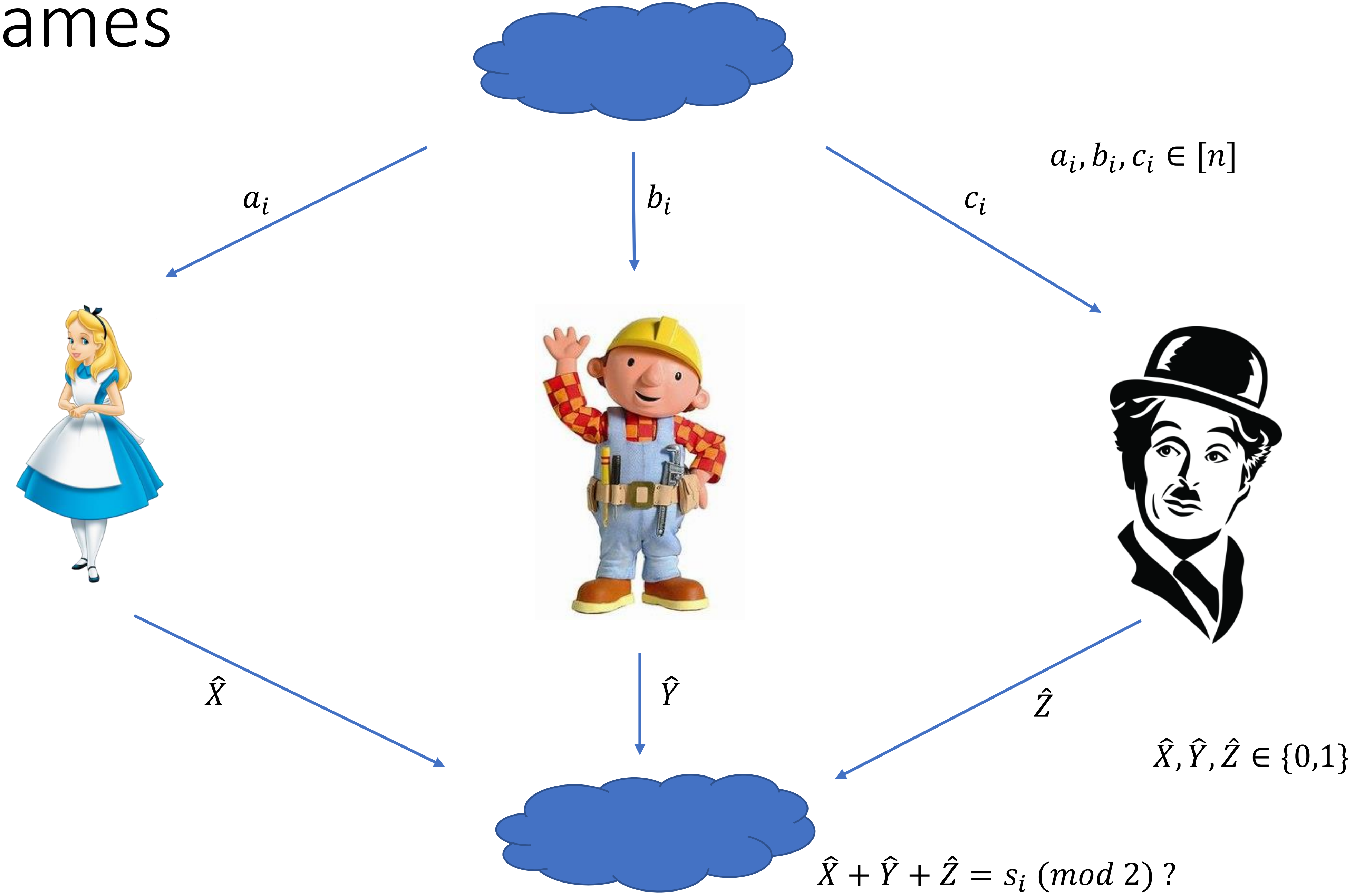
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$\hat{X} + \hat{Y} + \hat{Z} = s_i \ (mod\ 2)$?

XOR Games – Testing a linear system of equations

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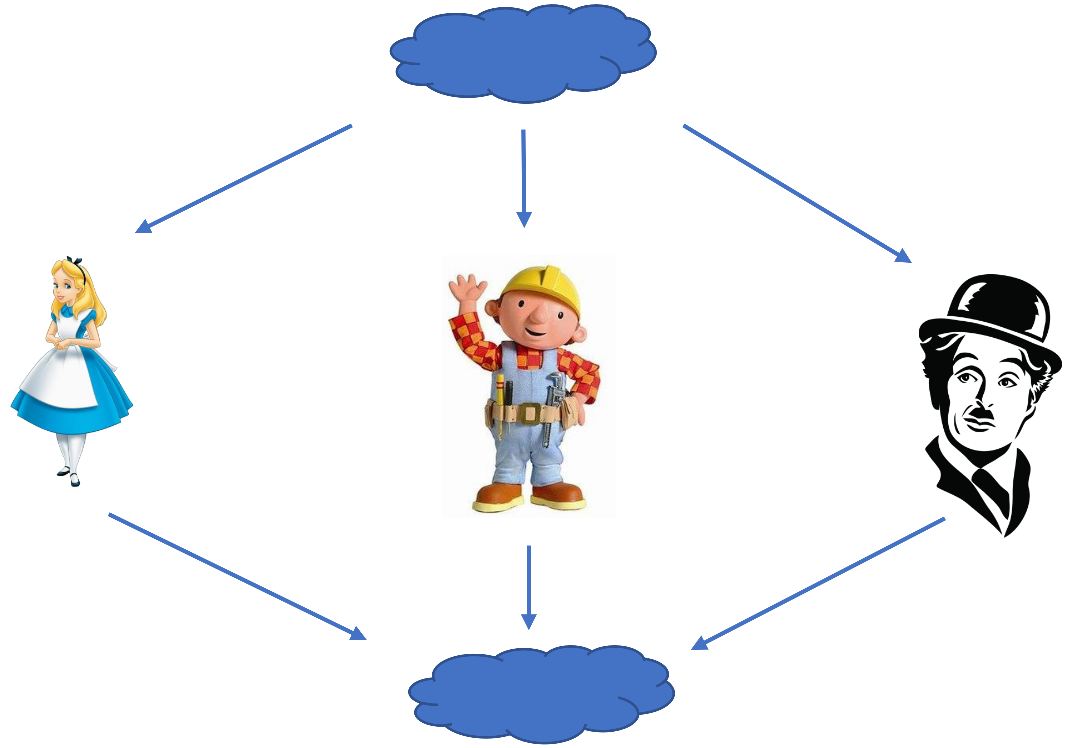

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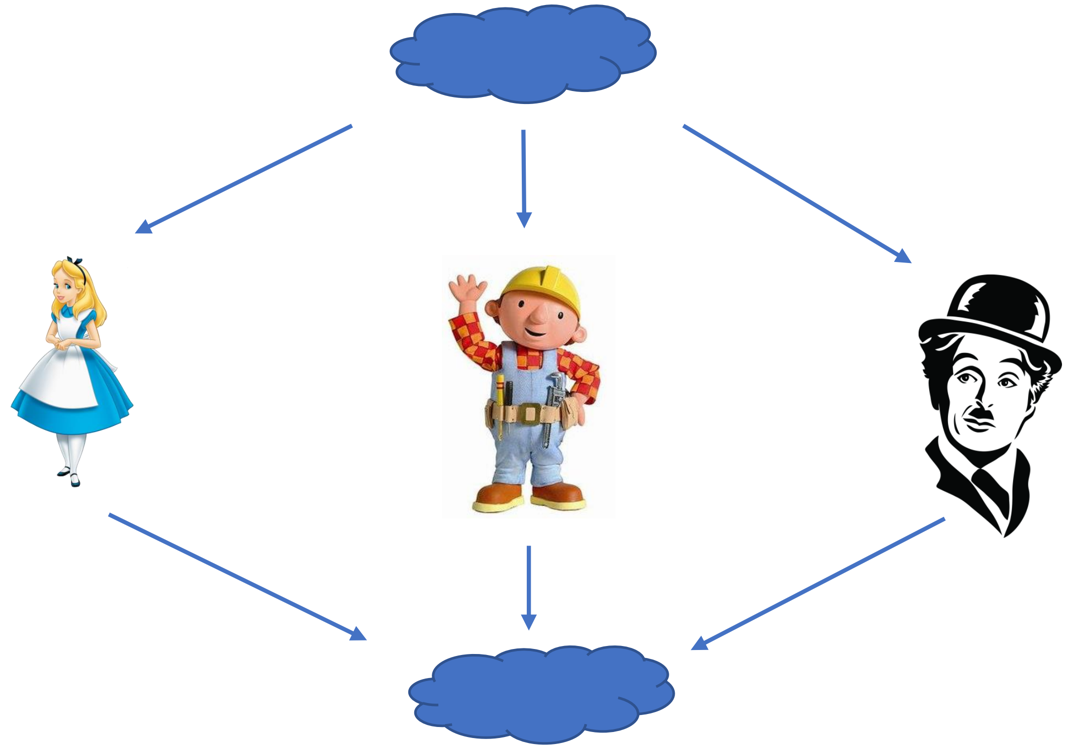
$i \in [m]$



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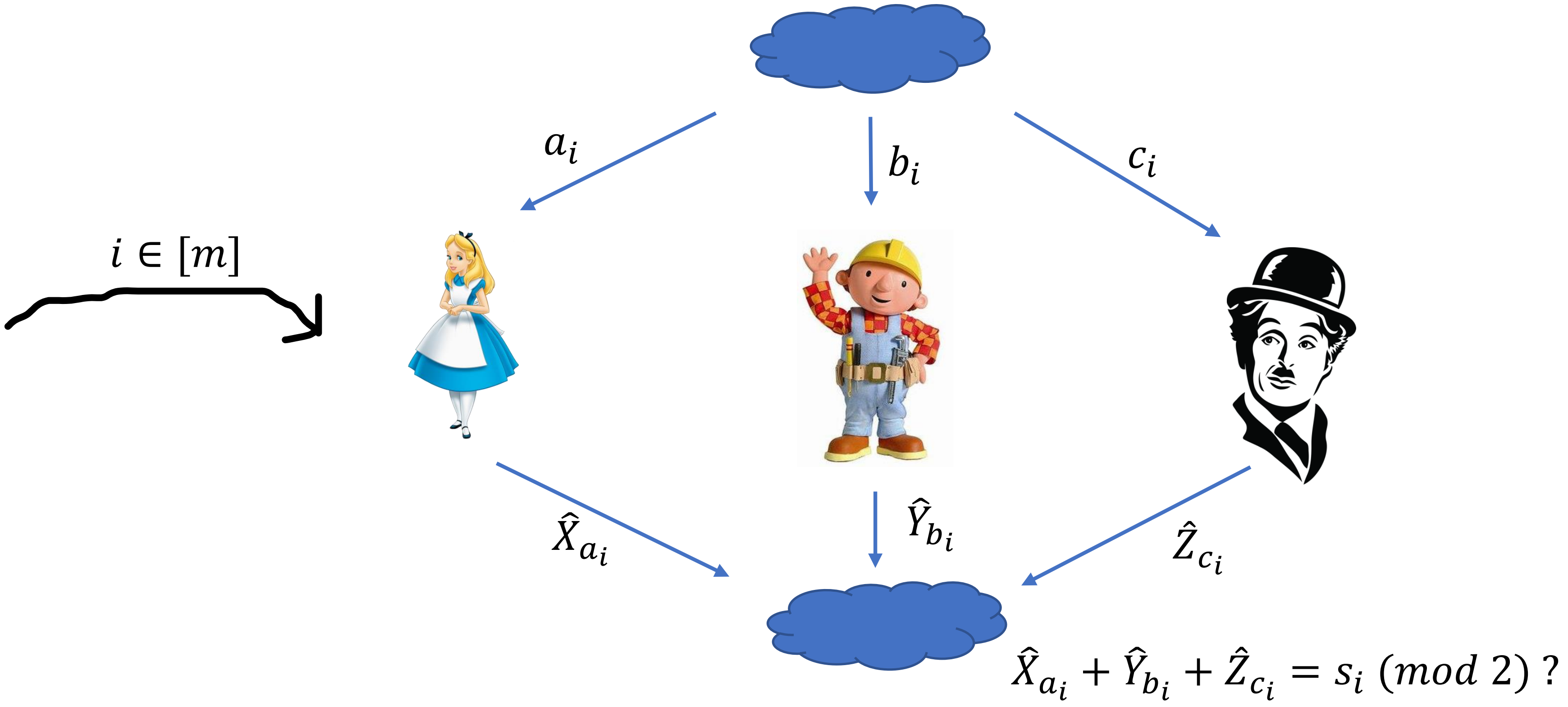

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Classical Value ω (the maximum win probability achievable by classical players) is the maximum fraction of satisfiable clauses in associated system of equations.

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Eigenvalues ± 1 $\quad X_j^2 = Y_j^2 = Z_j^2 = 1.$

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Entangled value ω_{co}^* can be larger than the max fraction of (classically) satisfiable clauses.

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This talk: We can decide if a 3XOR game has perfect commuting operator value ($\omega_{co}^* = 1$) in polynomial time. All 3XOR games with perfect commuting operator value have a perfect strategy where players share a 3 qubit GHZ state.

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- Group elements x_i, y_j, z_k which satisfy the same relations as the strategy observables X_i, Y_j, Z_k. $(x_i^2 = y_j^2 = z_k^2 = 1, \left[x_i, y_j\right] = \left[y_j, z_k\right] = [z_k, x_i] = 1)$

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If the game has $\omega_{co}^* = 1$, words in H correspond to products of operators (and -1) which fix $|\psi\rangle$

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Proof (Sketch):

Only if: $\sigma \in H \;\Rightarrow \omega_{co} < 1.$

Assume for contradiction that $\sigma \in H$ and $\omega_{co} = 1$.

- Since $\sigma \in H$ there exists a sequence of clauses $h_{r_1} h_{r_2} \dots h_{r_t} = \sigma \in H$.
- Since $\omega_{co} = 1$ all clauses in H correspond to products of operators which fix $|\psi\rangle$.

Then $|\psi\rangle = h_{r_1} h_{r_2} \dots h_{r_t} |\psi\rangle = \sigma |\psi\rangle = -|\psi\rangle$

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Proof (Sketch):

If: $\sigma \notin H \Rightarrow \omega_{co} = 1$.

Construct strategy observables X_i, Y_j, Z_k via representations of group elements x_i, y_j, z_k.
Representation is left action of group on (left) cosets of H.
$|\psi\rangle = |H\rangle - |\sigma H\rangle$

Check: $\sigma|\psi\rangle = -|\psi\rangle$, $h|\psi\rangle = |\psi\rangle \; \forall \; h \in H$.

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But H, σ has a lot of structure – we only care about specific instances of the subgroup membership problem.

Key Question: Are these instances decidable?

- For 2 players: YES (Tsirelson)
- For 3 players: YES (coming up)
- For >3 players: Completely open

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- If $[\sigma]_k \notin H\ (mod\ K) \Rightarrow \sigma \notin H$ so there exists a perfect commuting operator strategy ($\omega_{co}^* = 1$) with operators satisfying the additional relations imposed by K.
- If $[\sigma]_k \in H\ (mod\ K)$ there is no perfect commuting operator strategy with operators satisfying the additional relations imposed by K (but we can't, in general, conclude anything about ω_{co}^*).

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For nice choices of K, the group G/K has solvable subgroup membership problem.

High-level overview: Ask if there is a strategy where the strategy observables satisfy some additional constraint(s). Deciding if such a strategy exists can be **easier** than deciding if a general strategy exists.

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Ex 2 (important): $K = \gamma_2(G^E) = \left\langle \left\{ \left[x_i x_{i'}, y_j y_{j'}\right], \left[y_j y_{j'}, z_k z_{k'}\right], \left[z_k z_{k'}, x_i x_{i'}\right] \right\} \right\rangle^{G^E}$

- **Enforces that "Strategy observables commute in pairs."**
- We use this K for the rest of the talk.
- Sneaky detail: K is not a normal subgroup of G. But it is a normal subgroup of G^E and we can switch to thinking about G^E (even length words) and H^E (even length sequences of clauses) without breaking anything.

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If we mod out by K, the subgroup membership problem for G becomes decidable.

Theorem 2.4. *If a kXOR game corresponds to a subgroup H with $[\sigma]_K \notin H^E \pmod{K}$ then the game has $\omega^*_{co} = \omega^*_{tp} = 1$ with a perfect value MERP strategy. A description of this strategy can be found in polynomial time.*

If there is a perfect commuting operator strategy satisfying the K relations, then there is a perfect tensor product strategy where the players share a 3 qubit GHZ state.

Main Theorem 2

Main Theorem 2

For 3XOR Games

Theorem 2.6. *σ is contained in H iff, after modding out by K, the coset containing σ is contained in H^E. That is:*

$$\sigma \in H \iff [\sigma]_K \in H^E \pmod{K}.$$

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A 3XOR Game has a perfect commuting operator strategy iff it has a s

Putting everything together for 3XOR games:

$$\omega_{co}^* = 1 \Leftrightarrow \sigma \in H \Leftrightarrow [\sigma]_K \in H^E \Leftrightarrow 1 \text{ qubit tensor product strategy}$$

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Proof:

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For 3XOR Games

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Proof:

($\Rightarrow$) Clear

($\Leftarrow$) Involved!

Intuition: Why K?

Given a system of equations like:

$$X_1 Y_1 Z_1 |\psi\rangle = X_1 Y_2 Z_2 |\psi\rangle = X_3 Y_3 Z_3 |\psi\rangle = X_4 Y_3 Z_4 |\psi\rangle = |\psi\rangle$$

Combine them to get:

$$X_1 X_1 Y_1 Y_2 Z_1 Z_2 |\psi\rangle = Y_1 Y_2 Z_1 Z_2 |\psi\rangle = |\psi\rangle$$

$$X_3 X_4 Y_3 Y_3 Z_3 Z_4 |\psi\rangle = X_3 X_4 Z_3 Z_4 |\psi\rangle = |\psi\rangle$$

Then combine those to get:

$$(Y_1 Y_2 Z_1 Z_2)(X_3 X_4 Z_3 Z_4)(Y_1 Y_2 Z_1 Z_2)^{-1}(X_3 X_4 Z_3 Z_4)^{-1} |\psi\rangle$$

$$= (Z_1 Z_2)(Z_3 Z_4)(Z_1 Z_2)^{-1}(Z_3 Z_4)^{-1} = |\psi\rangle$$

Intuition: Why K?

$\Rightarrow$ Some elements of K naturally end up as fixing $|\psi\rangle$.

Modding out by K is restricting to a strategy where *all* elements of K fix $|\psi\rangle$.

You can get close to a proof that this works by repeating the previous slides construction to show lots of elements of K fix $|\psi\rangle$.

… but the full proof is a lot more work.

(Some) Open Questions

Can we decide whether $\omega^*_{co} = 1$ for k-XOR games with $k > 3$? Mod p games?

- One possible approach – K modding with general K.

More generally, for what games (resp. what classes of correlations) is it easy to compute ω^*_{co}? What do the strategies optimizing the value of those games look like?

- It is known that we can compute the value of symmetric XOR games, 2 question XOR games, and 3 player XOR games. *In all cases the optimal strategy looks the same.*

Even if we can't compute the value, can we easily compute the restricted value achievable by observables satisfying some relations? Is this restricted value useful?

Thanks!